Using *Mathematica* for linear algebra

**Solving simultaneous equations**

*Mathematica* will solve simultaneous equations for us, e.g.
(Note that one must use `==` in the relations, not `=`)

```mathematica
Solve[{2 x + 5 y - z == 2, x + y + 2 z == 1, x + 5 z == 3}, {x, y, z}]
```

This is how *Mathematica* deals with a case without a solution:
it gives an empty solution vector.

```mathematica
Solve[{x + y == 2, x + y == 5}, {x, y}]
```

Here's what happens if the equations have an infinite number of solutions:

```mathematica
Solve[{x + y == 2, 2 x + 2 y == 4}, {x, y}]
Solve::svars: Equations may not give solutions for all "solve" variables. 
```

We can also solve our original problem using the augmented matrix

```mathematica
eeaug = {{2, 5, -1, 2}, {1, 1, 2, 1}, {1, 0, 5, 3}}; eeaug // MatrixForm
```

We ask *Mathematica* to "row reduce" the augmented matrix (see Boas 3.2) and can read off the answer

```mathematica
RowReduce[eeaug] // MatrixForm
```

Finally, we can solve by writing the problem in matrix form, $E\ r=D$, with $E$ and $D$ as follows.

```mathematica
ee = {{2, 5, -1}, {1, 1, 2}, {1, 0, 5}}; \ fff = {2, 1, 3};
```

Here's the solution:
Vectors & operations

Vectors are entered in row form:

\[ v_1 = \{1, -1, 3\} \]
\[ v_2 = \{3, 1, 2\} \]

Dot product:

\[ \text{Dot}[v_1, v_2] \]
\[ 8 \]

It can also be written as just a Dot:

\[ v_1.v_2 \]
\[ 8 \]

Cross product:

\[ v_{1cr2} = \text{Cross}[v_1, v_2] \]
\[ \{-5, 7, 4\} \]

is antisymmetric, as it should be,

\[ \text{Cross}[v_2, v_1] \]
\[ \{5, -7, -4\} \]

and is orthogonal to \( v_1 \) and \( v_2 \):

\[ \{\text{Dot}[v_{1cr2}, v_1], \text{Dot}[v_{1cr2}, v_2]\} \]
\[ \{0, 0\} \]

You can pick out individual components of a vector as follows:

(Nota the double square parentheses.)

\[ v_1[[2]] \]
\[ -1 \]

Complex vectors

\[ v_3 = \{1 + I, 2, -3 I\}; v_4 = \{6 + I, -3 I, 4\}; \]

To get the complex inner product, need to explicitly put in the complex conjugation:
Conjugate[v3].v4
7 + i

Reversing order gives complex conjugate:
Conjugate[v4].v3
7 - i

The norm or length of v3 is
Sqrt[Conjugate[v3].v3]
$\sqrt{15}$

Mathematica has a built in Norm function (which takes lots of types of argument), but in this case gives the standard norm:
Norm[v3]
$\sqrt{15}$

Cross product works for complex vectors too:
Cross[v3, v4]
\{17, -1 - 22 i, -9 - 5 i\}

Matrices
Functions of Matrices
Eigenvalues and Eigenvectors