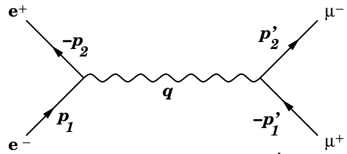
1. The Z^0 resonance.

(a) Label the diagram as: 

so that p_1 is the momentum of the incoming electron, p_2 that of the incoming positron, p_1' the momentum of the outgoing antimuon, and p_2' that of the outgoing muon. (Using p_1' for the outgoing muon, and p_2' for the antimuon might seem more natural, but for no particular reason, I used the above labeling.) The corresponding amplitude is $-i\mathcal{M} = \bar{v}^{(e)}_{s_2}(p_2)\gamma^\alpha(a+b\gamma_5)\,u^{(e)}_{s_1}(p_1)\,D_{\alpha\beta}(q)\,\bar{u}^{(\mu)}_{s_2'}(p_2')\gamma^\beta(a+b\gamma_5)\,v^{(\mu)}_{s_1'}(p_1')$, where $q \equiv p_1+p_2 = p_1'+p_2'$ and $D_{\alpha\beta}(q) \equiv -i(g_{\alpha\beta}+q_\alpha q_\beta/M_Z^2)/(q^2+M_Z^2-i\epsilon)$.

(b) The second part of $D_{\alpha\beta}(q)$ involving $q_\alpha q_\beta$ gives a term involving $\bar{v}^{(e)}_{s_2}(p_2)\not{q}(a+b\gamma_5)\,u^{(e)}_{s_1}(p_1)$ (times a similar second factor involving the muon spinors). Using $q = p_1+p_2$, together with the defining relations for massless free-particle spinors, $\not{p}\,u_s(p) = 0 = \bar{v}_s(p)\,\not{p}$, gives

$$\bar{v}^{(e)}_{s_2}(p_2)\not{q}(a+b\gamma_5)\,u^{(e)}_{s_1}(p_1) = \bar{v}^{(e)}_{s_2}(p_2)\not{p}_2(a+b\gamma_5)\,u^{(e)}_{s_1}(p_1) + \bar{v}^{(e)}_{s_2}(p_2)(a-b\gamma_5)\not{p}_1\,u^{(e)}_{s_1}(p_1) = 0\,,$$

where the anti-commutativity of γ_5 with γ^α was used in the second term. Therefore, the longitudinal $q_\alpha q_\beta$ piece of $D_{\alpha\beta}(q)$ does not contribute.

(c) The differential cross-section is

$$d\sigma = \frac{|\mathcal{M}|^2}{(2E_1)(2E_2)\,v_{\rm rel}}\,(2\pi)^4\delta^4(p_1+p_2-p_1'-p_2')\,\frac{d^3p_1'}{(2E_1')(2\pi)^3}\,\frac{d^3p_2'}{(2E_2')(2\pi)^3}\,.$$

In the center-of-mass frame, $\underset{\sim}{p}_1 = -\underset{\sim}{p}_2 \equiv \underset{\sim}{p}$ and $E_1 = E_2 \equiv E = |\underset{\sim}{p}|$. Similarly, $\underset{\sim}{p}_1' = -\underset{\sim}{p}_2' \equiv \underset{\sim}{p}'$ and $E_1' = E_2' \equiv E' = |\underset{\sim}{p}'|$. And $v_{\rm rel} \equiv |\frac{\underset{\sim}{p}_1}{E_1} - \frac{\underset{\sim}{p}_2}{E_2}| = 2$. Thus $d\sigma = \frac{\pi}{(4EE')^2}\,|\mathcal{M}|^2\,\delta(2E-2E')\,\frac{d^3\underset{\sim}{p}'}{(2\pi)^3}$. Integrating over $|\underset{\sim}{p}'| = E'$ eats the delta function setting $E' = E$ to give $\frac{d\sigma}{d\Omega} = |\mathcal{M}|^2/(16\pi E)^2$. Squaring the amplitude, summing over final spins, and averaging over initial spins gives

$$\begin{aligned}\overline{|\mathcal{M}|^2} &\equiv \frac{1}{4}\sum_{s_1,s_2,s_1',s_2'} |\mathcal{M}|^2 \\ &= \frac{1}{4|M_Z^2-4E^2|^2}\sum_{s_1,s_2,s_1',s_2'} \bar{v}^{(e)}_{s_2}(p_2)\gamma^\alpha(a+b\gamma_5)\,u^{(e)}_{s_1}(p_1)\,\bar{u}^{(\mu)}_{s_2'}(p_2')\gamma_\alpha(a+b\gamma_5)\,v^{(\mu)}_{s_1'}(p_1') \\ &\qquad\times\bar{v}^{(\mu)}_{s_1'}(p_1')\,(a-b\gamma_5)\,\gamma_\beta\,u^{(\mu)}_{s_2'}(p_2')\,\bar{u}^{(e)}_{s_1}(p_1)\,(a-b\gamma_5)\,\gamma^\beta\,v^{(e)}_{s_2}(p_2)\end{aligned}$$

where $\gamma^{\mu\dagger}(i\gamma^0) = -(i\gamma^0)\gamma^\mu$ was used (and the $-i\epsilon$ in the denominator is suppressed). Performing the spin sums using (for massless fermions) $\sum_s u_s(p)\bar{u}_s(p) = \sum_s v_s(p)\bar{v}_s(p) = -i\not{p}$ converts this to a product of two traces,

$$\begin{aligned}4|M_Z^2-4E^2|^2\,\overline{|\mathcal{M}|^2} &\equiv {\rm tr}\,(\gamma^\alpha(a+b\gamma_5)\not{p}_1(a-b\gamma_5)\,\gamma^\beta\not{p}_2)\,{\rm tr}\,(\gamma_\alpha(a+b\gamma_5)\,\not{p}_1'(a-b\gamma_5)\,\gamma_\beta\not{p}_2') \\ &= \Big[(a^2+b^2)\,{\rm tr}\,(\gamma^\alpha\not{p}_1\,\gamma^\beta\not{p}_2) - (2ab)\,{\rm tr}\,(\gamma^\alpha\not{p}_1\,\gamma_5\,\gamma^\beta\not{p}_2)\Big]\times \\ &\quad\big[(a^2+b^2)\,{\rm tr}\,(\gamma_\alpha\,\not{p}_1'\,\gamma_\beta\,\not{p}_2') - (2ab)\,{\rm tr}\,(\gamma_\alpha\,\not{p}_1'\,\gamma_5\,\gamma_\beta\,\not{p}_2')\big]\,.\end{aligned}$$

Now ${\rm tr}\,(\gamma^\alpha\not{p}_1\gamma^\beta\not{p}_2) = 4(p_1^\alpha p_2^\beta + p_1^\beta p_2^\alpha - g^{\alpha\beta}p_1\cdot p_2)$ and ${\rm tr}\,(\gamma^\alpha\not{p}_1\gamma_5\gamma^\beta\not{p}_2) = -4i\epsilon^{\alpha\beta\mu\nu}(p_1)_\mu(p_2)_\nu$

[using tr $(\gamma^5\gamma^\alpha\gamma^\beta\gamma^\mu\gamma^\nu) = 4i\epsilon^{\alpha\beta\mu\nu}$]. Therefore

$$\begin{aligned}
\tfrac{1}{4}|M_Z^2 - 4E^2|^2\,\overline{|\mathcal{M}|^2} \equiv & \Big[(a^2+b^2)\,(p_1^\alpha p_2^\beta + p_1^\beta p_2^\alpha - g^{\alpha\beta}\,p_1\cdot p_2) + (2iab)\,\epsilon^{\alpha\beta\mu\nu}\,p_{1\mu}p_{2\nu}\Big] \times \\
& \Big[(a^2+b^2)\,(p'_{1\alpha}p'_{2\beta} + p'_{1\beta}p'_{2\alpha} - g_{\alpha\beta}\,p'_1\cdot p'_2) + (2iab)\,\epsilon_{\alpha\beta\sigma\lambda}\,p_1'^{\sigma}p_2'^{\lambda}\Big] \\
= & \big\{2(a^2+b^2)^2\,\left[(p_1\cdot p'_1)\,(p_2\cdot p'_2) + (p_1\cdot p'_2)\,(p_2\cdot p'_1)\right] \\
& -\,(2ab)^2\,\epsilon^{\alpha\beta\mu\nu}\,\epsilon_{\alpha\beta\sigma\lambda}\,p_{1\mu}\,p_{2\nu}\,p_1'^{\sigma}\,p_2'^{\lambda}\big\} \\
= & \,2\left\{(a^2+b^2)^2\,\left[(p_1\cdot p'_1)\,(p_2\cdot p'_2) + (p_1\cdot p'_2)\,(p_2\cdot p'_1)\right]\right. \\
& \left. +\,(2ab)^2\,\left[(p_1\cdot p'_1)\,(p_2\cdot p'_2) - (p_1\cdot p'_2)\,(p_2\cdot p'_1)\right]\right\},
\end{aligned}$$

where $\epsilon^{\alpha\beta\mu\nu}\,\epsilon_{\alpha\beta\sigma\lambda} = -2(\delta^\mu_\sigma\delta^\nu_\lambda - \delta^\mu_\lambda\delta^\nu_\sigma)$ was used.

In the center of mass frame, $p_1 = (E, E\,\hat{z})$, $p_2 = (E, -E\,\hat{z})$, $p'_1 = (E', E'\,\hat{n})$, and $p'_2 = (E', -E'\,\hat{n})$, where $\hat{z}$ is the direction of the incoming electron and $\hat{n}$ is the direction of the outgoing muon. Therefore, $p_1\cdot p'_1 = p_2\cdot p'_2 = -(1-\cos\theta)\,EE'$ and $p_1\cdot p'_2 = p_2\cdot p'_1 = -(1+\cos\theta)\,EE'$ where $\cos\theta \equiv \hat{n}\cdot\hat{z}$. Inserting this gives

$$\overline{|\mathcal{M}|^2} \equiv \left|\frac{4EE'}{M_Z^2 - 4E^2 - i\epsilon}\right|^2 \left[(a^2+b^2)^2\,(1+\cos^2\theta) - 8a^2b^2\,\cos\theta\right]$$

or

$$\frac{d\overline{\sigma}}{d\Omega} \equiv \frac{1}{(4\pi)^2}\left|\frac{E}{M_Z^2 - 4E^2 - i\epsilon}\right|^2 \left[(a^2+b^2)^2\,(1+\cos^2\theta) - 8a^2b^2\,\cos\theta\right].$$

The $\cos\theta$ term generates a scattering asymmetry, so that the probability for a μ^- to emerge in some direction $\hat{n}$ is different than the probability for a μ^+ to emerge in the direction $\hat{n}$.

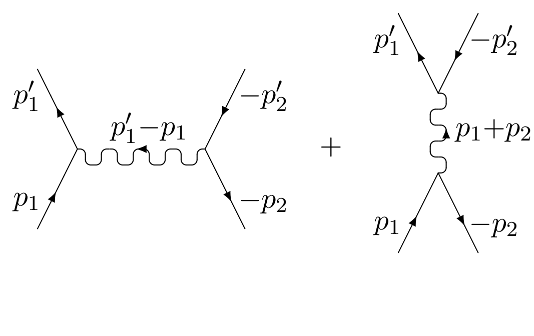
(d) If one defines the asymmetry $\mathcal{A}(\theta) \equiv \left[\frac{d\sigma}{d\Omega}(\theta) - \frac{d\sigma}{d\Omega}(\pi-\theta)\right] \Big/ \left[\frac{d\sigma}{d\Omega}(\theta) + \frac{d\sigma}{d\Omega}(\pi-\theta)\right]$, then this depends on a and b only through their ratio,

$$\mathcal{A}(\theta) = \frac{-8a^2b^2\,\cos\theta}{(a^2+b^2)^2\,(1+\cos^2\theta)} = \frac{-8(b/a)^2\,\cos\theta}{[1+(b/a)^2]^2\,(1+\cos^2\theta)}.$$

Given a measurement of $\mathcal{A}(\theta)$ at any value of θ (other than $\pi/2$), one may convert this relation to a quadratic equation in $(b/a)^2$ whose solution gives $(b/a)^2$ in terms of $\mathcal{A}(\theta)$. Measuring the asymmetry at a variety of angles will allow one to verify that the above functional form fits the data. There is a residual four-fold ambiguity: the asymmetry is independent of the sign of b/a, and is also unchanged if a and b are interchanged.

2. Bhabba Scattering.

(a) Let the incoming electron and positron momenta be p_1 and p_2, respectively, and the outgoing electron and positron momenta p'_1 and p'_2. At lowest order, there are two diagrams:



Note that the positron lines have been labeled so that the arrows follow the charge (*i.e.*, backwards relative to the electron lines), which is why the momenta (flowing in the direction of the arrows) is minus the physical 4-momenta of the positrons. Applying the usual Feynman rules, these diagrams generate the covariant amplitude:

$$
\begin{aligned}
-i\mathscr{M} &= (-e)^2 \left[\bar{u}_{s_1'}(p_1')\gamma^\mu u_{s_1}(p_1)\right] \frac{-ig_{\mu\nu}}{(p_1'-p_1)^2 - i\epsilon} \left[\bar{v}_{s_2}(p_2)\gamma^\nu v_{s_2'}(p_2')\right] \\
&\quad - (-e)^2 \left[\bar{v}_{s_2}(p_2)\gamma^\mu u_{s_1}(p_1)\right] \frac{-ig_{\mu\nu}}{(p_1+p_2)^2 - i\epsilon} \left[\bar{u}_{s_1'}(p_1')\gamma^\nu v_{s_2'}(p_2')\right],
\end{aligned}
$$

where, as always, the covariant amplitude omits the overall momentum conserving delta function, $(2\pi)^4\delta^4(p_1+p_2-p_1'-p_2')$. One tricky point is the relative minus sign between the two contributions. To determine this sign, one needs to consider the specific Wick contractions in the free fermion correlator $\langle 0|\bar{\psi}(2')\psi(1')\,\bar{\psi}(x)\gamma^\mu\psi(x)\,\bar{\psi}(y)\gamma^\nu\psi(y)\,\bar{\psi}(1)\psi(2)|0\rangle$ which lead to contributions represented by these diagrams. An even number of interchanges occur in contractions which give the s-channel diagram (on the right), while an odd number of interchanges occur in contractions which lead to the t-channel diagram (on the left).

(b) Squaring the amplitude, averaging over initial spins and summing over final spins gives

$$
\begin{aligned}
4\,\overline{|\mathscr{M}|^2} &= e^4\,\mathrm{tr}\left[\Lambda_+(p_1')\gamma^\mu\Lambda_+(p_1)\gamma^\nu\right]\mathrm{tr}\left[\Lambda_-(p_2)\gamma_\mu\Lambda_-(p_2')\gamma_\nu\right] \,/\, |(p_1'-p_1)^2 - i\epsilon|^2 \\
&\quad + e^4\,\mathrm{tr}\left[\Lambda_-(p_2)\gamma^\mu\Lambda_+(p_1)\gamma^\nu\right]\mathrm{tr}\left[\Lambda_+(p_1')\gamma_\mu\Lambda_-(p_2')\gamma_\nu\right] \,/\, [(p_1+p_2)^2]^2 \\
&\quad - e^4\,\mathrm{tr}\left[\Lambda_+(p_1')\gamma^\mu\Lambda_+(p_1)\gamma^\nu\Lambda_-(p_2)\gamma_\mu\Lambda_-(p_2')\gamma_\nu\right] \,/\, [(p_1'-p_1)^2 - i\epsilon](p_1+p_2)^2 \\
&\quad - e^4\,\mathrm{tr}\left[\Lambda_+(p_1)\gamma^\mu\Lambda_+(p_1')\gamma^\nu\Lambda_-(p_2')\gamma_\mu\Lambda_-(p_2)\gamma^\nu\right] \,/\, [(p_1'-p_1)^2 + i\epsilon](p_1+p_2)^2,
\end{aligned}
$$

with $\Lambda_+(p) \equiv -i\not{p} + m$ and $\Lambda_-(p) \equiv i\not{p} + m$.
Evaluating the traces gives

$$
\begin{aligned}
\mathrm{tr}\left[\Lambda_+(p_1')\gamma^\mu\Lambda_+(p_1)\gamma^\nu\right] &= 4\left[(p_1'\cdot p_1 + m^2)g^{\mu\nu} - {p_1'}^\mu p_1^\nu - {p_1'}^\nu p_1^\mu\right] \\
\mathrm{tr}\left[\Lambda_-(p_2)\gamma_\mu\Lambda_-(p_2')\gamma_\nu\right] &= 4\left[(p_2\cdot p_2' + m^2)g_{\mu\nu} - p_{2\mu}p_{2\nu}' - p_{2\nu}p_{2\mu}'\right] \\
\mathrm{tr}\left[\Lambda_-(p_2)\gamma^\mu\Lambda_+(p_1)\gamma^\nu\right] &= 4\left[(-p_2\cdot p_1 + m^2)g^{\mu\nu} + {p_2}^\mu p_1^\nu + {p_2}^\nu p_1^\mu\right] \\
\mathrm{tr}\left[\Lambda_+(p_1')\gamma_\mu\Lambda_-(p_2')\gamma_\nu\right] &= 4\left[(-p_1'\cdot p_2' + m^2)g_{\mu\nu} + p_{1\mu}'p_{2\nu}' + p_{1\nu}'p_{2\mu}'\right]
\end{aligned}
$$

and

$$
\begin{aligned}
&\mathrm{tr}\left[\Lambda_+(p_1')\gamma^\mu\Lambda_+(p_1)\gamma^\nu\Lambda_-(p_2)\gamma_\mu\Lambda_-(p_2')\gamma_\nu\right] \\
&\quad = 16\left[-2(p_1\cdot p_2')(p_1'\cdot p_2) + m^2(p_2\cdot p_1 - p_1'\cdot p_1 + p_2\cdot p_1' + p_2'\cdot p_1' + p_2'\cdot p_1 - p_2'\cdot p_2) - 2m^4\right] \\
&\quad = \mathrm{tr}\left[\Lambda_+(p_1)\gamma^\mu\Lambda_+(p_1')\gamma^\nu\Lambda_-(p_2')\gamma_\mu\Lambda_-(p_2)\gamma^\nu\right].
\end{aligned}
$$

To see that these two traces are the same, note that they differ by interchanging primed and unprimed momenta, but that the result is symmetric under this interchange. Or alternatively, note that these two traces must be complex conjugates of each other, but the answer is real. This trace evaluation is easiest if you first use the identities $\gamma^\mu\not{a}\gamma_\mu = -2\not{a}$, $\gamma^\mu\not{a}\not{b}\gamma_\mu = 4a\cdot b$, and $\gamma^\mu\not{a}\not{b}\not{c}\gamma_\mu = -2\not{c}\not{b}\not{a}$, to get rid of the contracted γ^μ's and γ^ν's.
When inserting these traces into the expression for $|\mathscr{M}|^2$, one may simplify the results by using overall momentum conservation, $p_1+p_2 = p_1'+p_2'$, plus the fact that all 4-momentum are on-shell, so that $p_1^2 = p_2^2 = {p_1'}^2 = {p_2'}^2 = -m_e^2$. By squaring the momentum conservation equation (or squaring it after moving terms from one side to the other) this implies that

$$
p_1\cdot p_2 = p_1'\cdot p_2', \qquad p_1\cdot p_2' = p_1'\cdot p_2, \qquad p_1\cdot p_1' = p_2\cdot p_2'.
$$

One can also ignore the $i\epsilon$'s in the denominators, since the momentum transfer $p_1' - p_1$ cannot be null if both p_1 and p_1' are on-shell (and $p_1 \neq p_1'$, so that some scattering occurs). A little plugging and chugging produces

$$\overline{|\mathscr{M}|^2} = 8\,e^4 \left(\frac{A}{[(p_1'-p_1)^2]^2} + \frac{B}{[(p_1+p_2)^2]^2} + \frac{C}{(p_1'-p_1)^2(p_1+p_2)^2} \right),$$

with

$$\begin{aligned} A &= (p_1\cdot p_2)^2 + (p_1\cdot p_2')^2 + 2m^2\, p_1\cdot p_1' + 2m^4\,, \\ B &= (p_1\cdot p_2')^2 + (p_1\cdot p_1')^2 - 2m^2\, p_1\cdot p_2 + 2m^4\,, \\ C &= 2(p_1\cdot p_2')^2 - 2m^2\,(p_1\cdot p_2 + p_1\cdot p_2' - p_1\cdot p_1') + 2m^4\,, \end{aligned}$$

The differential cross section is

$$d\sigma = \overline{|\mathscr{M}|^2}\,\frac{(2\pi)^4\delta^4(p_1'+p_2'-p_1-p_2)}{16\,v_{\rm rel}\,E_1E_2E_1'E_2'}\,\frac{d^3p_1'}{(2\pi)^3}\,\frac{d^3p_2'}{(2\pi)^3}\,.$$

In the c.m. frame, $E_1 = E_2 = E_1' = E_2'$ and $v_{\rm rel} = 2|\underset{\sim}{p}_1|/E_1$. Therefore,

$$d\sigma = \overline{|\mathscr{M}|^2}\,\frac{(2\pi)\delta(2E_1'-2E_1)}{32\,|\underset{\sim}{p}_1|\,E_1^3}\,|\underset{\sim}{p}_1'|^2dp_1'\,\frac{d\Omega}{(2\pi)^3} = \frac{\overline{|\mathscr{M}|^2}}{16\,E_1^2}\,\frac{d\Omega}{(4\pi)^2}\,,$$

using $\int dp_1'\,\delta(E_1'-E_1) = (\partial E_1'/\partial p_1)^{-1} = E_1/|\underset{\sim}{p}_1|$.
In the c.m. frame, we have

$$\begin{aligned} p_1\cdot p_2 &= -E_1^2 - |\underset{\sim}{p}_1|^2 = -E_1^2(1+\beta^2)\,, \\ p_1\cdot p_1' &= -E_1^2 + |\underset{\sim}{p}_1|^2\cos\theta = -E_1^2(1-\beta^2\cos\theta)\,, \\ p_1\cdot p_2' &= -E_1^2 - |\underset{\sim}{p}_1|^2\cos\theta = -E_1^2(1+\beta^2\cos\theta)\,, \\ (p_1+p_2)^2 &= -4E_1^2\,, \\ (p_1-p_1')^2 &= 2E_1^2\beta^2(1-\cos\theta)\,, \end{aligned}$$

where $\beta \equiv \frac{|p_1|}{E_1} = \sqrt{1-\frac{m^2}{E_1^2}}$. Putting everything together yields

$$\left.\frac{d\sigma^{e^+e^-}}{d\Omega}\right|_{\rm c.m.} = \frac{\alpha^2}{8E^2}\left\{ \frac{E^4(1+\beta^2)^2 + E^4(1+\beta^2\cos\theta)^2 - 2E^2m^2(1-\beta^2\cos\theta) + 2m^4}{E^4\beta^4(1-\cos\theta)^2} \right.$$
$$+ \frac{E^4(1+\beta^4\cos^2\theta) + E^2m^2(1+\beta^2) + m^4}{2E^4}$$
$$\left. - \frac{E^4(1+\beta^2\cos\theta)^2 + E^2m^2[1+\beta^2(1+2\cos\theta)] + m^4}{E^4\beta^2(1-\cos\theta)} \right\},$$

were $E \equiv E_1$ and (as always) $\alpha \equiv e^2/(4\pi)$. Judiciously using $|\underset{\sim}{p}| = \beta E$ and half angle trig formulas allows one to write this a little nicer as

$$\left.\frac{d\sigma^{e^+e^-}}{d\Omega}\right|_{\rm c.m.} = \frac{\alpha^2}{16E^2}\left\{ \frac{2|\underset{\sim}{p}|^4(1+\cos^4\frac{\theta}{2}) + 4m^2|\underset{\sim}{p}|^2\cos^2\frac{\theta}{2} + m^4}{|\underset{\sim}{p}|^4\sin^4\frac{\theta}{2}} \right.$$
$$+ \frac{|\underset{\sim}{p}|^4(1+\cos^2\theta) + 4m^2|\underset{\sim}{p}|^2 + 3m^4}{E^4}$$
$$\left. - \frac{4|\underset{\sim}{p}|^4\cos^4\frac{\theta}{2} + 8m^2|\underset{\sim}{p}|^2\cos^2\frac{\theta}{2} + 3m^4}{|\underset{\sim}{p}|^2E^2\sin^2\frac{\theta}{2}} \right\}.$$

In the ultra-relativistic limit, this becomes

$$\left.\frac{d\sigma^{e^+e^-}}{d\Omega}\right|_{\text{c.m.}}^{\text{u.r.}} = \frac{\alpha^2}{8E^2}\left\{\frac{1+\cos^4\frac{\theta}{2}}{\sin^4\frac{\theta}{2}} + \tfrac{1}{2}(1+\cos^2\theta) - 2\,\frac{\cos^4\frac{\theta}{2}}{\sin^2\frac{\theta}{2}}\right\},$$

which can be further simplified to obtain

$$\left.\frac{d\sigma^{e^+e^-}}{d\Omega}\right|_{\text{c.m.}}^{\text{u.r.}} = \frac{\alpha^2}{(8E)^2}\,\frac{(3+\cos^2\theta)^2}{\sin^4\frac{\theta}{2}}\,.$$

The non-relativistic limit is just

$$\left.\frac{d\sigma^{e^+e^-}}{d\Omega}\right|_{\text{c.m.}}^{\text{n.r.}} = \frac{\alpha^2\,m^2}{16|\underset{\sim}{p}|^4\sin^4\frac{\theta}{2}}\,.$$

(c) The differential cross-section is highly peaked at $\theta = 0$ (growing like $1/\theta^4$ for small angles), and falls monotonically with increasing θ in both the ultra-relativistic and non-relativistic limits. Because of this small-angle divergence, the total cross-section is infinite. This is not a failure of the theory — asking about the total cross section is a bad question which is not experimentally meaningful. Any experiment attempting to measure the total cross section will necessarily have some finite angular resolution (*i.e.* a smallest angle for which scattered and unscattered particles can be distinguished), coming from the finite size of the experiment and imperfect collimation of the incident beam. What is measurable, and theoretically calculable, is the finite but resolution dependent cross section for scattering by more than some minimal angle.