Recommended reading: Chapters 45–48 of Srednicki.

1. Yukawa theory (A). Consider a theory (in $D = 4$) of a Dirac fermion ψ and a real scalar field ϕ with $\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}m^2\phi^2 + \bar{\psi}(\not{\partial} + g\,\phi)\psi + (\text{const.})$.

   (a) Show that this theory will be invariant under a transformation taking $\phi \to -\phi$ provided ψ also transforms as $\psi \to A\psi$ for some matrix A. How must $\bar{\psi}$ transform? What is A?

   (b) Which one-loop 1PI diagrams are UV sensitive? What counterterms are needed to control this UV sensitivity? Is this theory renormalizable? Do any additional interactions need to be added to produce a renormalizable theory?

   (c) What are the one-loop renormalization group equations for the dimensionless coupling(s)?

   (d) Now suppose that ϕ is a complex scalar field. What is a renormalizable theory of ϕ and ψ for which there is a U(1) symmetry under which $\phi \to e^{i\alpha}\,\phi$. How must ψ transform under this symmetry?

2. Yukawa theory (B). Consider a theory (in $D = 4$) of a Dirac fermion ψ and a real scalar field ϕ with $\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}m^2\phi^2 + \bar{\psi}(\not{\partial} + g\,\phi)\psi + (\text{const.})$. Assume that m^2 is positive.

   (a) What is the particle content of the theory? Is there a stable fermion? A stable anti-fermion? A stable scalar particle?

   (b) Sketch (qualitatively) what the spectral densities of the $\langle\mathcal{T}(\psi(x)\bar{\psi}(y))\rangle$ and $\langle\mathcal{T}(\phi(x)\phi(y))\rangle$ propagators should look like. Identify any single particle delta-functions and multi-particle thresholds.

   (c) What $2 \to 2$ particle scattering processes are possible? What $2 \to 3$ particle scattering processes are possible? What (if any) $1 \to 2$ decay processes are possible?