

$L(xy)$ :  $x$  loves  $y$  . (Assume the universe of discourse is all people.)

1.  $\forall x \forall y L(xy)$       **Everyone loves everyone.**
2.  $\forall x \exists y L(xy)$       **Everyone loves someone.**
3.  $\exists x \forall y L(xy)$       **Someone loves everyone.**
4.  $\exists x \exists y L(xy)$       **Someone loves someone.**
  
5.  $\forall y \forall x L(xy)$       **Everyone is loved by everyone.**
6.  $\forall y \exists x L(xy)$       **Everyone is loved by someone.**
7.  $\exists y \forall x L(xy)$       **Someone is loved by everyone.**
8.  $\exists y \exists x L(xy)$       **Someone is loved by someone.**

Notice that where all the quantifiers are the same [all universal in (1) and (5); all existential in (4) and (8)] the sentences are equivalent -- just paraphrased from the active voice to the passive. But where the quantifiers are mixed, the sentences may differ in truth value. NB: Compare (2) and (7).

**Now assume the universe of discourse is all things.** Introduce a predicate letter "F" to abbreviate  
 $Fx$ :  $x$  is a person. The (1)-(4) above become.

- I.  $\forall x \forall y [Fx \wedge Fy \rightarrow L(xy)]$
- II.  $\forall x [Fx \rightarrow \exists y (Fy \wedge L(xy))]$
- III.  $\exists x [Fx \wedge \forall y (Fy \rightarrow L(xy))]$
- IV.  $\exists x \exists y [Fx \wedge Fy \wedge L(xy)]$

As an exercise you should try to relativize (5)-(8) to the predicate "F".

No one loves everyone.

$\sim \forall x \wedge y L(xy)$  (denial of (3))

By QN:  $\wedge x \forall y \sim L(xy)$

No one loves anyone

$\sim \forall x \forall y L(xy)$  (denial of (4))

By QN:  $\wedge x \wedge y \sim L(xy)$

Anyone who loves everyone is loved by everyone.

$\wedge x [\wedge y L(xy) \rightarrow \wedge z L(zx)]$

Noone who loves everyone is loved by anyone.

$\wedge x [\wedge y L(xy) \rightarrow \sim \forall z L(zx)]$  or

$\sim \forall x [\wedge y L(xy) \wedge \forall z L(zx)]$

Anyone who does not love their mother does not love anyone at all. [Use  $M(ab)$ :  $b$  is  $a$ 's mother.]

$\wedge x \wedge y [M(xy) \wedge \sim L(xy) \rightarrow \sim \forall z L(xz)]$

Only those who love their parents love themselves. [Use  $M$  as above and  $F(ab)$ :  $b$  is the father of  $a$ ]

$\wedge x [L(xx) \rightarrow \wedge y \wedge z (M(xy) \wedge F(xz) \rightarrow L(xy) \wedge L(xz))]$

$\forall y \wedge x F(xy) \Rightarrow \wedge x \forall y F(xy)$ , but not conversely.

E.g., Suppose  $F(xy)$ :  $y$  is the mother of  $x$ .

Then  $\wedge x \forall y F(xy)$  says  
everyone has a mother.

But  $\forall y \wedge x F(xy)$  says  
there is a mother of us all.

Show  $\wedge x \forall y F(xy)$

- |    |                   |                    |
|----|-------------------|--------------------|
| 1. | $\wedge x F(xa)$  | Prem, EI ("a" new) |
| 2. | $F(xa)$           | 1, UI              |
| 3. | $\forall y F(xy)$ | 2, EG              |

