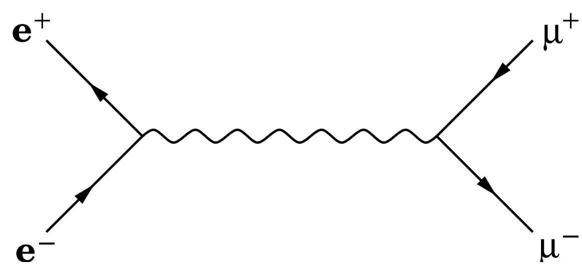
1. The Z^0 resonance. The neutral intermediate vector boson of the weak interactions, known as the Z^0, couples to electrons and muons through a term in the interaction Lagrangian density,

$$\mathcal{L}_{Z^0} = i\, Z^0_\alpha \Big\{ \bar{\psi}_e \gamma^\alpha\, (a + b\gamma_5)\psi_e + \bar{\psi}_\mu \gamma^\alpha\, (a + b\gamma_5)\psi_\mu \Big\}$$

where ψ_e and ψ_μ denote the Dirac fields for the electron and muon, respectively, and Z^0_α is a vector field, analogous to the vector potential for the photon, which creates (or destroys) Z^0 bosons. The (real) constants a and b are determined by the details of the weak interaction theory.

In e^+e^- colliders, at center-of-mass energies near the mass of the Z^0 ($M_Z \equiv 92$ GeV), the reaction $e^+e^- \to \mu^+\mu^-$ is dominated by Z^0 production as described by the Feynman diagram,



Since the Z^0 mass is so much larger than the electron or muon mass, you may treat the leptons as massless in the following.

(a) The propagator for the Z^0 boson may be shown to be

$$D_{\mu\nu}(q) = -i\left(g_{\mu\nu} + \frac{q_\mu q_\nu}{M_Z^2} \right) \Big/ \left(q^2 + M_Z^2 - i\epsilon\right).$$

(*I.e.*, similar to the covariant photon propagator, but with mass terms added.) Write down the expression for the amplitude corresponding to the diagram above.

(b) Show that the $q_\mu q_\nu$ term in the Z^0 propagator does not contribute to the amplitude.

(c) The center of mass angular distribution of the μ^- with respect to the direction of the (unpolarized) electron beam has a forward-backward asymmetry. [In other words, the amplitude for the μ^- to be emitted at an angle θ with respect to the incident e^- direction does not equal the amplitude to be emitted at the angle $(\pi - \theta)$.] Calculate the angular dependence of the (spin-averaged) scattering rate.

(d) Show how the ratio of coupling constants, b/a, may be extracted from the experimentally measured asymmetry.

2. Bhabba Scattering.

(a) Derive the covariant scattering amplitude, to lowest non-trivial order, for elastic electron-positron scattering, $e^+ + e^- \to e^+ + e^-$.

(b) Evaluate the spin-averaged square of the matrix element and derive the differential cross section. Express the various invariant dot products of 4-vectors in terms of the center of mass energy E, the scattering angle θ and the electron rest mass, and evaluate the spin-averaged differential cross section $d\sigma/d\Omega$ (in the c.m. frame). If you wish, you may simplify the algebra by focusing on the ultrarelativistic regime, $E \gg m$.

(c) Describe the angular distribution. How does it compare to the non-relativistic limit? What is the total scattering cross section?