Read Chapters 6 through 9 of Srednicki.

1. Consider the Dirac equation for an electron with mass m and charge e moving in the presence of a background electromagnetic potential $A_\mu(x)$. Assume that $|eA_0| \ll mc$ and (for simplicity) that $A_\mu(x)$ is static. Focus attention on stationary solutions to the Dirac equation with energy E close to mc^2 (*i.e.*, solutions where the magnitude of the non-relativistic energy $E_{\rm NR} \equiv E - mc^2$ is small compared to mc^2). For such solutions, show that the Dirac equation reduces to the non-relativistic Schrodinger equation, with the correct magnetic moment for the electron.

2. Let $I_N(g^2) \equiv \int d^N\phi \, e^{-S[\vec{\phi}]/g^2}$ with $\vec{\phi}$ an N-component real vector and $S[\vec{\phi}]$ a real-valued quartic polynomial of the form $S[\vec{\phi}] = \frac{1}{2}\phi_i M_{ij} \phi_j + \frac{1}{4!}\lambda_{ijkl}\,\phi_i\phi_j\phi_k\phi_l$, with $M \equiv \|M_{ij}\|$ symmetric and positive definite (and implied sums over repeated indices). Assume that the coefficients λ_{ijkl} have values which make the quartic term bounded below by zero.

   (a) Let $I_N^{(0)}$ denote the leading saddle-point approximation to $I_N(g^2)$. What is it?

   (b) Evaluate the first three sub-leading corrections in powers of g^2. That is, write $I_N(g^2) \sim I_N^{(0)} \left[1 + \sum_{k=1}^{\infty} c_k \, g^{2k}\right]$ and evaluate c_1, c_2, and c_3. Show how every (distinct) term may be naturally represented by a Feynman diagram in which each line is labeled by two indices (one at either end) and represents the indicated component of the matrix M^{-1}, and each quartic vertex is proportional to the indicated component of λ_{ijkl}. What is the symmetry factor of each diagram?

   (c) You should find both "connected" and "disconnected" diagrams. Show that, through the order evaluated, $\sum_{\text{all diagrams}} = \exp\{\sum_{\text{connected diagrams}}\}$. Can you explain why this is true in general?

   (d) Specialize your results to the case where $S[\vec{\phi}] = \frac{m}{2}\,\vec{\phi}\cdot\vec{\phi} + \frac{1}{4}(\vec{\phi}\cdot\vec{\phi})^2$. What is λ_{ijkl}? What is $\ln[I_N(g^2)/I_N^{(0)}]$ through $O(g^6)$?

   (e) Show, through the order which you have evaluated, that $W \equiv \lim_{N\to\infty} \frac{1}{N} \ln I_N(g^2/N)$ exists and is non-trivial. (Note the rescaling of the coupling: $g^2 \to g^2/N$.) Which diagrams survive in this limit?
   Optional food for thought: Can you identify, and sum, the diagrams (to all orders in g^2) which survive in this $N \to \infty$ limit? Can you evaluate the limit defining W without first doing a diagrammatic expansion in powers of g^2?