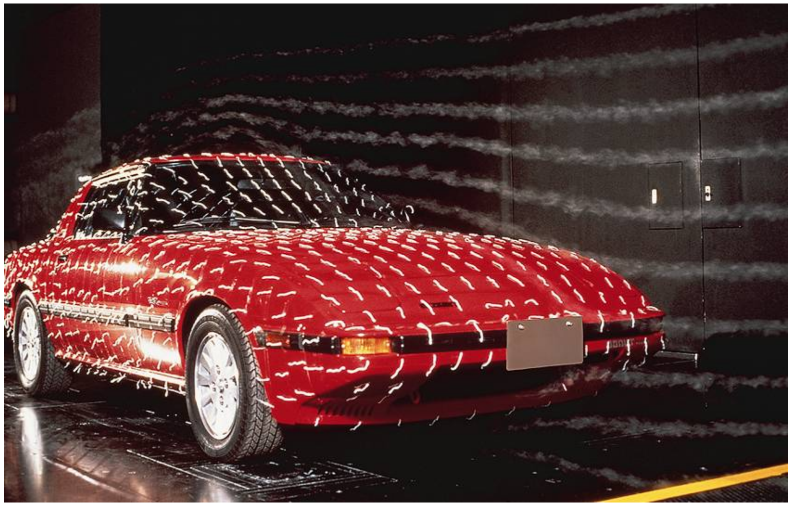
Physics 115A
General Physics II

Session 4

Fluid flow
Bernoulli's equation



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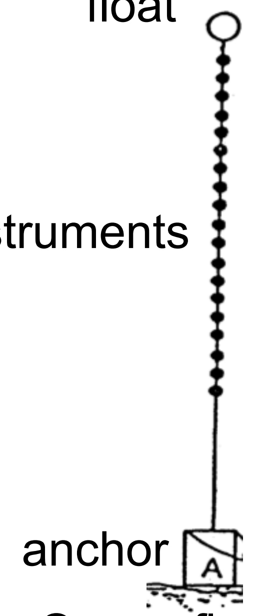
Lecture Schedule (up to exam 1)

Date	Day	Lect.	Topic	readings in Walker
31-Mar	Mon	1	Introduction, Preview	
1-Apr	Tues	2	Density & Pressure	15.1-15.3
3-Apr	Thurs	3	Static Fluids, Buoyancy	15.4-15.5
4-Apr	Fri	4	Fluid Flow, Bernoulli	15.6-15.8
7-Apr	Mon	5	Viscosity, Flow, Capillaries	15.9
8-Apr	Tues	6	Temperature, expansion	16.1-16.3
10-Apr	Thurs	7	Heat, Conduction	16.4-16.6
11-Apr	Fri	8	Ideal gas	17.1-17.2
14-Apr	Mon	9	Heat, Evaporation	17.4-17.5
15-Apr	Tues	10	Phase change	17.6
17-Apr	Thurs	11	First Law Thermodynamics	18.1-18.3
18-Apr	**Fri**		**EXAM 1 Ch 15,16,17**	

Today

Just joined the class? See course home page
courses.washington.edu/phy115a/
for course info, and slides from previous sessions

Example: Flotation

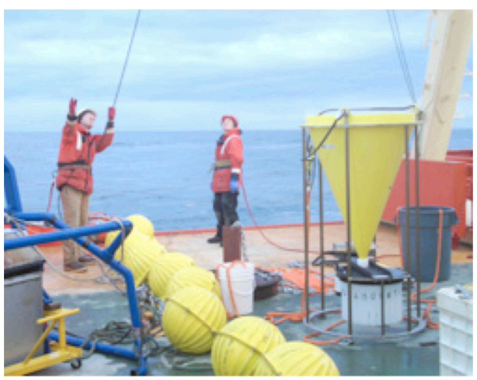


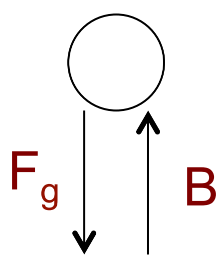
- A "string" of undersea oceanographic instruments is held down by an anchor, and pulled upright by a hollow steel sphere float.
  - The float displaces 1 m^3 of seawater
  - It weighs 1000N when in air

  What upward force can it supply to hold up the string?

$$F_g = m_{STEEL} g = 1000N$$

$$B = \rho_{\mathrm{SEAWATER}}\, g\, V_{FLOAT} = \left(1023kg/m^3\right)\left(9.8m/s^2\right)\left(1m^3\right) = 10{,}025N$$





Net upward force on sphere when free:
= B – F_g = 9,025 N
This is the max weight of instruments it can 'lift'

Combine several ideas: “Cartesian diver” demo

Demonstrates: Archimedes principle, Pascal’s Law, gas law (next week)

- You can do this at home with a plastic soda bottle and a small packet of soy sauce or ketchup
- “Diver” has some air in it, and just barely floats (*neutral buoyancy*)
  - Squeeze the bottle: increase P in water, *compress* air bubble
  - Compression → smaller V → *higher density* for diver: sinks
  - Release: reduced pressure →

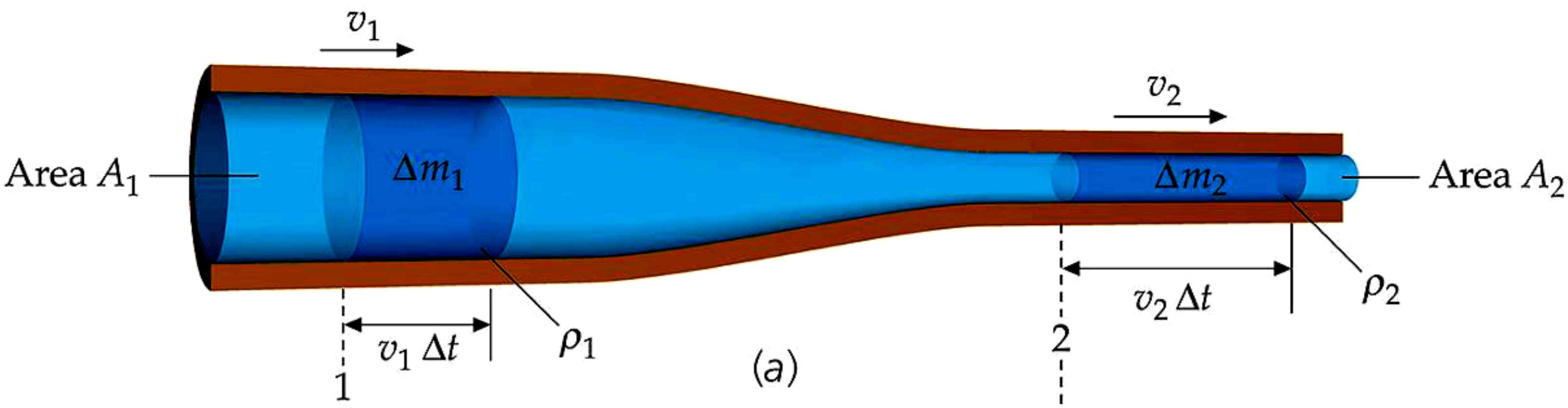
  bubble re-expands → lower density: floats again

(Toy said to have been invented by Descartes)

Rene Descartes, 1596-1650

For how-to instructions, see
http://www.stevespanglerscience.com/lab/experiments/cartesian-diver-ketchup

Fluids in motion: continuity equation



Incompressible fluid flows in a pipe that gets narrower. What happens?

Conservation of **matter**: mass going into pipe must = mass going out

Mass passing point 1 in time Δt: $\Delta m_1 = \rho_1 \Delta V_1 = \rho_1 A_1 v_1 \Delta t$

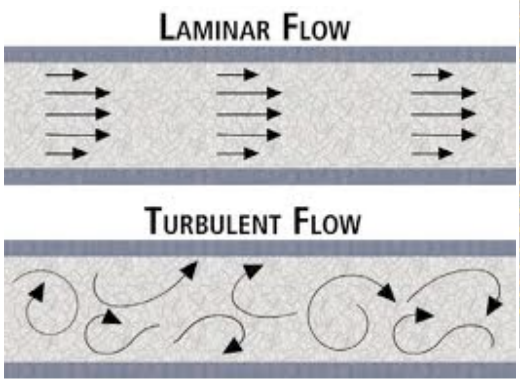
Mass passing point 2 in time Δt: $\Delta m_2 = \rho_2 A_2 v_2 \Delta t$

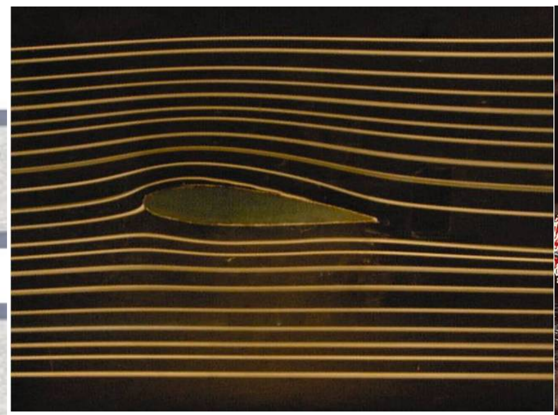
Fluid cannot disappear, or be compressed, so must have $\rho_1 A_1 v_1 = \rho_2 A_2 v_2$

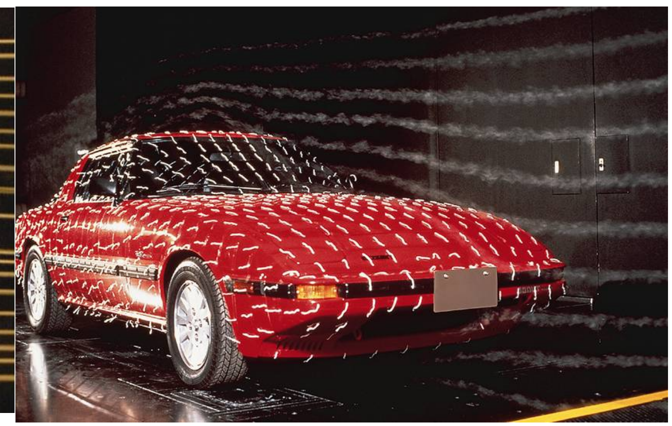
If density is constant, $\rho_2 = \rho_1 \quad \rightarrow \quad A_1 v_1 = A_2 v_2$

Laminar flow vs turbulent flow

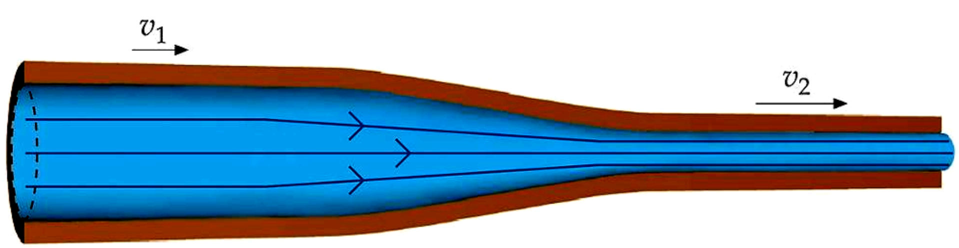
- As always in physics, we start with the simplest case, and add complications after we solve the easy stuff
- Laminar flow = smooth motion of fluid
  - Paths of particles of fluid (elements of mass Δm) do not cross
  - "streamline flow"





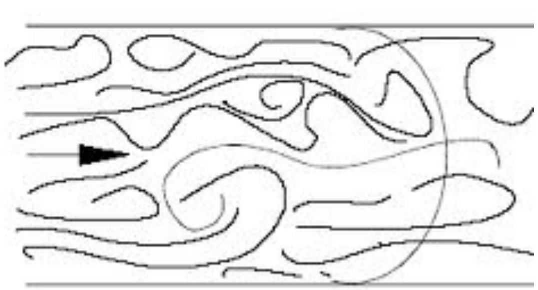


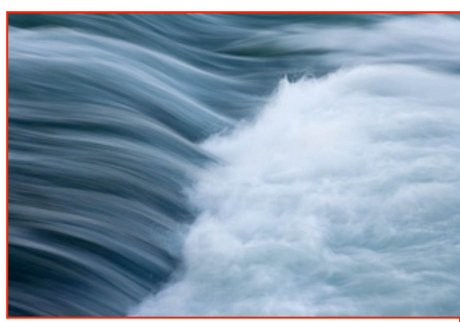
Streamlines: lines indicate the laminar flow path. Separation between streamlines correlates with pressure and flow speed (more on this later).

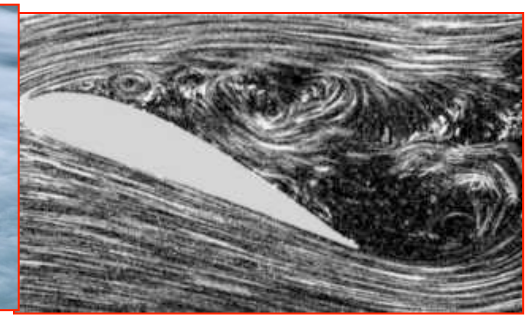


Turbulence – we wont go into...

- Turbulent flow = complex motion of fluid mass elements
  - Paths of water parcels may cross
    - Hard to analyze!
  - Important topic in weather prediction, oceanography, etc
  - Example of *chaotic behavior*
    - parcels of water that start out next to each other have unpredictable locations farther downstream
      - We have equations describing motion, but they cannot be solved uniquely in most cases
    - *Chaos theory* (mathematical toolkit) helps...

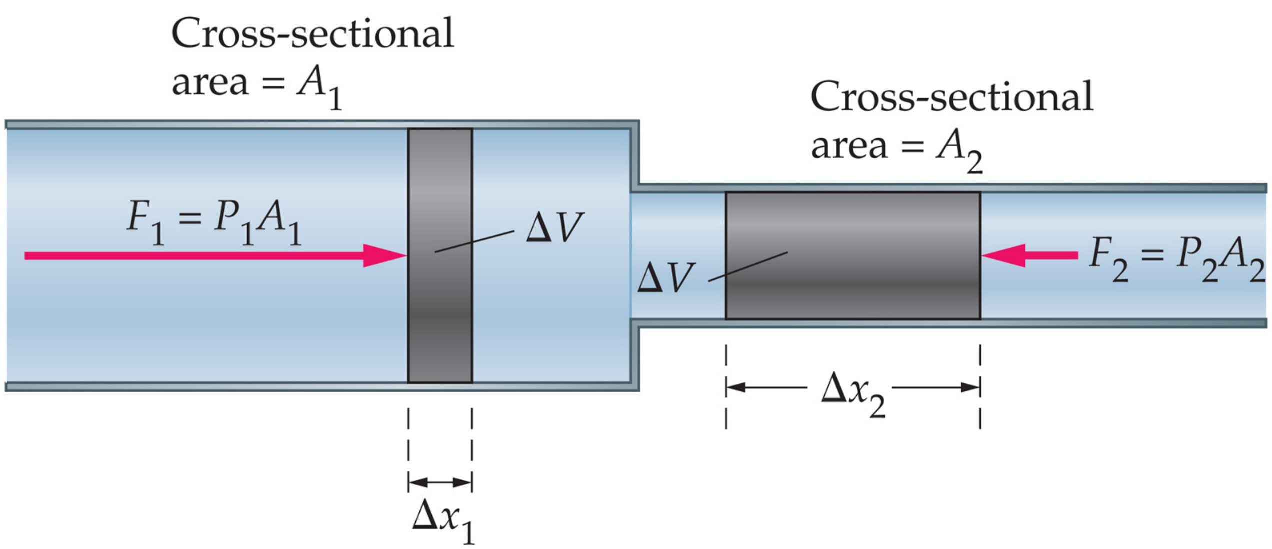






Bernoulli's equation: energy considerations in flow

- Apply conservation of energy to fluid flow
  - Suppose fluid speed changes (as usual: incompressible)
  - We know that $\rho_1 A_1 v_1 = \rho_2 A_2 v_2$
  - Consider a "parcel" of fluid ΔV : length Δx , area A
  - If speed changes, a force must have acted: pressure x area



Daniel Bernoulli (1700 – 1782)

Bernoulli's equation: changing speed

- F_1 = force due to pressure on parcel of fluid ΔV when at location 1, from fluid **behind** it $F_1 = P_1 A_1 \quad \Delta V_1 = \Delta x_1 A_1$
- F_2 = force due to pressure on same parcel of fluid ΔV when at location 2, from fluid **ahead** of it $F_2 = P_2 A_2 \quad \Delta V_2 = \Delta x_2 A_2$
- Work done by forces: $\Delta W_1 = F_1 \Delta x_1 = P_1 A_1 \Delta x_1 = P_1 \Delta V_1$

$$\Delta W_2 = -F_2 \Delta x_2 = -P_2 \Delta V_2$$

(F_2 points backward)

- Incompressible: $\Delta V_1 = \Delta V_2 = \Delta V$
- Net work is thus $\Delta W = \Delta W_1 + \Delta W_2 = P_1 \Delta V - P_2 \Delta V = \left(P_1 - P_2\right)\Delta V$
- Net work = change in KE of parcel

$$\Delta W = K_2 - K_1 , \quad where\ K_i = \tfrac{1}{2}\Delta m\, v_i^2 = \tfrac{1}{2}\rho \Delta V\ v_i^2$$

$$\Delta W = \Delta K \rightarrow \left(P_1 - P_2\right)\Delta V = \left(\tfrac{1}{2}\rho\ v_2^2 - \tfrac{1}{2}\rho\ v_1^2\right)\Delta V$$

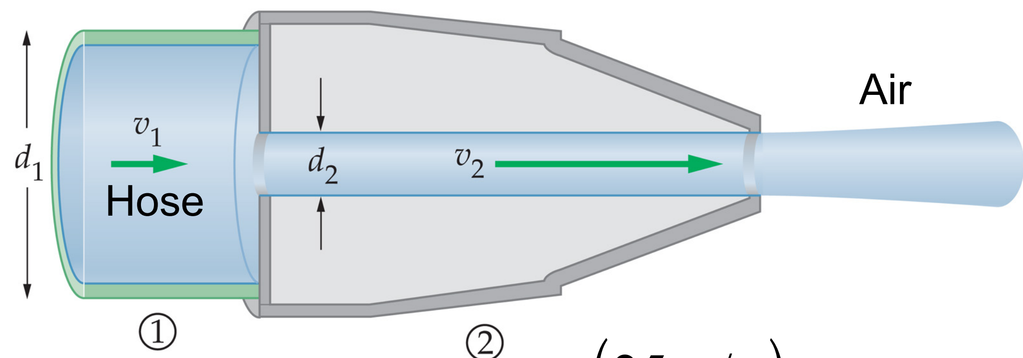
So $P_2 + \tfrac{1}{2}\rho v_2^2 = P_1 + \tfrac{1}{2}\rho v_1^2$ or $\boxed{P + \tfrac{1}{2}\rho v^2 = \text{constant}}$

The Bernoulli Equation (part of it...)

Pressure/speed of flow

- The part of Bernoulli's eq'n we did so far tells us that pressure must drop ($P_2 < P_1$) when speed increases
  - Makes sense: P_1 pushes fluid forward, P_2 pushes backward!
- Example: **Hose nozzle:** circular cross-section, diameter reduced ½

P_2 = 110 kPa (water)
$d_1 = 2\,d_2 \rightarrow A_1 = 4A_2$
If v_2 = 25 m/s,
what is P_1 and v_1 ?



Continuity eqn says: $\rho A_1 v_1 = \rho A_2 v_2 \rightarrow v_1 = \frac{A_2}{A_1} v_2 = \frac{(25m/s)}{4} = 12.5m/s$

Bernoulli: $P_2 + \frac{1}{2}\rho v_2^2 = P_1 + \frac{1}{2}\rho v_1^2 \rightarrow P_1 = P_2 + \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$

$$P_1 = \left(110kPa\right) + \tfrac{1}{2}\left(1000kg/m^3\right)\left(\left[25m/s\right]^2 - \left[6.25m/s\right]^2\right)$$

$$= 110kPa + \left(500kg/m^3\right)(586m^2/s^2) = 403kPa$$

Another part of Bernoulli's eqn: changing height

- Suppose the pipe changes height but not diameter:
  - The gravitational PE of the fluid changes
  - Raising the fluid: pressure forces have to do work on the parcel of fluid
    - Or: gravity does negative work on it (falling)
    - Work done raising a mass distance y = -mgy

$$\Delta W_{GRAV} = -mg(y_2 - y_1) = -\rho \Delta V g (y_2 - y_1)$$

- Total work done on parcel = W done by gravity + W by forces (pressure difference)

If pipe area A does not change,

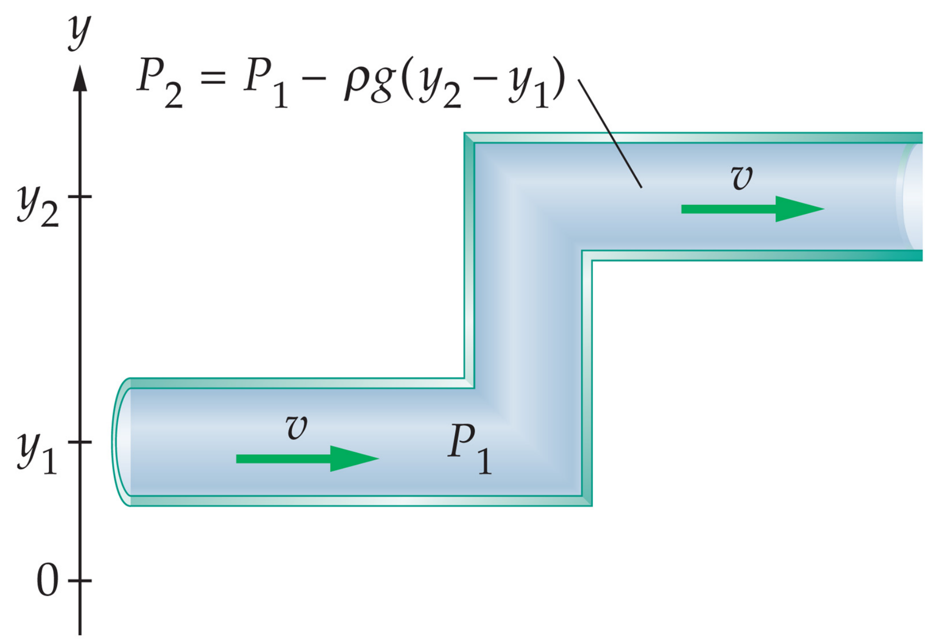
v = constant, so no change in KE

$$\Delta W_{TOTAL} = \Delta W_{GRAV} + \Delta W_{FORCES} = \Delta K = 0$$

$$(P_1 - P_2)\Delta V - \rho \Delta V g (y_2 - y_1) = 0$$

$$P_2 + \rho g y_2 = P_1 + \rho g y_1$$ or

$$\boxed{P + \rho g y = \text{constant}}$$



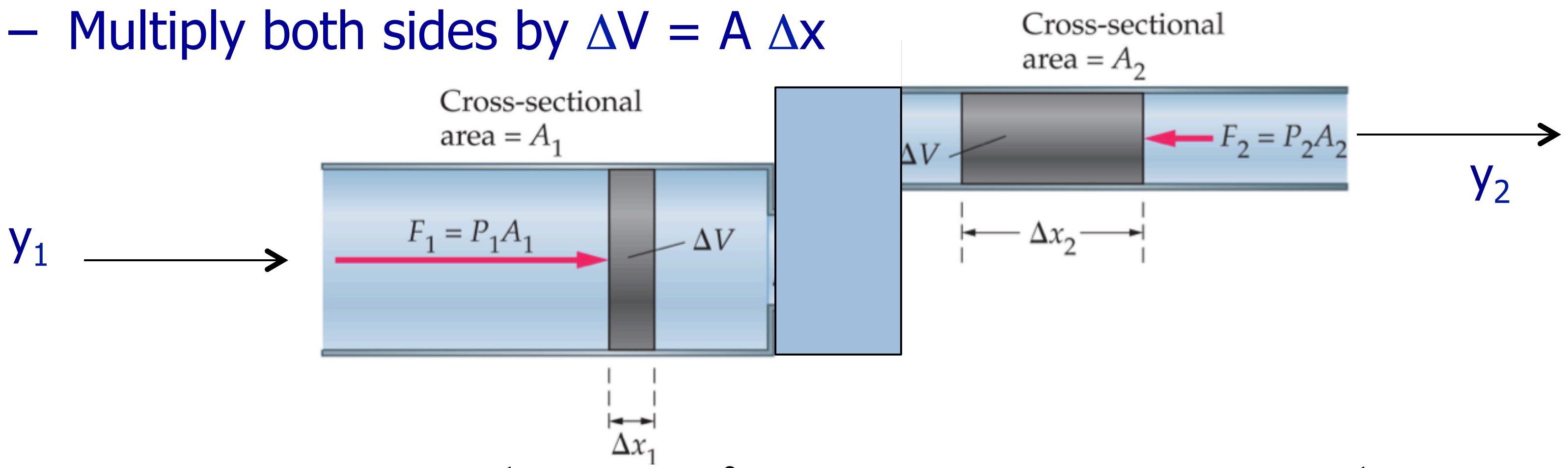
Doing The Full Bernoulli

- We can combine both pieces: The Bernoulli Equation
  - Pressure vs speed
  - Pressure vs height

$$\boxed{P_2 + \rho g h_2 + \tfrac{1}{2}\rho v_2^2 = P_1 + \rho g h_1 + \tfrac{1}{2}\rho v_1^2}$$

or $$\boxed{P + \rho g h + \tfrac{1}{2}\rho v^2 = \text{constant}}$$

- This turns out to be conservation of total energy
  - Multiply both sides by $\Delta V = A\,\Delta x$

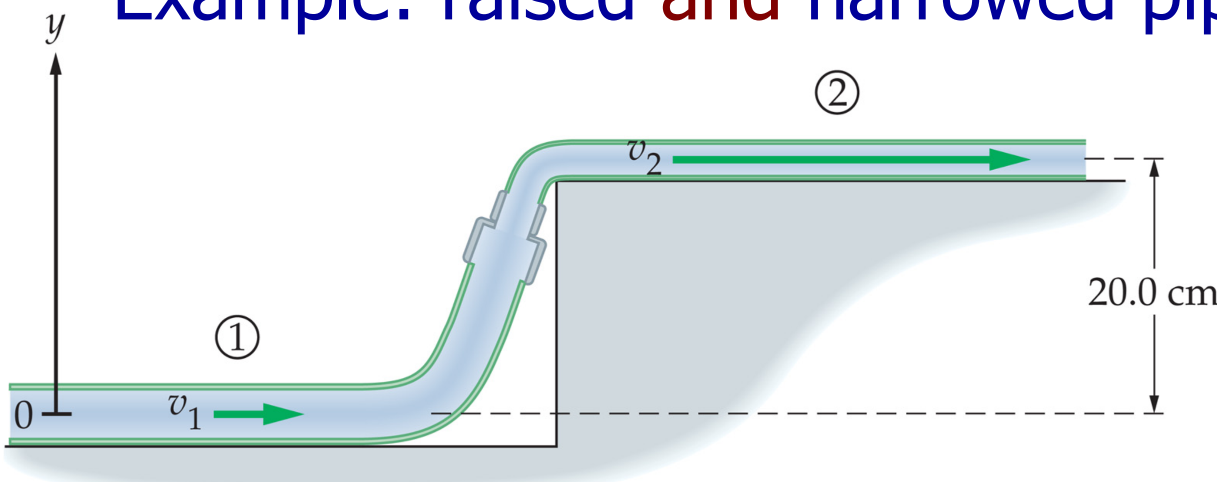


$$P_1 A_1 \Delta x_1 + \rho A_1 \Delta x_1 g h_1 + \tfrac{1}{2}\rho A_1 \Delta x_1 v_1^2 = P_2 A_2 \Delta x_2 + \rho A_2 \Delta x_2 g h_2 + \tfrac{1}{2}\rho A_2 \Delta x_2 v_2^2$$

$$F_1 \Delta x_1 + m g h_1 + \tfrac{1}{2} m v_1^2 = F_2 \Delta x_2 + m g h_2 + \tfrac{1}{2} m v_2^2$$

Work on m by upstream mass | PE | KE = Work by m on downstream mass | PE | KE

Example: raised and narrowed pipe



- Garden hose connected to raised narrower hose

  $A_1 = 2\,A_2$ (so what is ratio of diameters?)

  $\Delta y = h = 20$ cm, $v_1 = 1.2$ m/s, $P_1 = 143$kPa , fluid is fresh water

  Find P and v at location 2

  – Use continuity eqn to get v_2 : $A_1 v_1 = A_2 v_2 \rightarrow v_2 = \dfrac{A_1 v_1}{A_2} = 2v_1 = 2.4m/s$

  – Use Bernoulli to relate P's :

$$P_1 + \rho g h_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \tfrac{1}{2}\rho v_2^2 \rightarrow P_2 = P_1 + \rho g\left(h_1 - h_2\right) + \tfrac{1}{2}\rho\left(v_1^2 - v_2^2\right)$$

$$P_2 = 143kPa + \left(1000kg/m^3\right)\left[9.8m/s^2\left(-0.2m\right) + \tfrac{1}{2}\left(\left\{1.2m/s\right\}^2 - \left\{2.4m/s\right\}^2\right)\right]$$

$$= 143kPa + \left(-1{,}960Pa - 2{,}160Pa\right) = \boxed{138.9kPa}$$

Torricelli's Law

A large tank of water, open at the top, has a small hole through its side a distance h below the surface of the water.

Find the speed of the water as it flows out the hole. $v_a = 0$ $P_a = P_b = P_{at}$

(P at surface, and outside the hole)

$$P_a + \rho g h_a + 0 = P_b + \rho g h_b + \tfrac{1}{2}\rho v_b^2$$

$$\rho g \Delta h = \tfrac{1}{2}\rho v_b^2 \qquad \boxed{v_b = \sqrt{2g\Delta h}}$$

Torricelli's Law:
Speed of water jet at depth h is same as speed of object dropped from height h

Evangelista Torricelli (1608 - 1647)