What about identity?

Identity "=" is a special 2-place predicate that satisfies the following rules, where α and β are any variables or name letters. (no quantifiers in (1) and (2) means quantify universally over any free variables).

1. *Reflexivity* (Aristotle: "Each thing is what it is.")

$$\alpha=\alpha$$

2. *Substitutivity* (Leibniz' Law:"Identicals are indiscernible.")

$$\{\alpha=\beta,\phi(\alpha)\}\vdash\phi(\beta)$$

where $\phi(\beta)$ comes from $\phi(\alpha)$ by properly substituting β for one or more free occurrences of α.

For example, symmetry: $\vdash\bigwedge x\bigwedge y(x=y\rightarrow y=x)$.
Here's a proof. Assume x=y.
From (1) x=x. Call this $\phi(x)$.
From (2) by properly substituting 'y' for 'x' on the left in $\phi(x)$ yields $\phi(y)$, y=x.

Homework: prove that '=' is transitive.

"Predicate logic with identity" or "first order logic with identity" is what you get if you add the above rules (or their equivalent) to pure predicate logic.
To adapt our proof of completeness to this, notice that when you make a model I of sentences that include '=', the 2-place relation '≈' that corresponds to '=' on the universe U of the model may not be the identity relation on U. (If it is, the model is called "normal".) But '≈' will have to be an equivalence relation. (A model preserves truths!) And more.

For example, suppose we have a two place predicate F(xy) applied to the names 'A' and 'B' to get F(AB). If we also have A=C and B=D, then F(CD) follows from substitutivity. Hence on the universe U

$$\langle I(A), I(B)\rangle \in I(F)\ \&\ I(A)\approx I(C)\ \&\ I(B)\approx I(D)$$
$$\Rightarrow \quad \langle I(C), I(D)\rangle \in I(F).$$

Thus the '≈' relation that corresponds to identity satisfies an "equivalence principle": If one n-tuple $\langle\alpha_1, \alpha_2, \ldots, \alpha_n\rangle$ belongs to the I-extension of some n-place predicate then so does every "equivalent" n-tuple $\langle\beta_1, \beta_2, \ldots, \beta_n\rangle$; i.e., where each β_i falls into the equivalence class $[\alpha_i]$ determined by '≈'. (Recall that

$$[\alpha_i]=\{\beta \in U|\ \alpha_i\approx\beta\}.$$

So $\beta_i\in[\alpha_i]$ is a fancy way of saying that $\alpha_i\approx\beta_i$.)

This principle applies to '≈', which is the extension of '='. Thus $\alpha\approx\beta$ iff $[\alpha]=[\beta]$. So '≈' between the elements of U turns into identity between the equivalence classes. This suggests that we could make a normal model if we replace U by the set of ≈-equivalence classes of elements of U.

So in the proof of completeness instead of defining the universe to be the set of name letters in our maximally consistent and reliable Δ (which would then make identity non-normal) we let the universe U of our model consist of all the $\approx$-equivalence classes $[\alpha]$, where α is a name letter in the vocabulary of Δ and where '$\approx$' is defined by $\alpha\approx\beta$ iff '$\alpha=\beta$' is a sentence in Δ. (Need to show that this is an equivalence relation!)

Call our new interpretation over this universe [I]. Then for each n-place predicate F^n and n-tuple of name letters we can let
$<[\alpha_1], [\alpha_2], \ldots, [\alpha_n]> \in [I](F^n)$ iff
$<\alpha_1, \alpha_2, \ldots, \alpha_n> \in I(F^n)$.

Sentence letters come out to be true or false under [I] just as under I. So our induction to show that all the sentences in the vocabulary of Δ are happy goes through and we get not just a model but a normal model, where "identity" is really identity.