Homework 8.

8A. 2D Bose Condensate Issues. (from Sturge 11.4).

a) Show that a 2D perfect Bose gas in a box (i.e., confined to a certain area of a plane by a potential which is uniform within that area and infinite outside it) does not undergo Bose-Einstein condensation.

b) Now suppose that the Bose gas is still confined to a plane (i.e., 2D), but within the plane it is confined by a potential of the form $\phi(r)=ar^2$, where r is measured from some point on the plane. This is equivalent to a harmonic oscillator potential. Show that Bose-Einstein condensation is now possible, and find the critical density at which this occurs.

8B. Fountain Effect (from Sturge 11.3)

In a normal/superfluid mixture, the normal fluid of He consists of phonon-like excitations of the superfluid and its thermal properties can be approximated by a Deby models with heat capacity $C_V \sim 0.08\ T^3$ J/mole (see Schroeder problem 7.61). The particle density of liquid He is 2×10^{22} cm^{-3} and the mass is 4 amu.

Estimate the pressure difference Δp in the fountain effect (two containers connected by a membrane permeable only to the superfluid) for a temperature difference $\Delta T = 0.2$ K if the average value of the temperature is 2 K. If the vessels are open to the atmosphere, to approximately what height will the He level in the hotter vessel rise?

Hint for 8A:

Recall from Chapter 2 that the multiplicity of states in $N = 2$ oscillators with $i = n_x + n_y$ quanta shared between them is given as $\Omega(2,i) = \frac{(2+i-1)!}{i!(2-1)!} = \frac{(i+1)!}{i!} = i+1$. The density of states $\Omega(2,i)di = (i+1)di$ can be converted to energy as $g(E)dE = \Omega di$