Definition of a *derivation* for axiomatic sentential logic.

$$\Sigma \vdash \phi$$

There is a *finite* sequence of formulas L1, L2 , ..., Ln such that

(1) Ln is ϕ

and (2) for $i \leq n$, either Li is an axiom, or Li is in Σ, or

(3) there are j, k < i such that Li follows from Lj and Lk by *Modus Ponens*.

We write $\vdash \phi$ for $\emptyset \vdash \phi$,

and $(\Sigma + \psi) \vdash \phi$ for $\Sigma \cup \{\psi\} \vdash \phi$, and we often omit the set builder braces and write

$\phi 1, \phi 2, \phi 3, \ldots, \phi n \vdash \phi$ for $\{\phi 1, \phi 2, \phi 3, \ldots, \phi n\} \vdash \phi$

(1) For any formula ϕ, $\phi \vdash \phi$.

(2) $(\Sigma + \psi) \vdash \phi$ and $\vdash \psi \Rightarrow \Sigma \vdash \phi$.

(3) $\Sigma \vdash \psi$ and $\Sigma \subseteq \Sigma' \Rightarrow \Sigma' \vdash \psi$
Cor. $\vdash \phi \Rightarrow \Sigma' \vdash \phi$ for **any** set Σ' of formulas.

(4) **Finiteness Condition for $\vdash$.**
$\Sigma \vdash \phi \Rightarrow$ there is a **finite** subset $\Sigma_f \subseteq \Sigma$ such that $\Sigma_f \vdash \phi$.

(5) If $\Sigma \vdash \phi$ and for every formula ψ in Σ, $\Sigma' \vdash \psi$, then $\Sigma' \vdash \phi$.
(Transitivity)

(6) **Deduction Theorem** (sentential logic)
If $(\Sigma + \psi) \vdash \phi$, then $\Sigma \vdash (\psi \rightarrow \phi)$.

(7) If $\Sigma \vdash (\phi \rightarrow \psi)$ and $\Sigma \vdash \phi$, then $\Sigma \vdash \psi$. (Modus Ponens!)

Definition: A set Σ of formulas is **consistent** iff there is *some* formula ψ such that $\Sigma \nvdash \psi$. Otherwise, Σ is **inconsistent** (that is, iff for *every* formula ψ, $\Sigma \vdash \psi$)

(8) **The Basic (Syntactic) Connection.**
$\Sigma \vdash \phi$ iff $\Sigma \cup \{\sim\phi\}$ is inconsistent. (Equivalently, $\Sigma \vdash \sim\phi$ iff $\Sigma \cup \{\phi\}$ is inconsistent.)