Below is a sketch of the Hardy Paradox.*

Let ψ be the state of the separated system for a typical 2x2 EPRB experiment. Suppose we can expand it in eigenstate of the component observables A,B; A',B; A,B'; and A',B' respectively as follows

(1) $\psi = a_1 10 + a_2 01 + a_3 00$

where 10 is the product $|A=1\rangle|B=-1\rangle$ of A,B eigenstates; 01 is the product $|A=-1\rangle|B=1\rangle$ and 00 is the product $|A=-1\rangle|B=-1\rangle$. (We assume also that the coefficients a_n are non-zero and the sum of their absolutes values squared is 1.) Since there is no 11 term(i.e., $|A=1\rangle|B=1\rangle$ that coefficient is zero. Hence the probability P(AB)=0 that A=1 and B=1 in state ψ.

(2) $\psi = b_1 11 + b_2 01 + b_3 00$ where here the 11,01 and 00 terms are products of the A' with B eigenstates. That there is no 10 term means that the probability is zero that A'=1 and B=0. Hence, where P(B|A') is the conditional probability that B=1given that A'=1, then P(B|A')=1.

(3) $\psi = c_1 11 + c_2 10 + c_3 00$ where here the 11,10 and 00 terms are products of the A with B' eigenstates. That there is no 01 term means that the probability is zero that A=-1 and B'=1. Hence, P(A|B')=1.

(4) $\psi = d_1 11 + d_2 10 + d_3 01 + d_4 00$ where these terms are products of the A' with B' eigenstates and where $|d_4|^2 \neq 0$. Hence P(A'B')≠0.

Thus we have P(AB)=0, P(B|A')=1 and P(A|B')=1. So we can argue as follows.

In the state ψ not both A and B are 1. But A'=1 implies that B=1. And B'=1 implies that A=1. Hence we can't have that both A'=1 and B'=1. Yet according to (4) there is some non-zero probability for exactly that!

Problem. Find the assumptions that resolve the fallacy. (Hint: What entitles us to put the A, B, B' probabilities together with the A', B, B' ones??)

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* L. Hardy, Nonlocality For Two Particles Without Inequalities For Almost All Entangled States.*Physical Review Letters* **71**, 1665-68 (1993). See also S. Goldstein, Nonlocality without Inequalities for Almost All Entangled States for Two Particles , *Physical Review Letters* **72**, 1951-3 (1994); N. David Mermin, Quantum Mysteries Refined. *American Journal of Physics* **62**, 880-887 (1994) and A. Fine Locality and the Hardy Theorem, in Jeremy Butterfield and Constantine Pagonis (eds.), *From Physics to Philosophy*, Cambridge: Cambridge University Press, 1999, pp. 1-11.