Physics 115
General Physics II

Session 18

Lightning
Gauss's Law
Electrical potential energy
Electric potential V

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Lecture Schedule
(up to exam 2)

21-Apr	Mon	12	Specific Heats	18.4-18.6
22-Apr	Tues	13	Second Law	18.7-18.10
24-Apr	Thurs	14	Entropy	18.8-18.10
25-Apr	Fri	15	Charges	19.1-19.4
28-Apr	Mon	16	E field	19.5-19.66
29-Apr	Tues	17	Gauss law	19.7
1-May	Thurs	18	Electrical potential	20.1-20.3
2-May	Fri	19	Potential, conductors	20.4
5-May	Mon	20	Capacitors	20.5-20.6
6-May	Tues	21	Current	21.1-21.2
8-May	Thurs	22	Power, Series & Parallel Circuits	21.3-21.4
9-May	**Fri**		**EXAM 2 - Ch. 18,19,20**	

Today

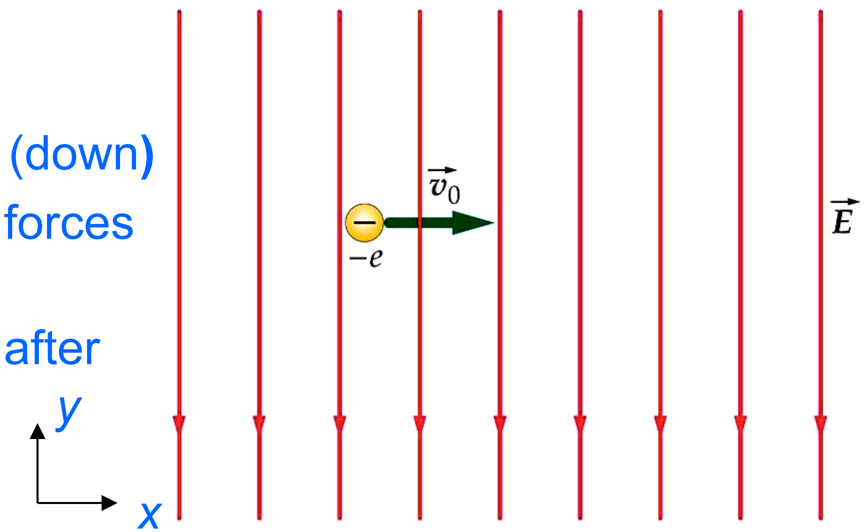
Example: Electron Moving in a Perpendicular Electric Field

...similar to prob. 19-101 in textbook

- Electron has $\mathbf{v}_0$ = 1.00x10^6 m/s **i**
- Enters uniform electric field $\boldsymbol{E}$ = 2000 N/C (down)

(a) Compare the electric and gravitational forces on the electron.

(b) By how much is the electron deflected after travelling 1.0 cm in the x direction?



$$\frac{F_e}{F_g} = \frac{eE}{mg}$$

$$= \frac{(1.60\times10^{-19}\ \text{C})(2000\ \text{N/C})}{(9.11\times10^{-31}\ \text{kg})(9.8\ \text{N/kg})}$$

$$= 3.6\times10^{13}$$

$$\Delta y = \frac{1}{2}a_y t^2, \quad a_y = F_{net}/m = (eE\uparrow + mg\downarrow)/m \approx eE/m$$

$$\Delta y = \frac{1}{2}\left(\frac{eE}{m}\right)t^2, \quad v_x >> v_y \rightarrow t \approx \frac{\Delta x}{v_x} \rightarrow \Delta y = \frac{eE}{2m}\left[\frac{\Delta x}{v_x}\right]^2$$

$$= \frac{(1.60\times10^{-19}\ \text{C})(2000\ \text{N/C})}{2(9.11\times10^{-31}\ \text{kg})}\left[\frac{(0.01\ \text{m})}{(1.0\times10^{6}\ \text{m/s})}\right]^2$$

$$= 0.018\ \text{m} = 1.8\ \text{cm}\ \ (\text{upward})$$

(Math typos corrected)

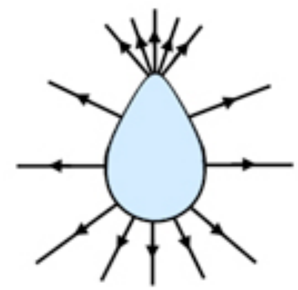
Big Static Charges: About Lightning

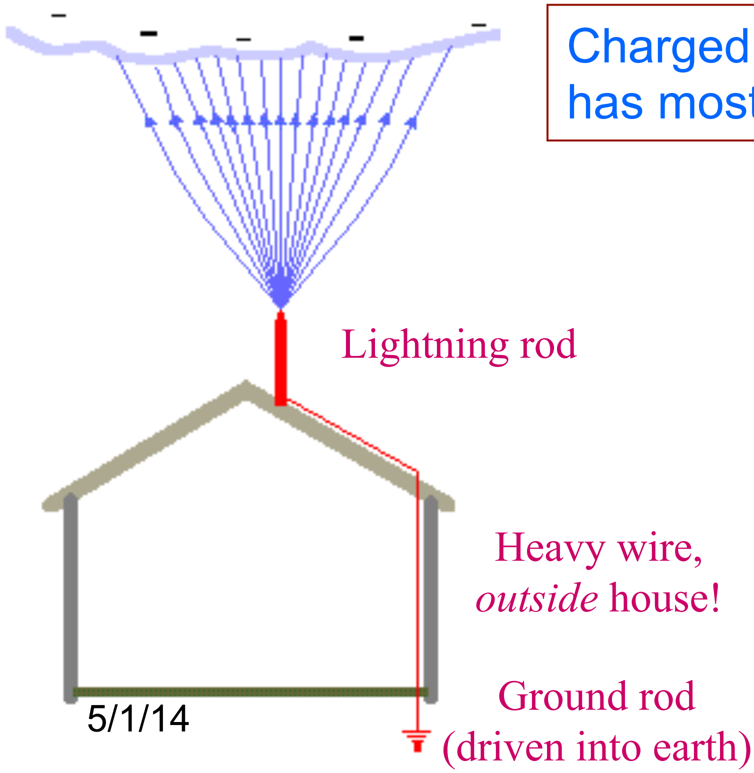
- Lightning = huge electric discharge
- Clouds get charged through friction
  - Clouds rub against mountains
  - Raindrops/ice particles carry charge
- Discharge may carry 100,000 amperes
  - What's an *ampere*? Definition soon…
- 1 kilometer long arc means 3 billion volts!
  - What's a *volt*? Definition soon…
  - High voltage breaks down air's resistance
  - What's *resistance*? Definition soon...
- Ionized air path stretches from cloud to ground and also ground toward cloud
- Path forms temporary "wire" along which charge flows
  - often bounces a few times before settling

Lightning Rods

- Ben Franklin invented lightning rods (1749) to protect buildings
  - Provide safe conduit for lightning away from house, in case of strike
  - Discharge electric charge accumulation on house *before* lightning channel forms, via "corona discharge" (diffuse, localized ionization)
    - Corona discharge (air plasma) sometimes seen on tops of boat masts

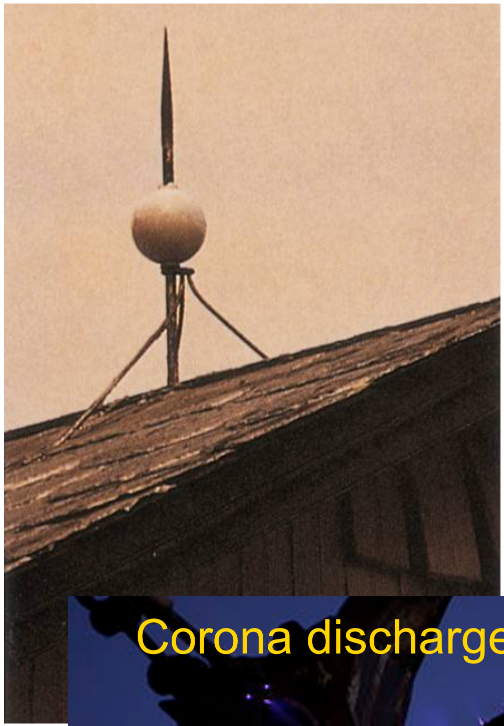
Charged object with a sharp point has most intense E field there:

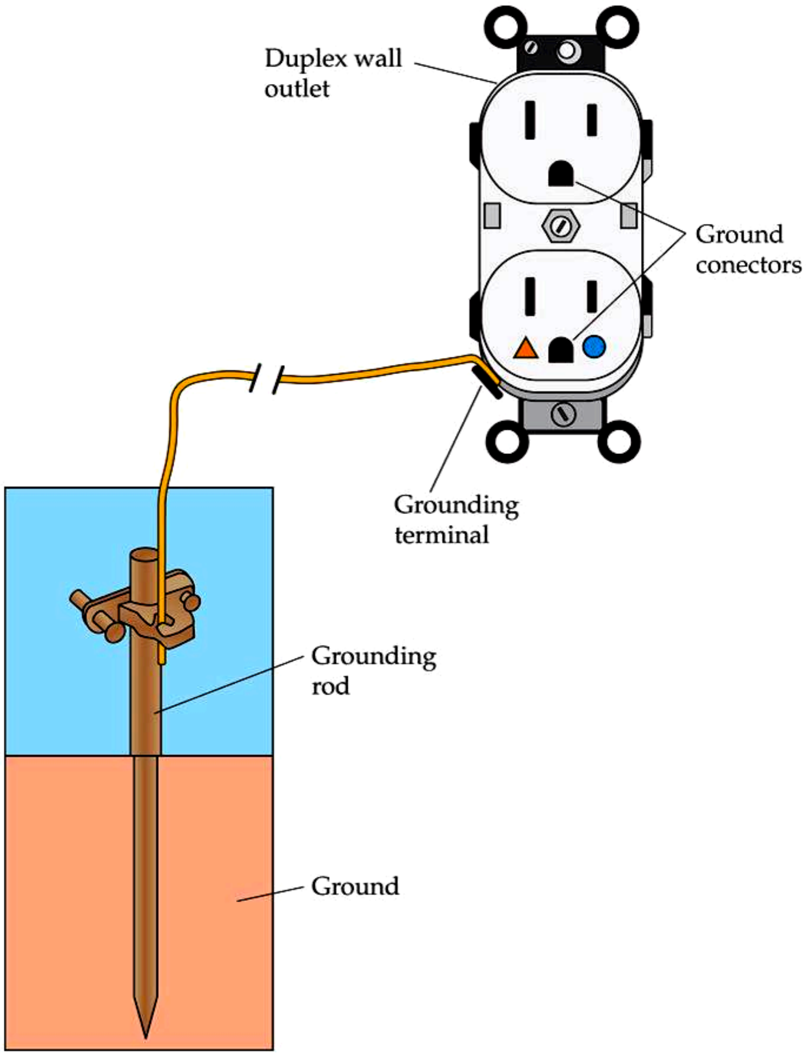




Charge concentrates at sharp tip of lightning rod, because *electric field lines* are very dense there (intense E).
(Recall demo of van de Graaf generator)
Charge "leaks" away, diffusing charge, via what is sometimes called "St. Elmo's Fire (ball lightning)", or "coronal discharge"

Grounding and Lightning Rods





Last time **Gauss's Law**: exploiting the flux concept

- Carl Friedrich Gauss (Germany, c. 1835)

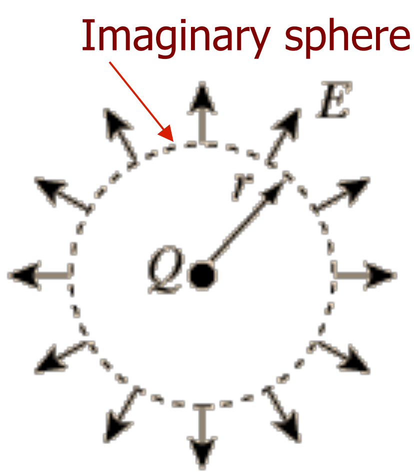
(possibly the greatest mathematician of all time)

The electric flux through any **closed surface** is proportional to the **enclosed electric charge**

Imagine a spherical surface surrounding charge +Q

- E field must be **uniform** due to symmetry
  - No reason for any direction to be "special"
  - So: Each patch of area on sphere has **same** E
- E field points outward (or opposite, for –Q)
  - Perpendicular to surface, so cos θ = 1

Carl Friedrich Gauss
(1777 – 1855)



$$\Phi_E = \vec{E} \cdot \vec{A} = EA\cos\theta = EA = \left(k\frac{Q}{r^2}\right)A_{SPHERE}$$

$$A_{SPHERE} = 4\pi r^2 \Rightarrow \Phi_E = E\left(4\pi r^2\right) = 4\pi k Q$$

$$\Phi_E = (const)Q \rightarrow \Phi_E \propto Q$$

Notice: we could reverse the calculation to get E from flux:

$$\Phi_E = E\left(4\pi r^2\right) \rightarrow E = \frac{4\pi kQ}{4\pi r^2} = \frac{kQ}{r^2}$$

Field lines and Electric Flux

Intensity of E field is indicated by density of field lines

- Double Q → double the magnitude of E at any given point

SO: Each charge **Q** has number of field lines proportional to **Q**

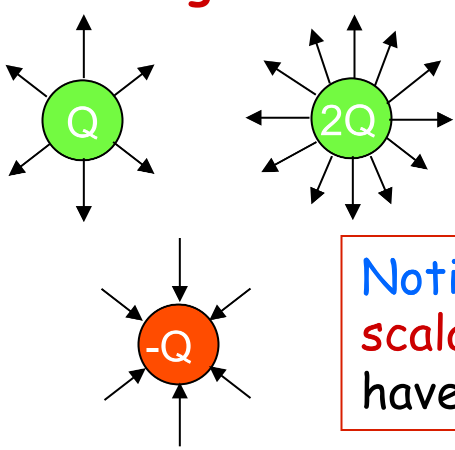
- Positive charge has lines going out
- Negative charge has lines going in

- Surround any set of charges with a closed surface (any shape!)

Net number of field lines coming out ~ the charge inside:

- (lines going out – lines going in) ~ Q_{net} inside

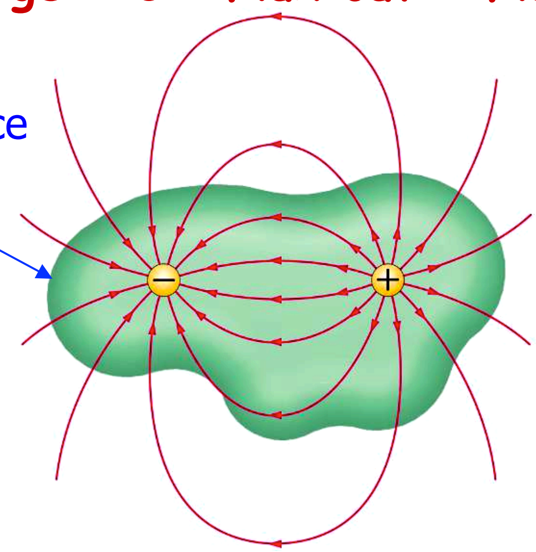
Net charge: net flux out or in



Notice: electric flux is a scalar quantity, but can have + or - sign

Net charge = 0: Flux out = Flux in

Imaginary surface



Gauss's Law restated: a new constant

- For a spherical surface enclosing charge Q we found $\Phi_E = 4\pi k Q$
  - Net electric flux exiting sphere
- Instead of k, a more commonly-used constant is $\varepsilon_0 = \dfrac{1}{4\pi k}$
  - Pronounced "epsilon-naught" (British) or epsilon-zero"
  - Recall k= 9 x 10^9 N m^2/C^2, so ε_0 = 8.85 x 10^{-12} $C^2/(N\ m^2)$
  - ε_0 = "permittivity of free space"
  - Now Coulomb's Law for point charges separated by r is written

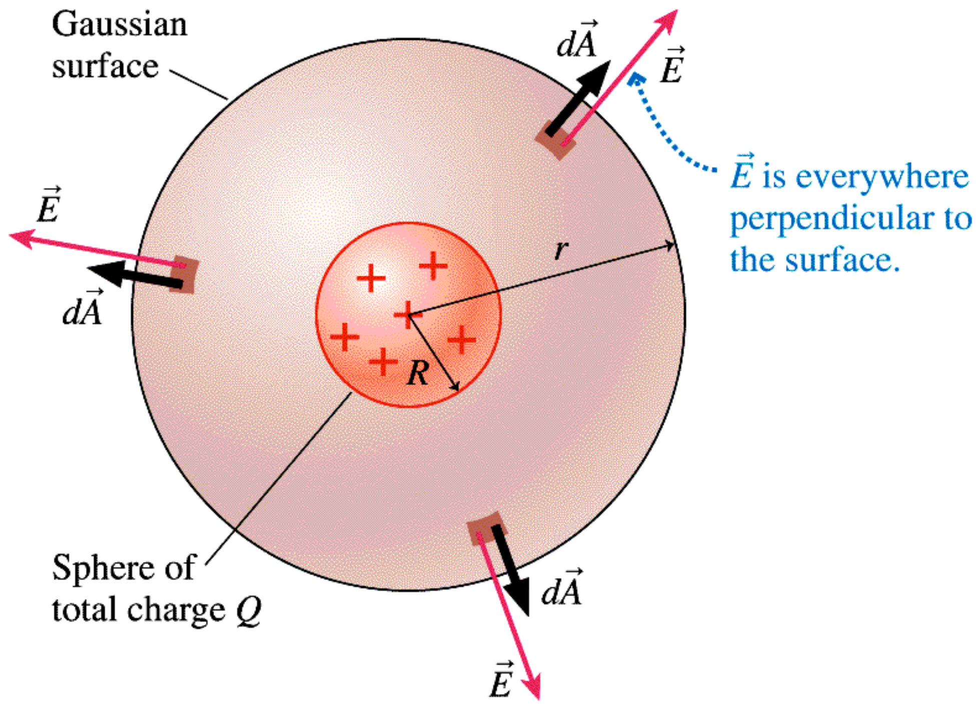
$$F_E = \frac{k|q_1 q_2|}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r^2},$$ and E(r) for a point charge is $$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

- Gauss's Law: If net charge Q is inside any closed surface, the net flux through the surface is

$$\Phi_E = \frac{Q_{ENCLOSED}}{\varepsilon_0}$$

  - Notice: ANY closed surface will do, not just spheres
  - If surface does not contain Q, net flux = 0 (as much out as in)
  - Use this to find E easily, for special cases with symmetrical charge arrangements: choose a handy **Gaussian surface**

Choosing Gaussian Surfaces



Gaussian surfaces are just mathematical concepts **we** create – no connection to any real surfaces in space!

They can help simplify calculating E fields from charge distributions

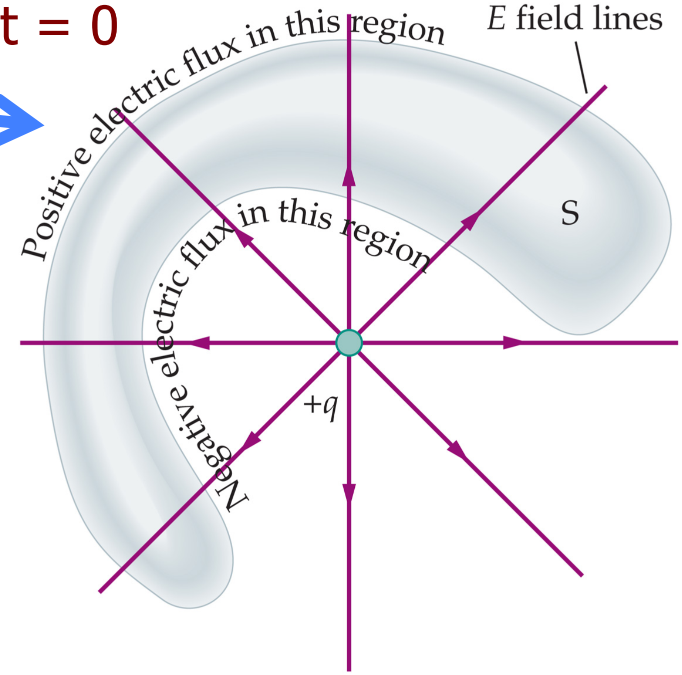
Choose Gaussian surfaces with convenient shapes matching the symmetry of the electric field (or charge distribution):

- point charge: (or any spherically symmetric arrangement): use a sphere,
- line of charge: use a cylinder; sheet of charge: use a box ...etc

Gauss's Law is most useful when field lines are either ***perpendicular*** or ***parallel*** to the Gaussian surfaces that they cross or lie inside.

Applying Gauss's Law with Gaussian surfaces

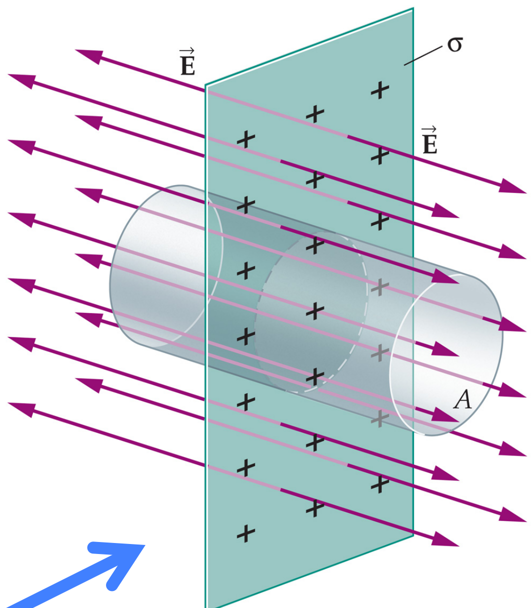
- If surface encloses 0 charge, NET flux out = 0
  - All E field lines that enter, also exit
  - Add up flux on opposite sides, net = 0



- Symmetry example:

Find E near an infinite uniformly charged sheet using Gauss's Law

- *Direction* of E must be perpendicular to sheet (symmetry)



Choose a cylinder with end-cap area A

- Sides are parallel to E: flux = 0
- Ends are perpendicular to E: flux = EA

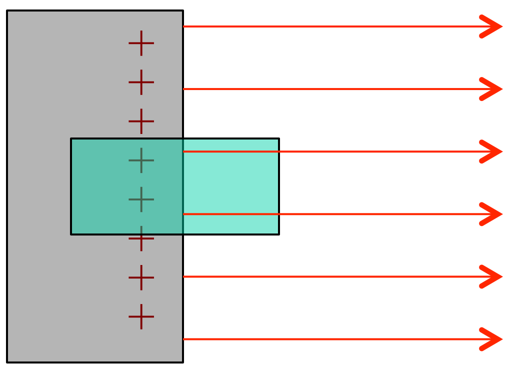
$$\Phi_E = \frac{Q_{ENCLOSED}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0},$$

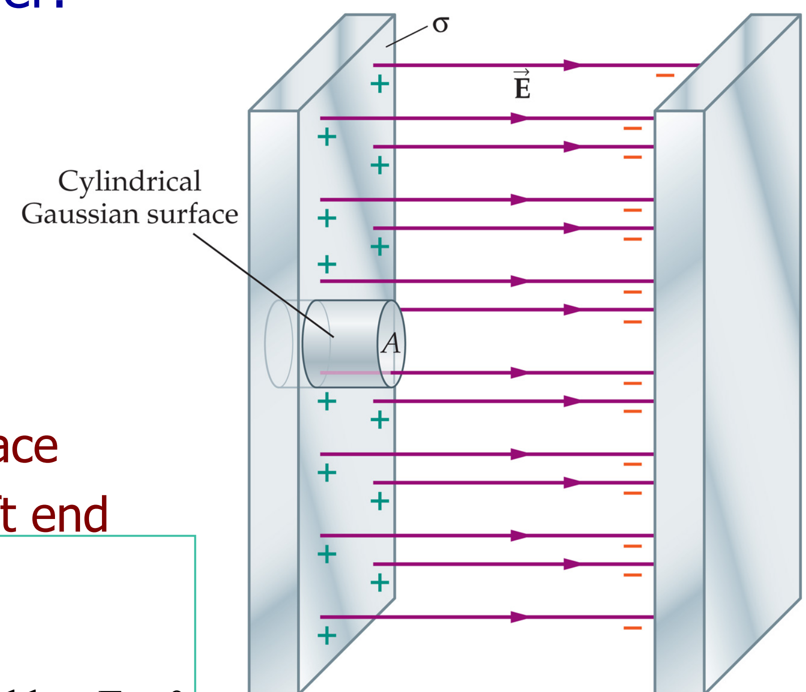
σ= charge density on surface, C/m^2

$$E(2A) = \frac{\sigma A}{\varepsilon_0} \rightarrow E = \frac{\sigma}{2\varepsilon_0} = const$$

Parallel conducting **plates**, revisited

- Notice: **not** the same as infinite sheet of charge!
  - Plates have thickness, and are conductors
  - Choose a Gaussian cylinder surrounding plate surface with one end **inside** the conductor (there, E=0)
  - Close-up of left end of cylinder:





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- All charge lies on plate surface
- E=0 inside → flux =0 on left end

$$\Phi_E = \frac{Q_{ENCLOSED}}{\varepsilon_0}, \quad \text{same as for sheet}$$

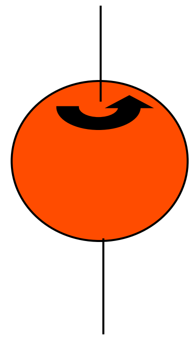
But now, no factor of 2 because only one end has $E \neq 0$

$$\Phi_E = EA = \frac{\sigma A}{\varepsilon_0} \rightarrow E = \frac{\sigma}{\varepsilon_0} = const$$

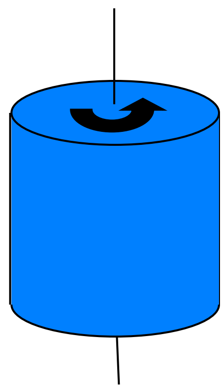
BTW: Symmetries are important in physics

Rotational symmetries

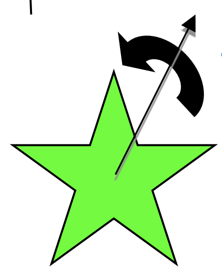
Spherical (full) symmetry:
rotation about ***any*** axis does not change the object:
looks the same from any viewpoint



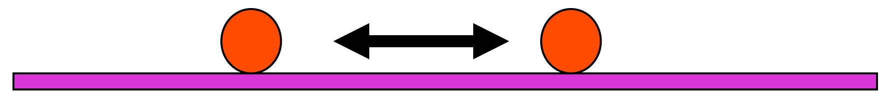
Cylindrical symmetry:
any rotation about ***one*** axis does not change object:
looks the same from any viewpoint in a plane perpendicular to axis of symmetry



Partial rotational symmetry:
specific rotation about ***one*** axis does not change object:
Example: 5-pt star looks the same under 72^0 rotations about its axis



Translational symmetry:
translation along a coordinate axis does not change object.



Reflection (“Parity”) **symmetry:**
mirror image reflection is *same* as object.

A (yes) A

E (no) Ǝ

Deep thought: Quantum theory tells us that mathematical symmetries in equations are connected to conservation laws

Quiz 11

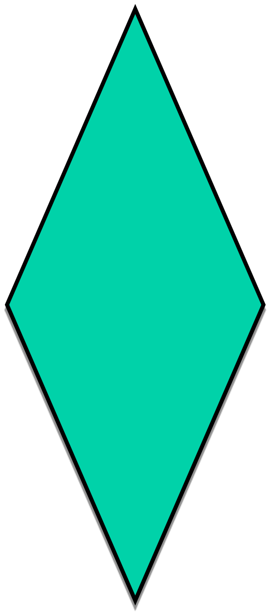
What kind of symmetry does a diamond (2D shape on a plane) have?

(a) Spherical

(b) Cylindrical

(c) Reflection

(e) none of the above



Potential Energy and Potential Difference

$$\Delta U_g = -W_g = -F_g \Delta s$$

$$\Delta U_E = -W_E = -F_E \Delta s = -q_0 E \Delta s$$

Recall potential energy in mechanics: Work done on or by an object (= KE gained or lost)

Example: work done BY gravity when object moved around (distance Δs) in a gravity field

Sign convention: W is work done BY field, so Δs is + if it is in the same direction as the field, negative if Δs is in opposite direction

Example: ball falls distance d, it loses U

Lift the ball distance d, it gains U

Same goes for work done by electrostatic force on a charge $+q_0$

New term: Electric potential is the electrical potential energy *difference* per unit charge between two points in space, due to work done by E fields: $\Delta V = \Delta U_E / q_0$

We always say V = potential difference (not PE): units are J/C

Volts

- Definition of electric potential describes only changes in V
- We can **choose** to put V=0 wherever we want – *differences* from place to place will remain the same
- Units for V are J/C: 1.0 J/C = 1.0 volt (V)
  - After Alessandro Volta (Italy, c. 1800) who invented the battery
- Note: Joules are useful for human-scale, not "micro" objects
  - For subatomic particles we use for energy units the electron-volt (eV) = energy gained by one electron charge, falling through one volt of potential difference:

$$1.0eV = (1.6\times10^{-19}C)(1V) = 1.6\times10^{-19}J$$

- Work done by E, and potential difference, for a test charge q_0 moved in the same direction as **E**:

$$W = q_0 E\Delta s$$

$$\Delta V = \frac{-W}{q_0} = -E\Delta s \rightarrow E = -\frac{\Delta V}{\Delta s}$$

This tells us:

1) Another unit for E can be volts per meter, so 1 N/C = 1 V/m.
2) E is given by the slope on a plot of V versus position