L(xy): x loves y. (Assume the universe of discourse is all people.)

1. \( \forall x \forall y \ L(xy) \) \hspace{1cm} Everyone loves everyone.
2. \( \forall x \exists y \ L(xy) \) \hspace{1cm} Everyone loves someone.
3. \( \exists x \forall y \ L(xy) \) \hspace{1cm} Someone loves everyone.
4. \( \exists x \exists y \ L(xy) \) \hspace{1cm} Someone loves someone.

5. \( \forall y \forall x \ L(xy) \) \hspace{1cm} Everyone is loved by everyone.
6. \( \forall y \exists x \ L(xy) \) \hspace{1cm} Everyone is loved by someone.
7. \( \exists y \forall x \ L(xy) \) \hspace{1cm} Someone is loved by everyone.
8. \( \exists y \exists x \ L(xy) \) \hspace{1cm} Someone is loved by someone.

Notice that where all the quantifiers are the same [all universal in (1) and (5); all existential in (4) and (8)] the sentences are equivalent -- just paraphrased from the active voice to the passive. But where the quantifiers are mixed, the sentences may differ in truth value. NB: Compare (2) and (7).

Now assume the universe of discourse is all things. Introduce a predicate letter "F" to abbreviate \( Fx: x \) is a person. The (1)-(4) above become.

I. \( \forall x \forall y \ [Fx \wedge Fy \rightarrow L(xy)] \)
II. \( \forall x [Fx \rightarrow \exists y \ (Fy \wedge L(xy))] \)
III. \( \exists x [Fx \wedge \forall y \ (Fy \rightarrow L(xy))] \)
IV. \( \exists x \exists y \ [Fx \wedge Fy \wedge L(xy)] \)

As an exercise you should try to relativize (5)-(8) to the predicate "F".
No one loves everyone.
\[ \neg \forall x \forall y \, L(xy) \, \text{(denial of (3))} \]
By QN: \[ \neg \exists y \, \forall x \, \neg L(xy) \]

No one loves anyone
\[ \neg \forall x \forall y \, L(xy) \, \text{(denial of (4))} \]
By QN: \[ \neg \exists x \, \forall y \, \neg L(xy) \]

Anyone who loves everyone is loved by everyone.
\[ \forall x \forall y \, L(xy) \rightarrow \forall z \, L(zx) \]

No one who loves everyone is loved by anyone.
\[ \exists x \forall y \, L(xy) \rightarrow \neg \exists z \, L(zx) \]
\[ \exists x \forall y \, L(xy) \rightarrow \neg \exists z \, L(zx) \]

Anyone who does not love their mother does not love anyone at all. [Use \( M(ab) \): \( b \) is \( a \)'s mother.]
\[ \neg \exists x \forall y \, L(xy) \rightarrow \neg \exists z \, L(zx) \]

Only those who love their parents love themselves. [Use \( M \) as above and \( F(ab) \): \( b \) is the father of \( a \)]
\[ \forall x \forall y \, L(xy) \rightarrow \forall z \, L(zx) \]
\[ (M(xy) \land F(xz) \rightarrow L(xy) \land L(xz)) \]
\[ \forall y \land x \ F(xy) \Rightarrow \land x \lor y \ F(xy) \text{, but not conversely.} \]

E.g., Suppose \( F(xy) \): \( y \) is the mother of \( x \).

Then \( \land x \lor y \ F(xy) \) says everyone has a mother.

But \( \lor y \land x \ F(xy) \) says there is a mother of us all.

Show \( \land x \lor y \ F(xy) \)

1. \[ \land x \ F(xa) \] Prem, EI ("a" new)
2. \[ F(xa) \] 1, UI
3. \[ \lor y \ F(xy) \] 2, EG