What about identity?

Identity "=" is a special 2-place predicate that satisfies the following rules, where α and β are any variables or name letters.

I. *Reflexivity* (Aristotle: "Each thing is what it is.")

'$\alpha=\alpha$' is a theorem

II. *Substitutivity* (Leibniz :"Identicals are indiscernible.")

$$\alpha=\beta, \phi(\alpha) / \therefore \phi(\beta) \quad \text{is valid}$$

where $\phi(\beta)$ comes from $\phi(\alpha)$ by properly substituting β for one or more free occurrences of α.

Ex. 1. Alice is smart. Alice is Barbara.
Therefore, Barbara is smart.

2. Alice is smart. Bob is not.
Therefore, Alice is not Bob.

(*Note the different uses of "is".*)

Can you use I and II to prove that identity is
symmetric ($x=y \Rightarrow y=x$) and
transitive ($x=y \& y=z \Rightarrow x=z$) ?

3. It is possible that you exist and that your particular body doesn't. So you have a property – possibly existing without your body – that your body doesn't have. ∴ You are not identical to your body.

A: you; B: your particular body

Fx: it is possible that x exists without x's particular body

$$FA \wedge {\sim}FB \;/\therefore\; A \neq B$$

4. Only Alice and Bob solved the problem.

A: Alice; B: Bob; Fx: x solved the problem

$$\bigwedge x(Fx \rightarrow x=A \vee x=B)$$

BUT: (4) implies that Alice and Bob did solve the problem, which this translation does not imply. So try

$$\bigwedge x(Fx \leftrightarrow x=A \vee x=B)$$

Given reflexivity ('$\alpha=\alpha$'), this will work!

5. Greensleeves can jump farther than any other frog. *C: Greensleeves. Fx: x is a frog*

J(xy): x can jump farther than y

$$\bigwedge x[Fx \rightarrow J(Cx)] \wedge FC$$

BUT: this translation implies that Greensleaves can jump farther than herself. So try

$$\bigwedge x[Fx \wedge x \neq C \rightarrow J(Cx)] \wedge FC$$

Numerical Quantifiers

At least one student will get a grade of 4.0.
(*Fx: student x will get a grade of 4.0*)

(1) $\bigvee x Fx$

At most one student will get a grade of 4.0.

(2) $\bigwedge x \bigwedge y\, (Fx \wedge Fy \rightarrow x=y)$

Exactly one student will get a grade of 4.0.
Take the conjunction of (1) and (2); or

(3) $\bigvee y \bigwedge x\, (Fx \leftrightarrow x=y)$

Every quadratic equation has two solutions.
Fx: x is a quadratic equation
G(xw): w is a solution to x

$$\bigwedge x(Fx \rightarrow \bigvee y \bigvee z[y \neq z \wedge \bigwedge w(G(xw) \leftrightarrow w=y \vee w=z)])$$

So, if x is free in the formula $\phi(x)$:

$(1x)\phi(x)$: $\bigvee y \bigwedge x\, (\phi(x) \leftrightarrow x=y)$

$(2x)\phi(x)$: $\bigvee y \bigvee z[y \neq z \wedge \bigwedge w(\phi(w) \leftrightarrow w=y \vee w=z)]$

…

$(nx)\phi(x)$: $\bigvee y_1 \bigvee y_2 \ldots \bigvee y_n[y_i \neq y_j$ *(for $i \neq j$)* $\wedge$
$\bigwedge w(\phi(w) \leftrightarrow w=y_1 \vee w=y_2 \vee \ldots \vee w=y_n)]$