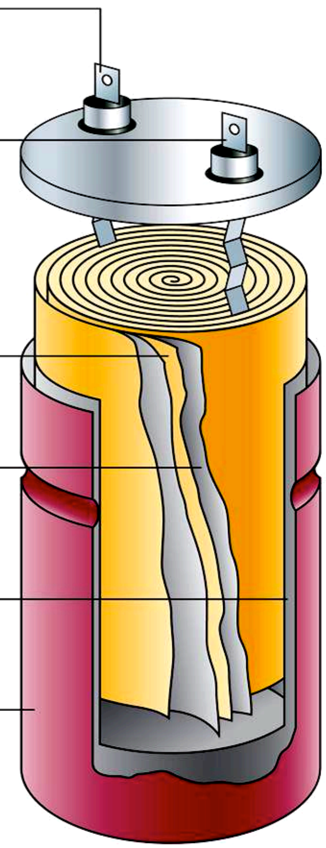
Physics 115
General Physics II

Session 20



Capacitors
Dielectrics

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Lecture Schedule
(up to exam 2)

21-Apr	Mon	12	Specific Heats	18.4-18.6
22-Apr	Tues	13	Second Law	18.7-18.10
24-Apr	Thurs	14	Entropy	18.8-18.10
25-Apr	Fri	15	Charges	19.1-19.4
28-Apr	Mon	16	E field	19.5-19.66
29-Apr	Tues	17	Gauss law	19.7
1-May	Thurs	18	Electrical potential	20.1-20.3
2-May	Fri	19	Potential, conductors	20.4
5-May	Mon	20	Capacitors	20.5-20.6
6-May	Tues	21	Current	21.1-21.2
8-May	Thurs	22	Power, Series & Parallel Circuits	21.3-21.4
9-May	**Fri**		**EXAM 2 - Ch. 18,19,20**	

Today

Announcements

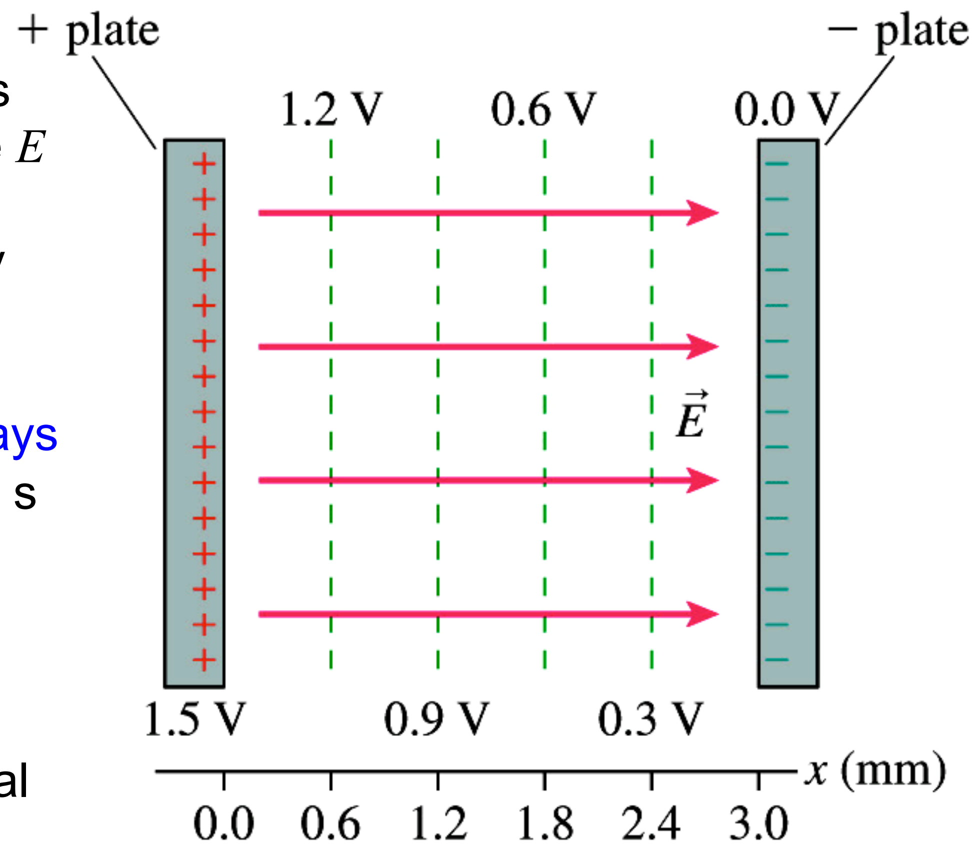
- Exam 2 is this Friday 5/9
  - Covers material discussed in class from Chs 18, 19, 20
    - NOT Ch. 21
  - Same format and procedures as last exam
    - If you arranged to take exam 1 with section B, please do same for all remaining exams, OR email us to say you want to change
  - Practice questions will posted Tuesday, and we will review them in class Thursday

Field Lines and Equipotentials

Field maps can have equipotentials drawn, simultaneously showing the E field and the electric potential V.
Here: map for equal and oppositely charged plates.

Remember: equipotentials will always be perpendicular to field lines: that's how we define potentials! (no work done moving along one)

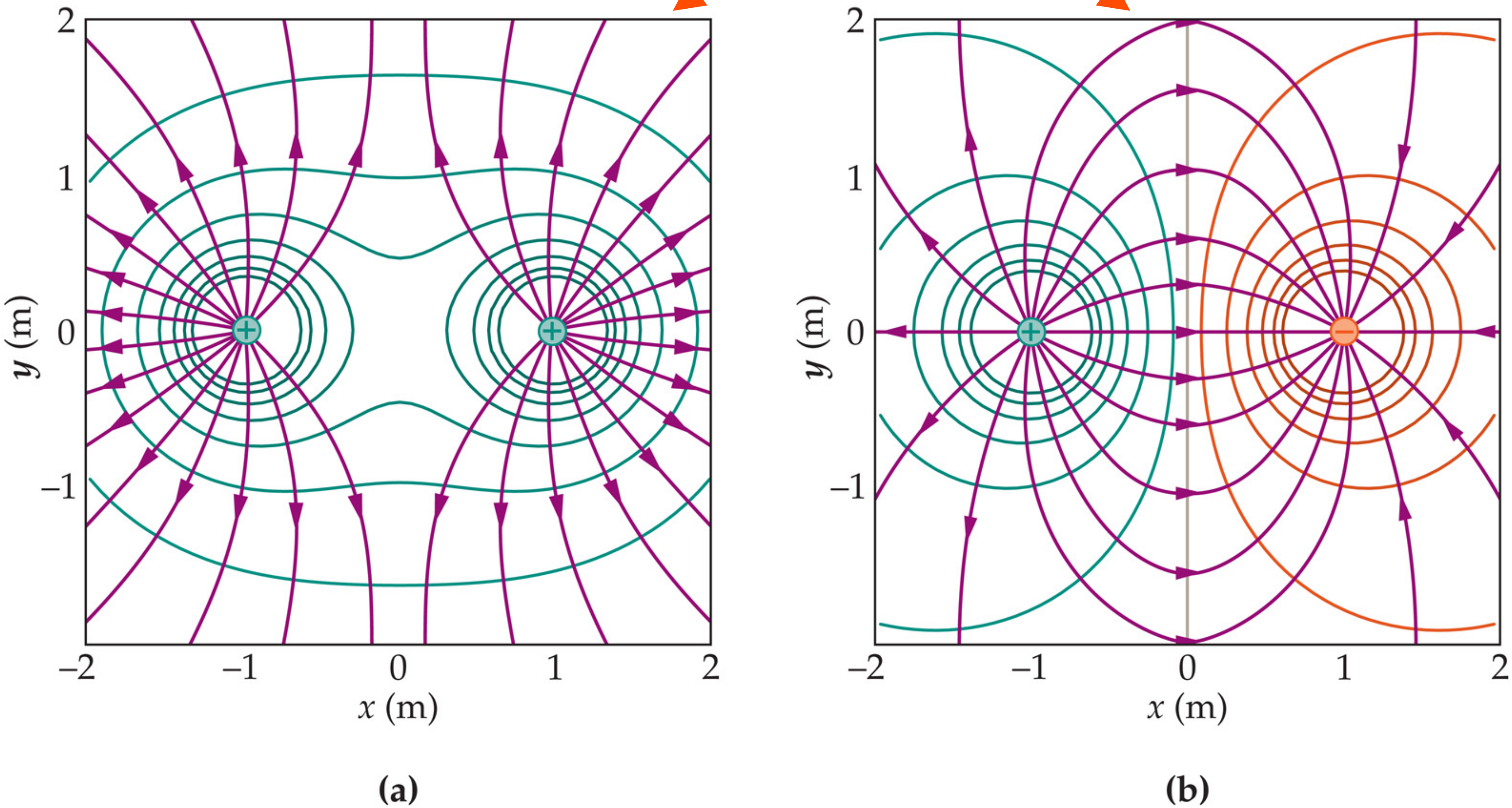
Remember: both field lines and V contours are "just pictures", not real objects.
Spacing of lines, etc, is just a matter of choice.



Notice: V increases in opposite direction to motion of a "falling" +q

Equipotentials around sets of charges

- For 2 point charges (both +, or a dipole)



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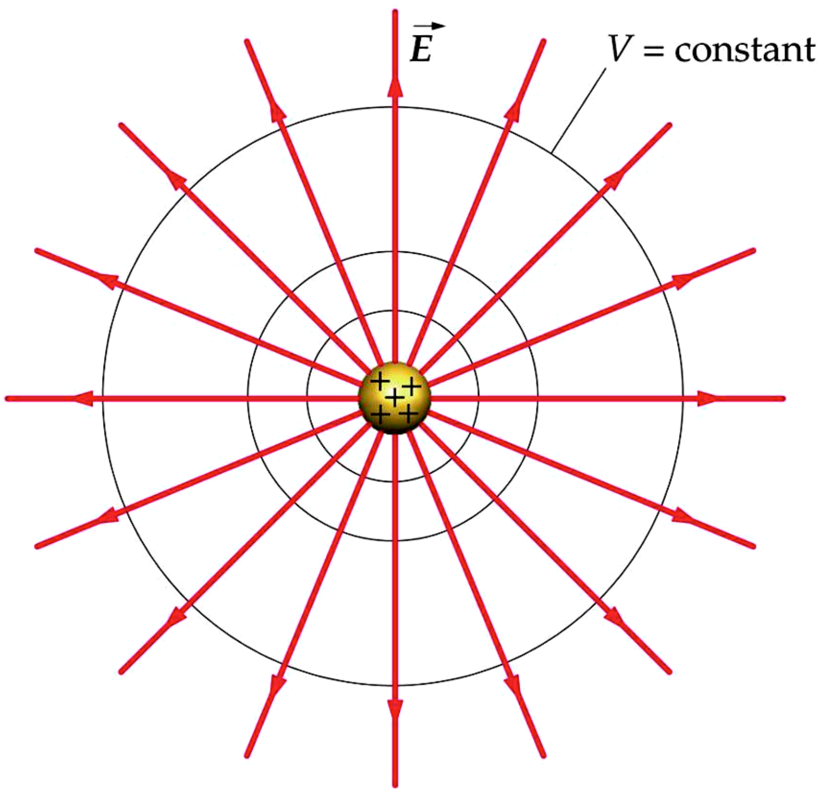
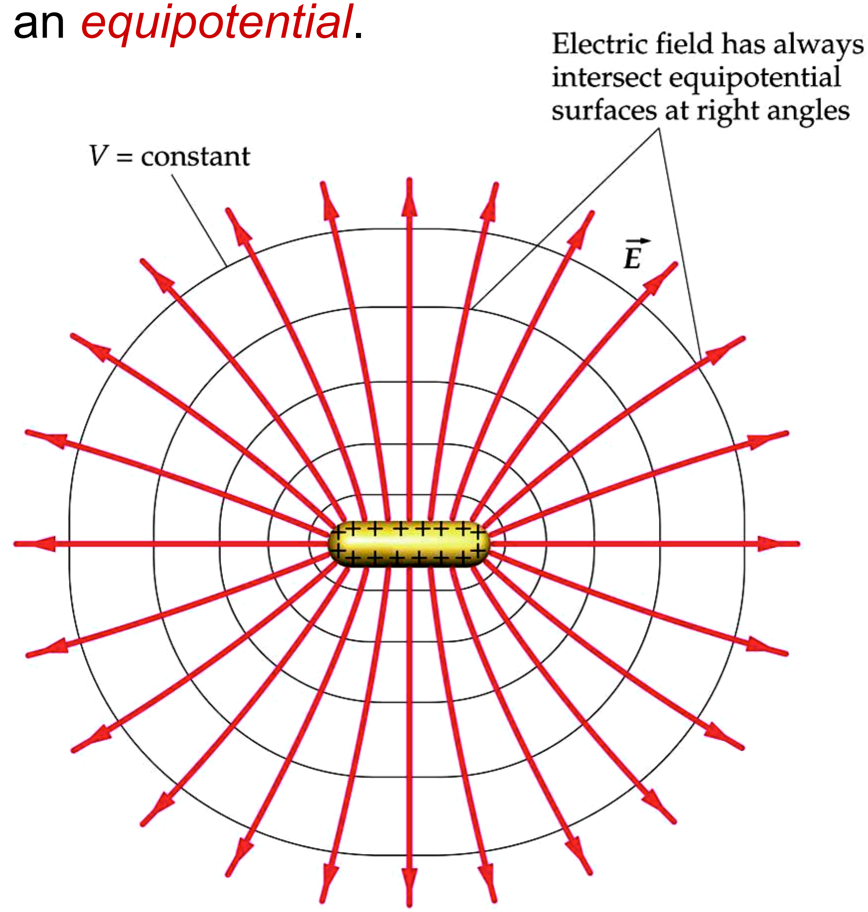
Conductors and Equipotentials

Recall: All points on or inside a conductor in electrostatic equilibrium must be at the same potential.

(Otherwise, mobile charges will move around until the potential IS constant.)

Therefore, the surface of a conductor is an *equipotential*.





Why is charge concentrated near sharp spots?

- Conducting sphere creates external E field as if it were a point charge at its center (same as gravity of Earth)

Surface area $A = 4\pi R^2$, charge density $\sigma = Q / A$

$$V(R) = \frac{kQ}{R} = 4\pi k\sigma R \rightarrow V \propto R$$

- If we want 2 conducting spheres of different radii to have the same potential at their surfaces, we have to give the smaller one larger charge **density** (not more Q):

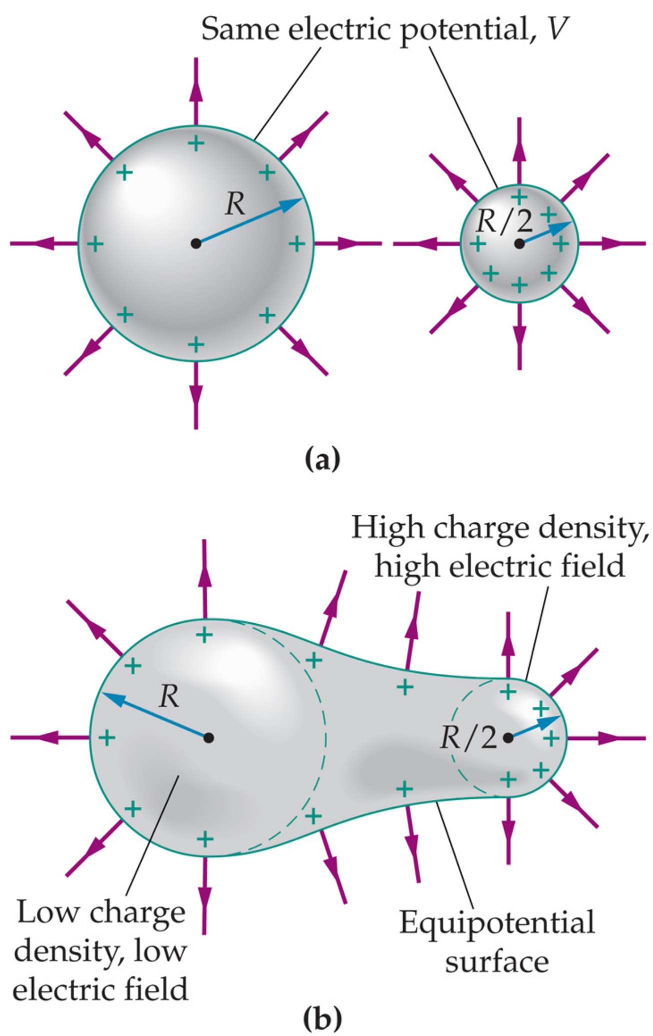
Surface area $A = 4\pi r^2$, charge density $\sigma_1 = Q_1 / A$

$$V = 4\pi k\sigma_1 R \Rightarrow R/2 \text{ needs } 2\sigma_1 \text{ to have same } V$$

$$\left(\text{But notice: } Q_2 = \sigma_2 A = 2\sigma_1 4\pi (R/2)^2 = Q_1/2\right)$$

$$E_{SURFACE} = \frac{kQ}{r^2} = \frac{k\left(4\pi r^2\sigma\right)}{r^2} = 4\pi k\sigma$$

So higher σ → higher E



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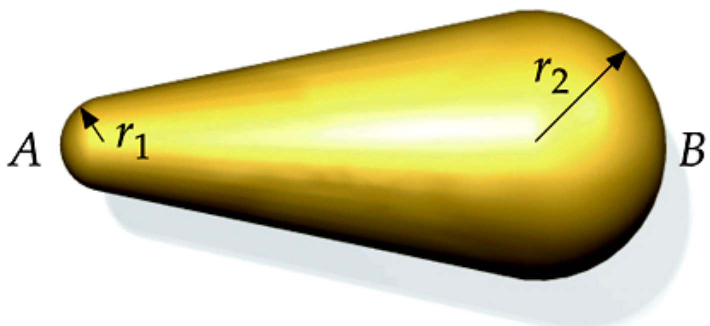
Charge density and E are larger where radius of curvature is smaller, to keep V=constant

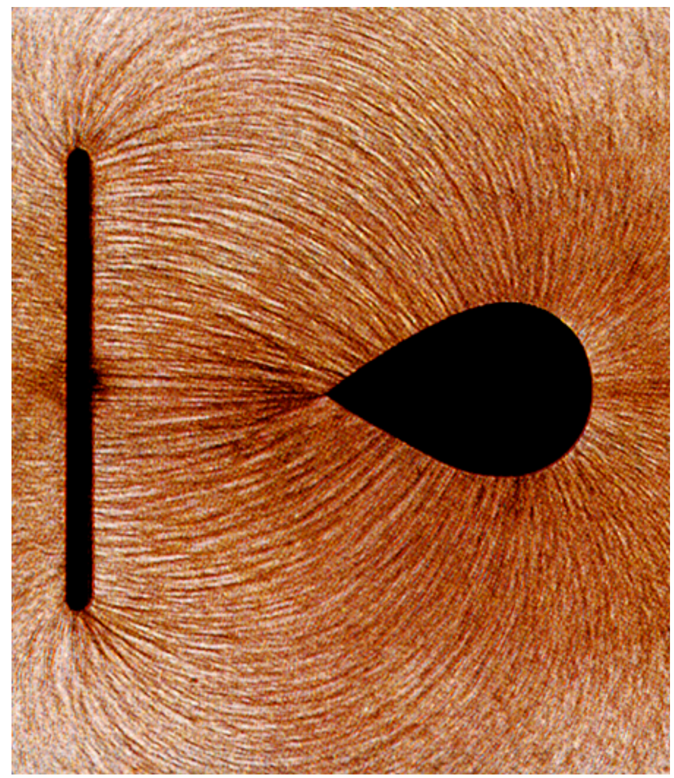


SO: On the surface of a non-spherical conductor, regions with a *small* radius of curvature must have *high* surface E field and charge density (~1/R).

In terms of permittivity instead of k, same equations are

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R} = \frac{1}{4\pi\varepsilon_0}\frac{4\pi R^2\sigma}{R} = \frac{R\sigma}{\varepsilon_0}$$

$$\sigma = \frac{V\varepsilon_0}{R} \qquad E_{\text{surface}} = \frac{\sigma}{\varepsilon_0} = \frac{V}{R}$$



Capacitance, charge and potential

- For parallel plates, Q on plates is proportional to ΔV between them
  - For 2Q, you get 2E, and 2ΔV, for the same Δs
  - Proportionality means...

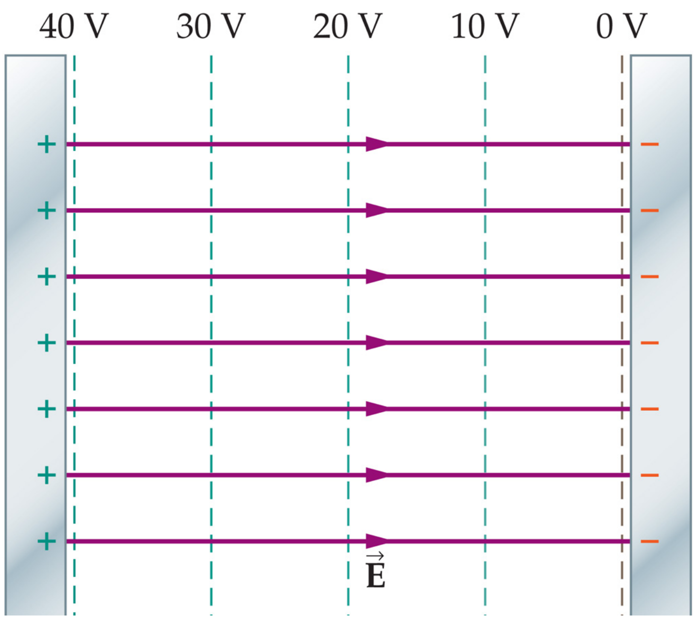
$$Q = CV$$

$$\text{Define capacitance: } C \equiv \frac{Q}{V}$$

$$\text{Units of capacitance: } 1 \text{ farad} = 1 \text{ F} = 1 \text{ C/V}$$

  - Named after Michael Faraday

$$1 \ \mu\text{F} = 10^{-6} \text{ F}; 1 \text{ nF} = 10^{-9} \text{ F}; 1 \text{ pF} = 10^{-12} \text{ F}$$



We can also give k and ε_0 in units of farads -- sometimes handier:

V= J/C = N-m/C, so k (units: N•m²/C²)→ V•m/C = m/F, and ε ~ (1/k)

$$\varepsilon_0 = 8.85 \times 10^{-12} \text{ F/m} = 8.85 \text{ pF/m}; \ k = 8.99 \times 10^{9} \text{ m/F}$$

Quiz 13

- "Equipotentials can never intersect" --   Why?

A. If they intersected, that point in space would have 2 values of V at the same time!
B. Field lines never intersect, and equipotentials are always perpendicular to field lines
C. Neither A nor B are true
D. Both A and B are true

Quiz 13

- "Equipotentials can never intersect"

Why?

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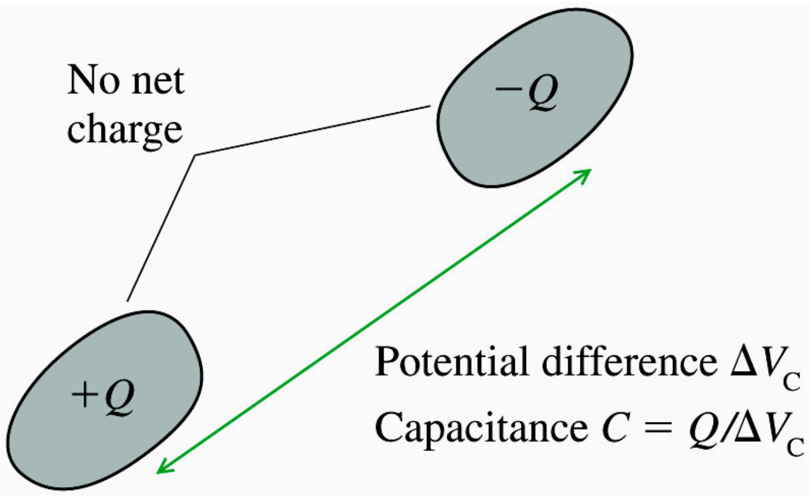
Not just for parallel plates

ANY two conducting electrodes holding equal and opposite Q will form a capacitor, regardless of their shape or arrangement.

$$C = \frac{Q}{\Delta V}$$



Notice: Capacitance depends ***only on the geometry*** of the electrodes, ***not*** on their Q or potential difference at any time.

The same arrangement of electrodes located in a different place in space may have different V and Q, but has the same C.

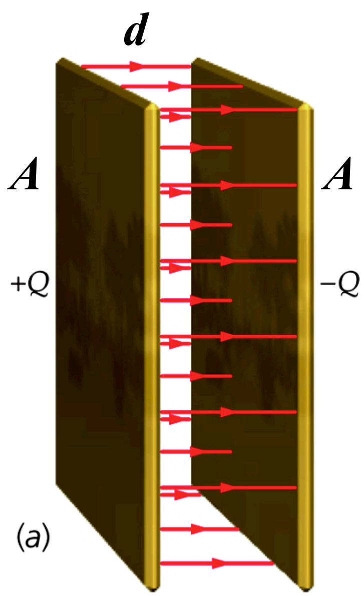
C for Parallel Plate Capacitors

Common example of capacitors:

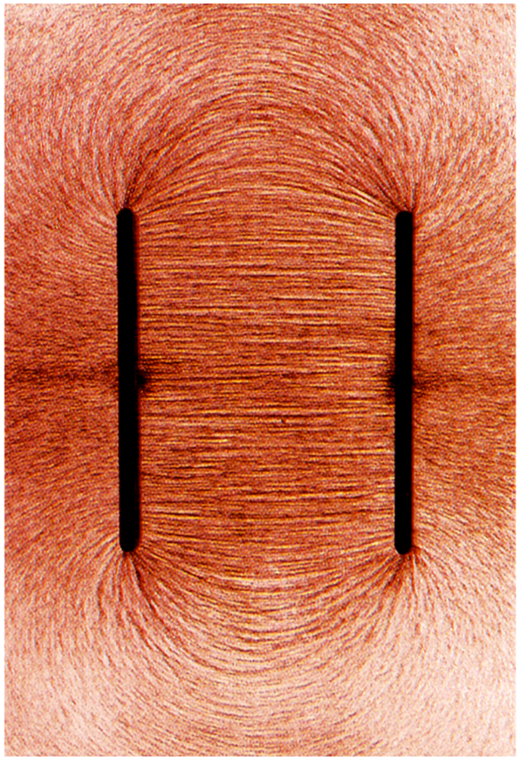
Parallel plate capacitor has equal and opposite charges on two plates of area A separated by a gap of width d.

Notice: E field is uniform, as long as you stay away from the edges

Assume we can "neglect edge effects" and take E to be uniform,



(a)


(b)

Then:

$\Delta V = -E\Delta s$ going in the direction of $\vec{E}$

Then ΔV is negative as you go from + to − electrode

If we take $+V$ = potential of the + electrode

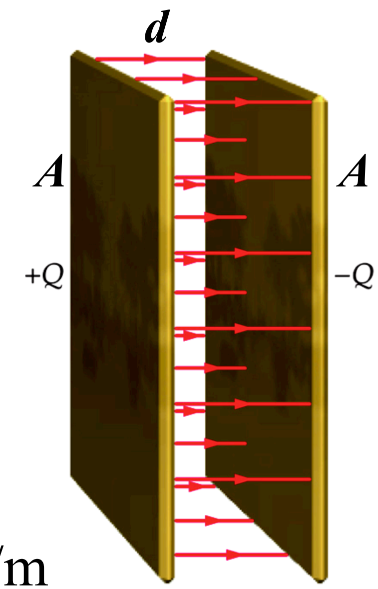
then $\Delta s = -d \rightarrow V = \left(\frac{\sigma}{\varepsilon_0}\right) d = \left(\frac{Q}{A}\right)\frac{d}{\varepsilon_0}$

$$C = \frac{Q}{V} = \frac{Q}{Qd/(\varepsilon_0 A)} = \varepsilon_0 \frac{A}{d}$$

Capacitance ~ Area/spacing

Example: Capacitance of a Parallel Plate Capacitor

A parallel-plate capacitor has square metallic plates of edge length of 10.0 cm separated by 1.0 mm.



(a) Calculate the capacitance of the device.

(b) If the capacitor is "charged up" to ΔV = 12 V, how much charge must be transferred* from one plate to the other?

For capacitance problems, it is handy to express ε_0 in terms of farads:

$$\varepsilon_0 = 8.85 \text{ x } 10^{-12} \text{ C}^2/\left(\text{N}\cdot\text{m}^2\right) = 8.85 \text{ x } 10^{-12} \text{ F/m} = 8.85 \text{ pF/m}$$

$$C = \frac{\varepsilon_0 A}{d} = \frac{(8.85 \text{ pF/m})(0.10 \text{ m})^2}{(0.001 \text{ m})} = 88.5 \text{ pF}$$

$$Q = CV = (88.5 \text{ pF})(12 \text{ V}) = 1062 \text{ pC} = 1.06 \text{ nC}$$

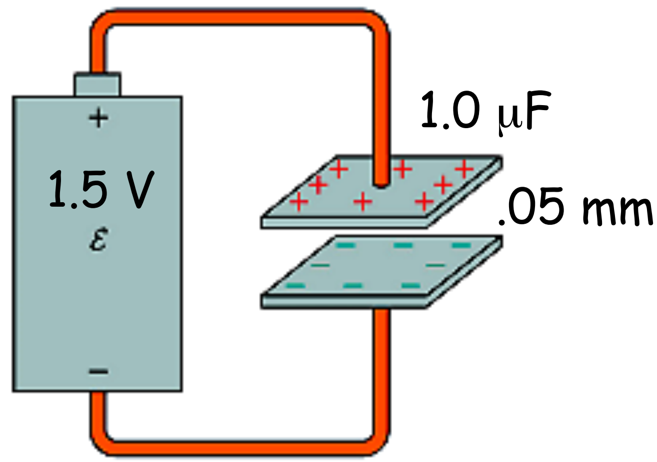
* How's it done? **Work** must be done on the charge to move it – provided by some source of energy (e.g, a battery)

Example: Charging a Capacitor

The spacing between the plates of a 1.0 μF capacitor is 0.05 mm.

(a) What must the surface area A of the plates be?

(b) How much charge is on the plates if this capacitor is attached to a 1.5 V battery?

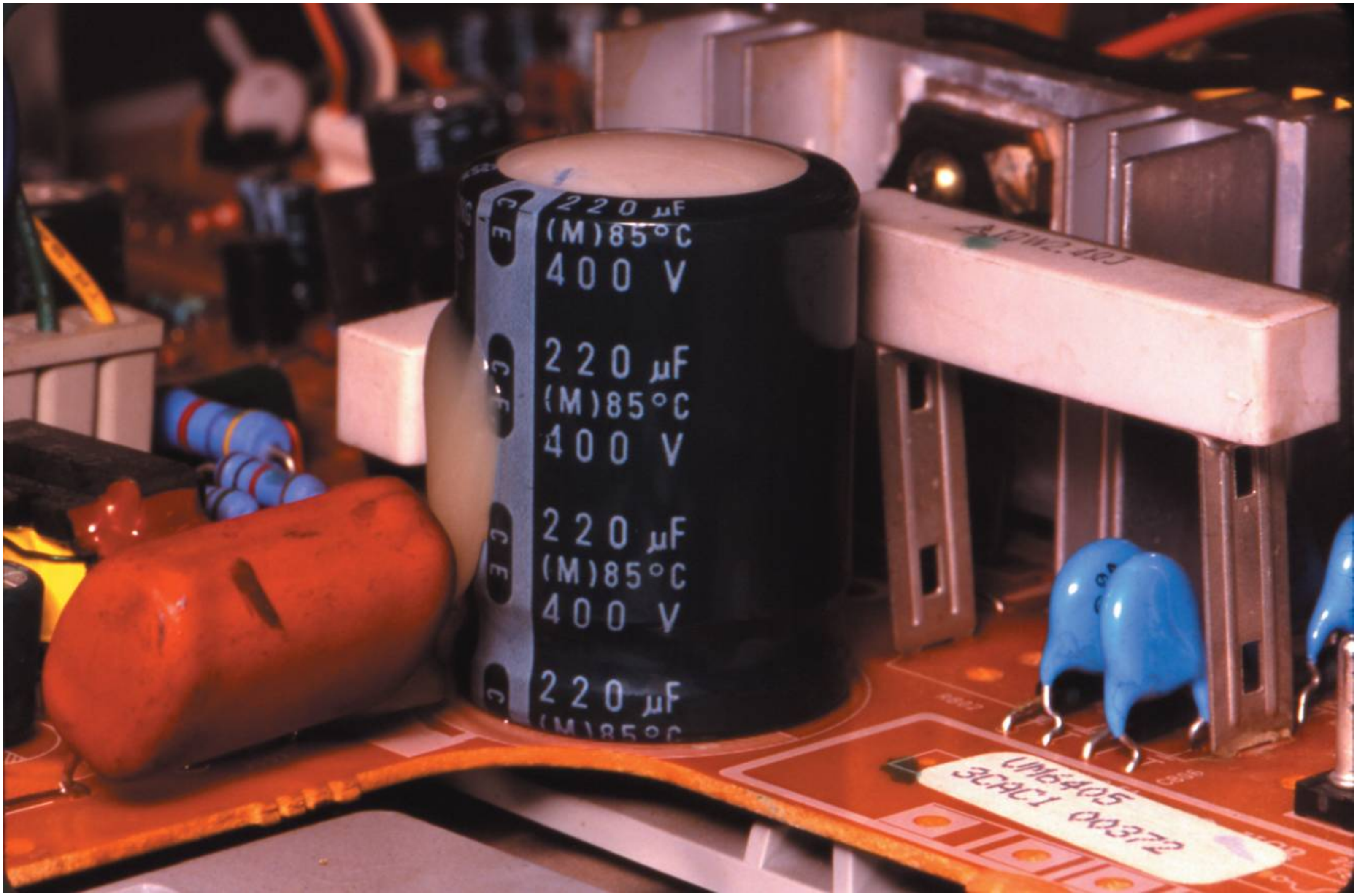


$$A = \frac{dC}{\varepsilon_0} = (5.0\times10^{-5}\text{ m})(1.0\times10^{-6}\text{ F})/(8.85\times10^{-12}\text{ F/m}) = 5.65\text{ m}^2$$

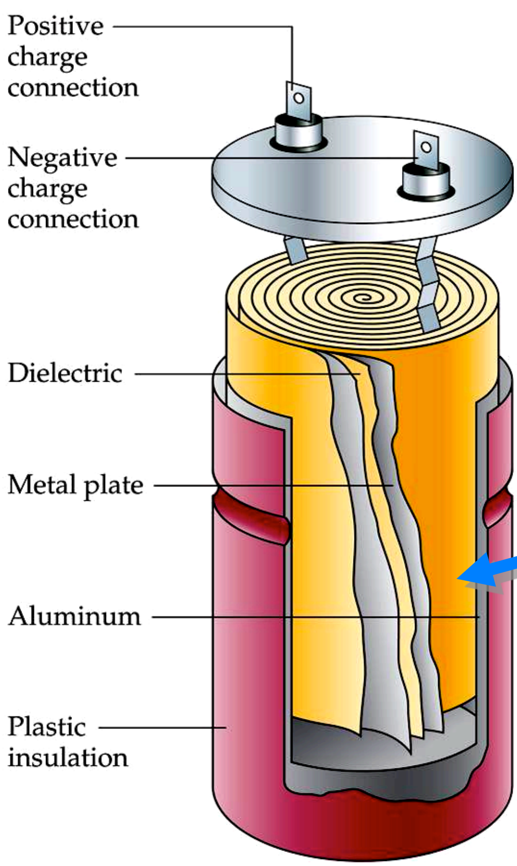
(Large surface area – how's it done in a small package?)

$$Q = C\Delta V_C = (1.0\times10^{-6}\text{ F})(1.5\text{ V}) = 1.5\times10^{-6}\text{ C} = 1.5\ \mu\text{C}$$

Examples of Capacitors in electronic circuits

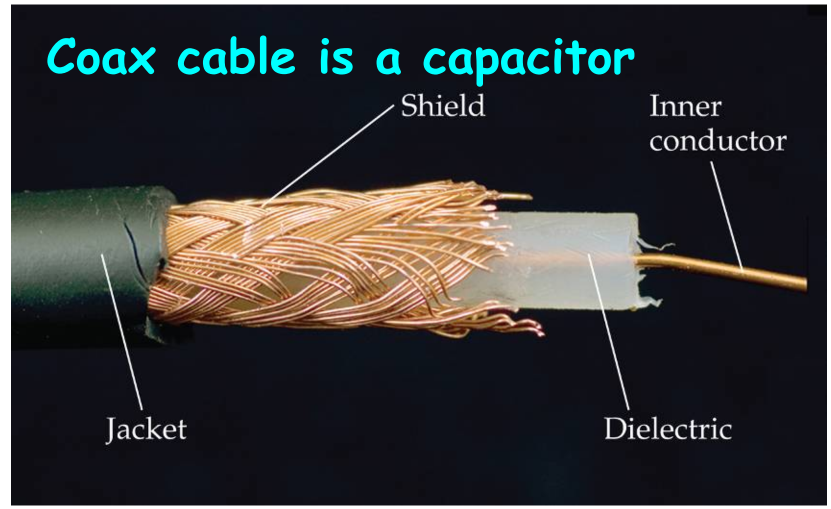


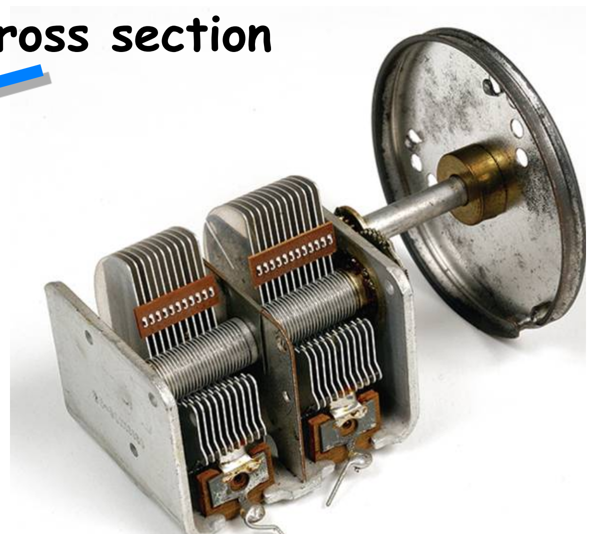
Real Capacitors



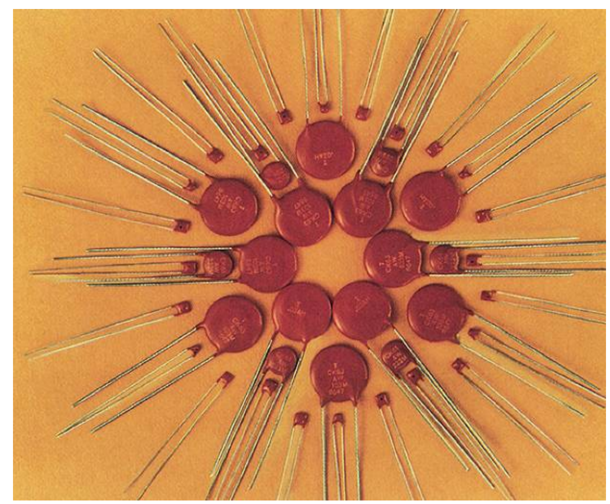
Large-C capacitor

Cross section





Variable capacitor



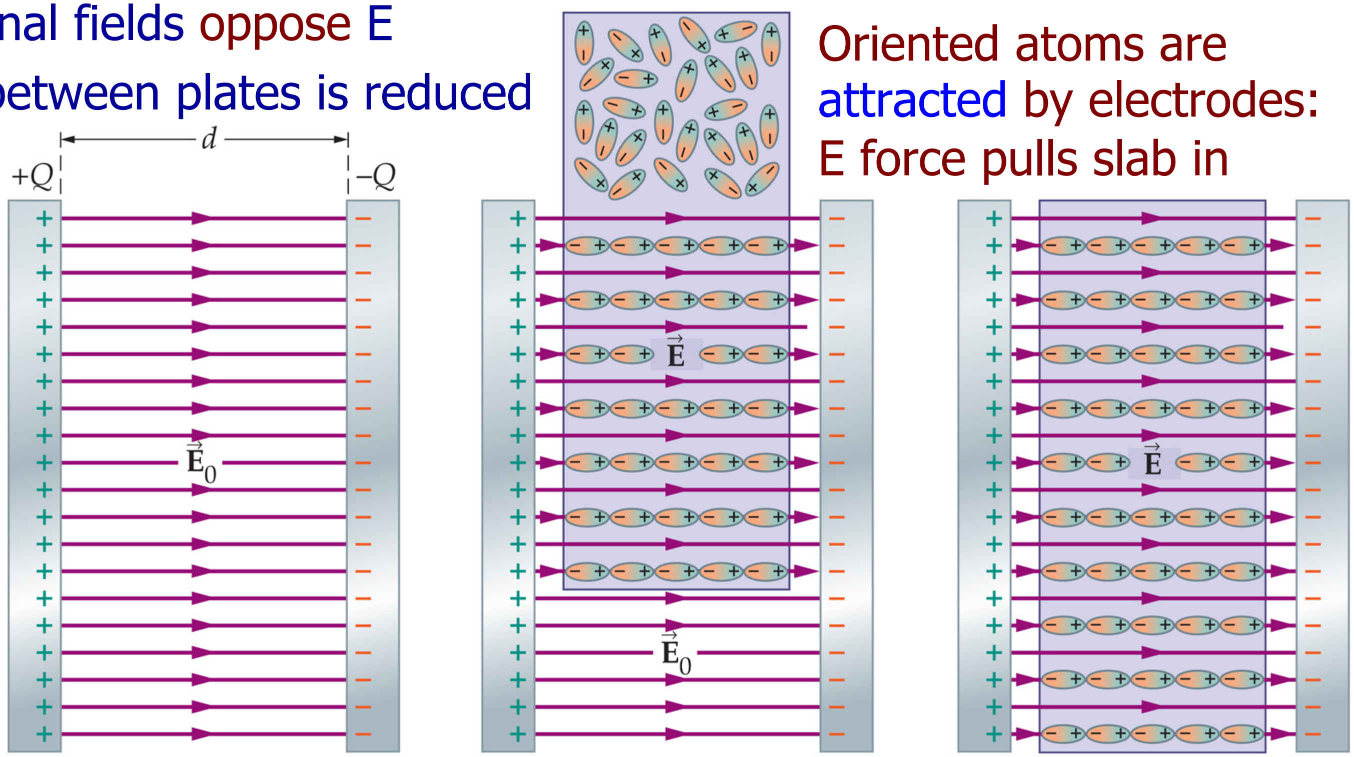
Disk capacitors

Dielectrics in capacitors

- For ideal capacitors we assumed vacuum between plates – or air, which makes very little difference
- However if the insulator between plates is polarizable, there is a big difference
  - E field of separated charges on plates orients the polar molecules
  - Oriented atoms have their – end toward + charged capacitor plate
  - Atoms' internal fields oppose E
  - Effective E between plates is reduced

Oriented atoms are attracted by electrodes: E force pulls slab in

If E between plates is reduced, V = -E Δs → V between plates is also smaller: Then C=Q/V → larger C for same Q on plates



Dielectric constant

- Polarizable materials ("dielectrics") increase C compared to vacuum, for given geometry of capacitor:

$$C_0 = \frac{Q}{V_0}, \quad V_0 = E_0 d, \text{ for vacuum between plates}$$

$$\text{with dielectric, } E_0 \rightarrow E = \frac{E_0}{\kappa}, \quad \kappa = \text{dielectric constant}$$

$$V = Ed = \frac{E_0}{\kappa} d = \frac{V_0}{\kappa} \Rightarrow C = \frac{Q}{V} \rightarrow C = \kappa \frac{Q}{V_0} = \kappa C_0$$

- Notice:
  - For an isolated capacitor (fixed charge already in place), V drops
  - If capacitor is connected to a battery (maintains constant V) the charge will increase to match the increased C

Substance	Dielectric constant, κ
Water	80.4
Neoprene rubber	6.7
Pyrex glass	5.6
Mica	5.4
Paper	3.7
Mylar	3.1
Teflon	2.1
Air	1.00059
Vacuum	1

Dielectric strength: breakdown

- We can't just keep stuffing charge into a capacitor:
  - At some V, the insulator breaks down
  - Breakdown is usually at some very high E field intensity
  - Air can handle ~3 million volts per meter (3000 volts/mm)
  - When you touch a light switch and feel a 1 mm spark:
    - Your body + light switch = capacitor
    - You have accumulated enough Q to make your half of the capacitor ~ 3000V higher potential than ground

Substance	Dielectric strength (V/m)
Mica	100×10^6
Teflon	60×10^6
Paper	16×10^6
Pyrex glass	14×10^6
Neoprene rubber	12×10^6
Air	3.0×10^6

Energy Storage in a Capacitor

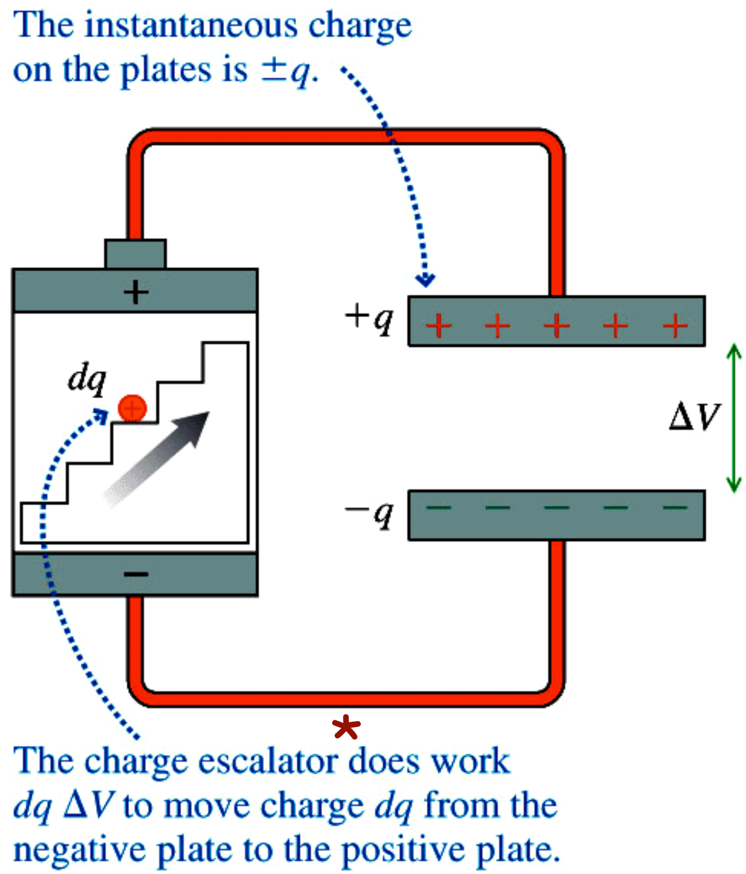
In capacitors, charge is stored on electrodes with potential difference ΔV. It takes work to move charge against the E field represented by ΔV !

The first bit of charge is easy to move: for an uncharged capacitor, V=0

Thereafter each bit of charge takes more work: V grows linearly with total Q on the capacitor, since V=Q/C.

The stored charge represents the work done, in potential energy: U = Q ΔV

Using calculus we find the total work done is $W_{TOTAL} = U_C = \frac{1}{2} Q \Delta V_C$



$$C = \frac{Q}{\Delta V_C} \rightarrow U_C = \frac{1}{2} C \Delta V_C^2 = \frac{Q^2}{2C}$$

Or, without calculus: since V grows linearly with total Q, average V $= \frac{1}{2}$ Q/C, so total W = $QV_{AVG} = \frac{1}{2} Q^2 / C$

* What's this "charge escalator"? A source of energy (e.g, a battery) that "lifts" charge through the potential difference (against E force)