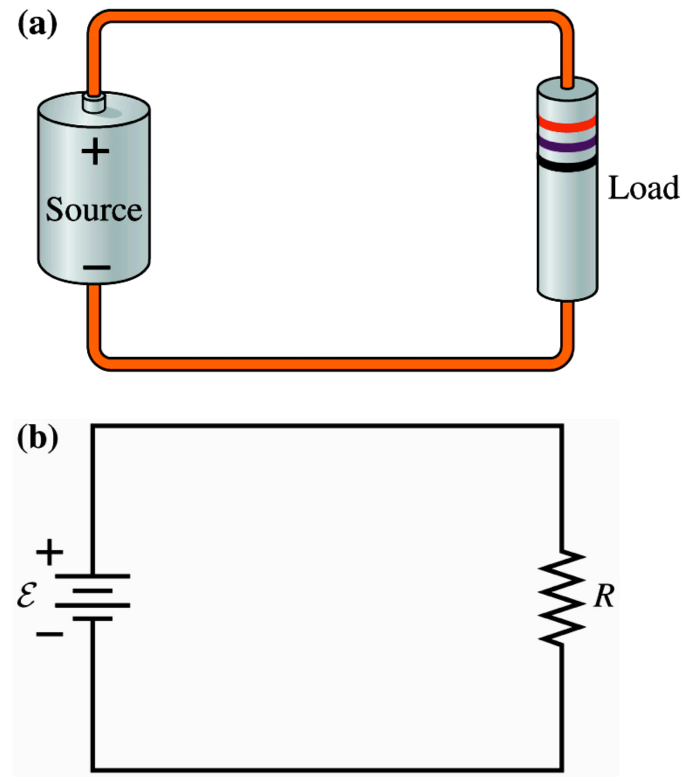
Physics 115
General Physics II



Session 24
Circuits
Series and parallel R
Meters
Kirchoff's Rules

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Lecture Schedule

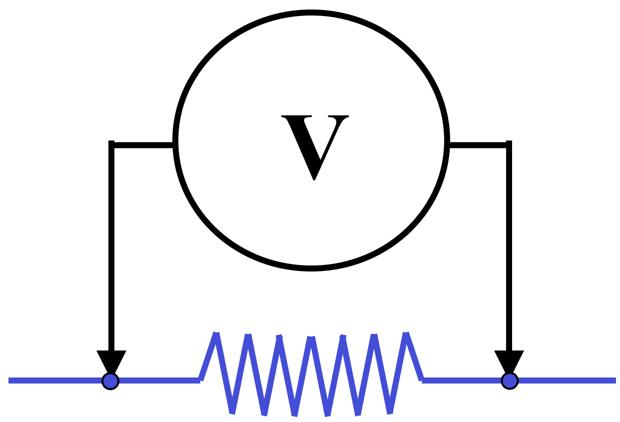
12-May	Mon	23	DC Circuits & Meters	21.5-21.8
13-May	Tues	24	DC Circuits	21.5-21.8
15-May	Thurs	25	RC circuits	21.6-21.7
16-May	Fri	26	Circuits - Neurons	
19-May	Mon	27	Magnetism	22.1
20-May	Tues	28	Magnetic Force	22.2-22.5
22-May	Thurs	29	Magnetic Fields	22.6-22.7
22-May	Fri	30	Induced EMF, Applications	23.1-23.3
26-May	***holiday***		NO CLASS	
27-May	Tues	31	Energy, RL circuits	23.4-23.8
29-May	Thurs	32	Transformer	23.9-23.10
30-May	**Fri**		**EXAM 3 - Chapters 21,22,23**	
2-Jun	Mon	33	AC circuits	24.1-24.3
3-Jun	Tues	34	AC circuits	24.4-24.5
5-Jun	Thurs	35	Resonance, Applications	24.6
6-Jun	Fri	36	Last class - review	
June 9	**FINAL EXAM**		**2:30-4:20 p.m.**	**Comprehensive**
	Mon		**Monday, June 9, 2014**	

Today

Voltmeters vs. Ammeters

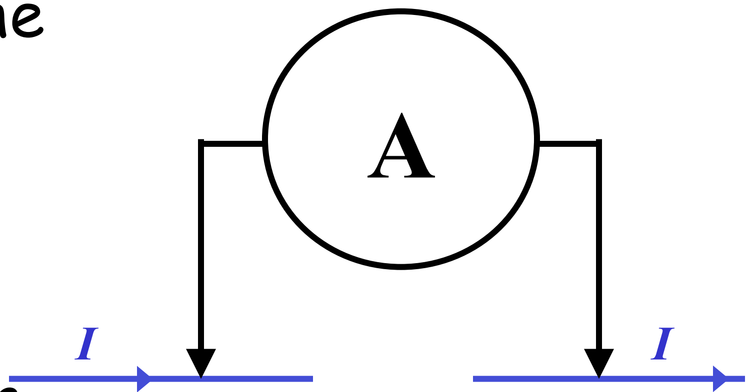
A voltmeter is connected **across** circuit elements to measure the potential difference between two points in the circuit.

An ideal voltmeter has ***infinite*** internal resistance, so it draws no current from the circuit.



An ammeter is **inserted** by breaking a circuit connection, to measure the current flowing through that connection in the circuit.

An ideal ammeter has ***zero*** internal resistance, so it does not affect the current passing through it.



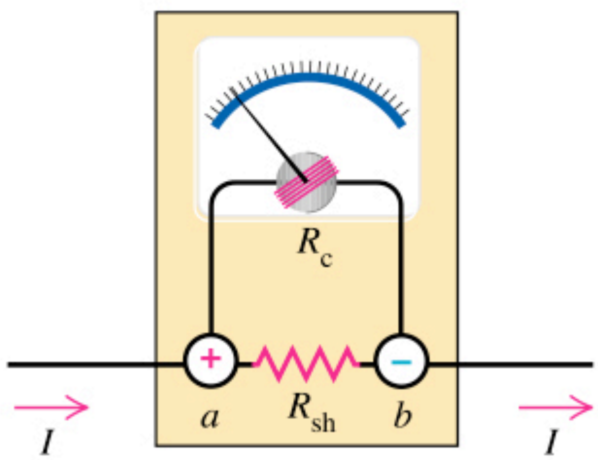
What's inside ammeter, voltmeter

Galvanometer deflects in proportion to current through it

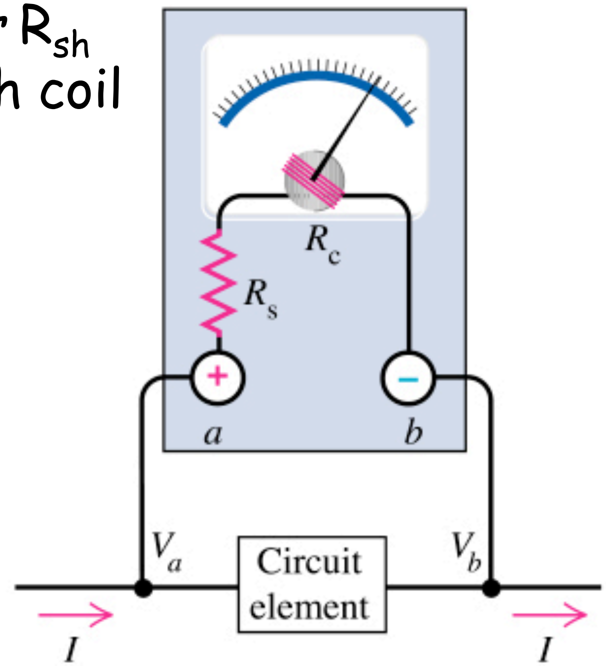
- Assume its coil has negligible resistance, $R_c \sim 0$

1. Ammeter:

- Voltage drop across *"shunt" resistor* R_{sh}
  $V_{sh}=I\,R_{sh}$ determines current through coil
- Want R_{sh} as small as possible



(a)



(b)

2. Voltmeter:

- Voltage drop $V_a - V_b$ determines current through R_S and coil
- Want *series resistance* R_S as large as possible

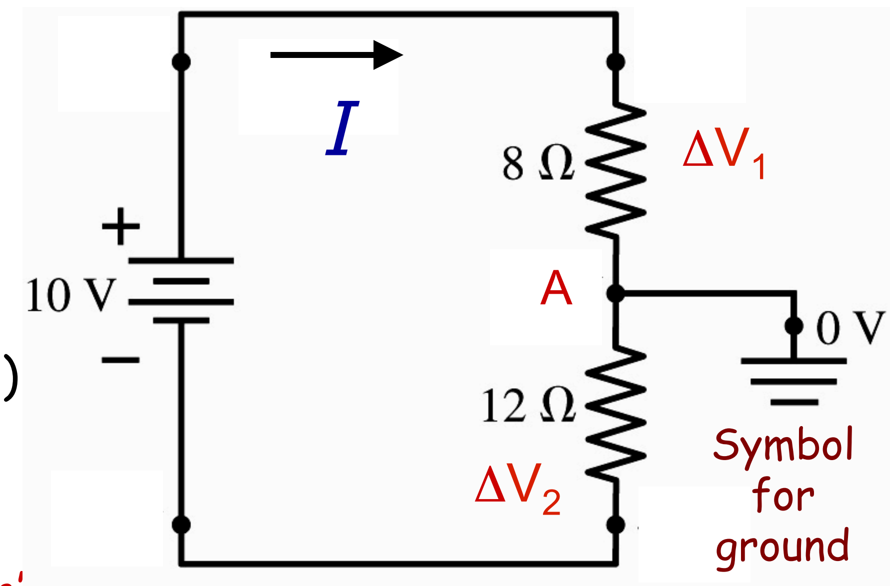
More about resistor circuits: ground points

Earth = infinite charge reservoir
Ground wire = connected to the earth
Ground = zero potential for circuits

This circuit is grounded at the junction between the two resistors. This becomes the "zero" for the V scale, rather than the negative terminal of the battery. (With no ground, we say the circuit is "floating")

Find the potential difference ΔV across each resistor:

Example: A Grounded Circuit



$$I = \frac{\mathcal{E}}{R} = \frac{10\ \text{V}}{8\ \Omega + 12\ \Omega} = 0.5\ \text{A}$$ Ohm's Law

$$\mathcal{E} + \Delta V_1 + \Delta V_2 = 0$$ Cons. of energy

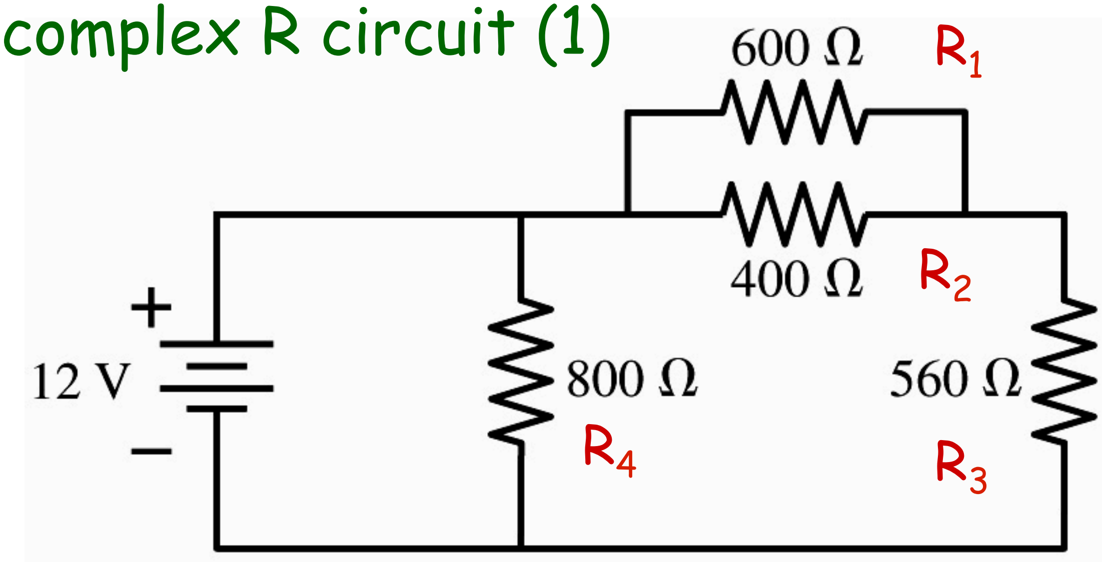
$$\Delta V_1 = (8\ \Omega)(0.5\ \text{A}) = 4\ \text{V}$$

$$\Delta V_2 = (12\ \Omega)(0.5\ \text{A}) = 6\ \text{V}$$

Notice: no current flows through the ground wire: no ΔV across it
(Point A is +6V relative to battery's – terminal, but 0V relative to ground)
Choice of zero doesn't matter: only ΔV's have physical meaning

Example:
Analyzing a more complex R circuit (1)

Four resistors are connected to a 2 V battery as shown. Find the current through the battery.



Step 1: combine parallel resistors into R_{eq1} :

$$\frac{1}{R_{eq1}} = \frac{1}{R_1} + \frac{1}{R_2}$$

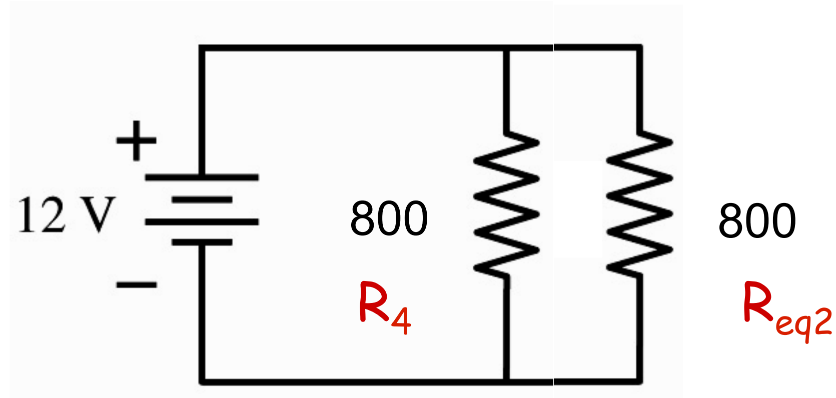
$$R_{eq1} = \frac{1}{1/R_1 + 1/R_2} = \frac{R_1 R_2}{R_1 + R_2} = \frac{600\Omega(400\Omega)}{(600+400)\Omega} = 240\Omega$$

Step 2: combine series resistances R_{eq1} and R_3 into an equivalent R:

$$R_{eq2} = R_{eq1} + R_3 = 240\Omega + 560\Omega = 800\Omega$$

Analyzing a more complex circuit (2)

Now we have this equivalent circuit:



Step 3: combine these parallel resistors into one R_{eq} :

$$R_{eq3} = \frac{R_4 R_{eq2}}{R_4 + R_{eq2}} = \frac{800\Omega(800\Omega)}{(800+800)\Omega} = 400\Omega$$

$$I_{battery} = \mathcal{E} / R_{eq3} = 12V / 400\Omega = 0.03A$$

How to analyze even more complex circuits

To deal with greater complexity, we can use physical laws to organize our work:

As usual: first, some definitions
Junction = place where wires connect
Closed loop = part of a circuit that forms a single closed path for current

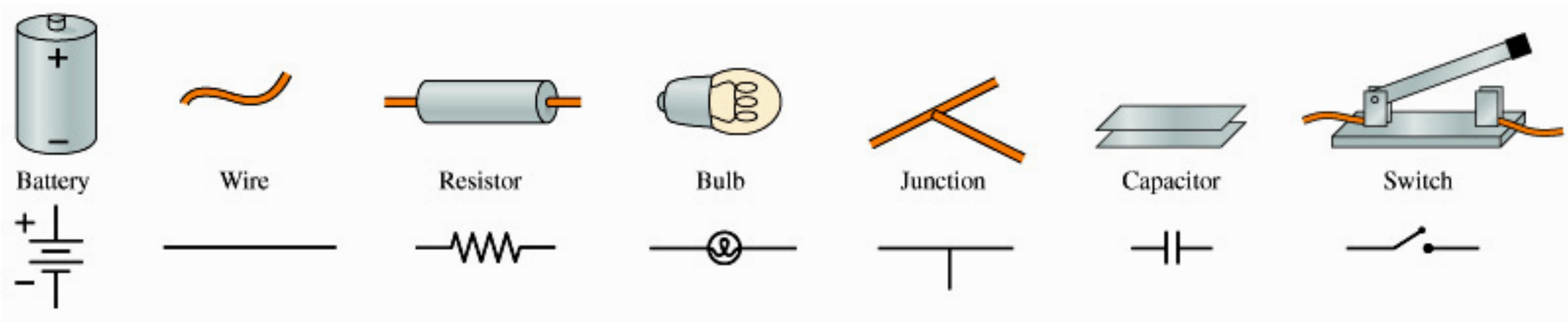
Then

1. Conservation of matter (charge): for any junction in a circuit, current in must equal current out: net current = 0
2. Conservation of energy: for any closed loop in a circuit, the sum of potential differences must be 0: net ΔV around the loop = 0

= Kirchoff's rules for electric circuits

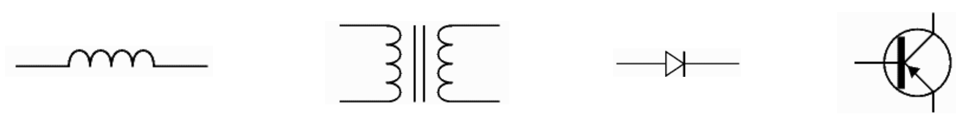
Gustav Kirchoff, 1824-1887

About Circuit Elements & Diagrams



These are some of the symbols commonly used to represent components in circuit diagrams.

Other components (coming soon) and their symbols:
inductance, transformer, diode, transistor

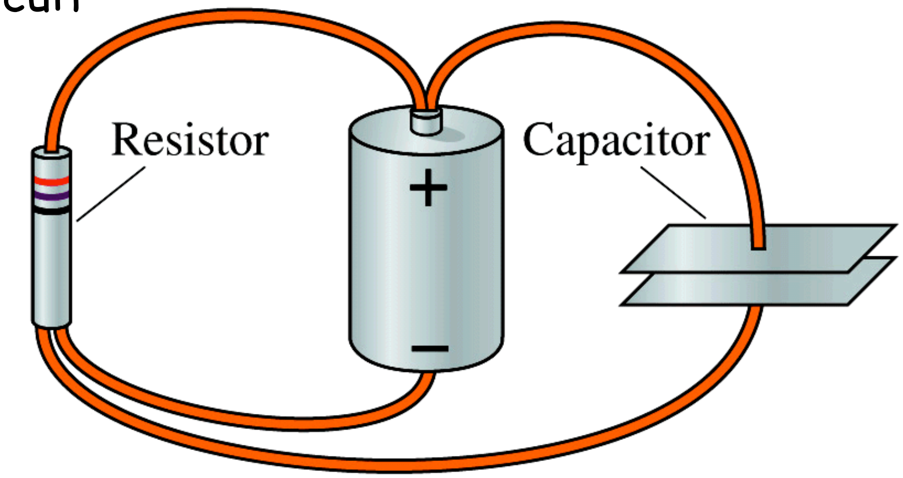


Circuit Diagrams

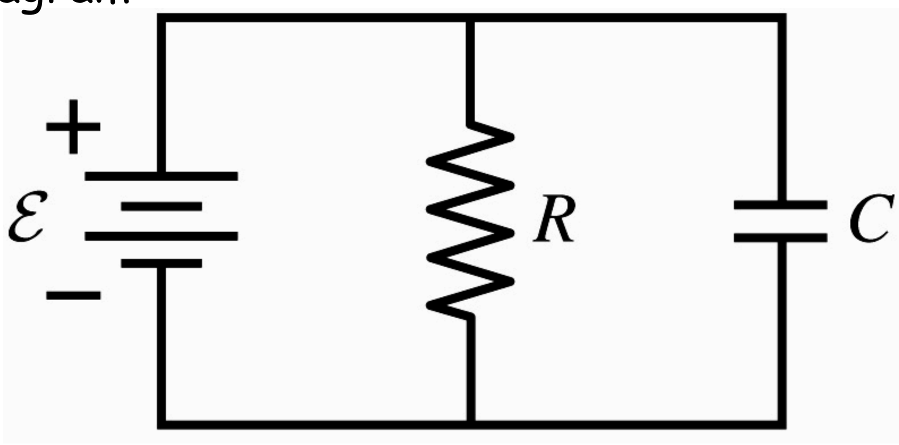
In discussing circuits, we will make the following assumptions:

1. Wires have very small resistance, so that we can take $R_{wire}=0$ and $\Delta V_{wire}=0$ in circuits. All wire connections are ideal.
2. Resistors have constant resistance values, regardless of current
3. Insulators are ideal non-conductors, with $R=\infty$ and $I=0$ through the insulator.

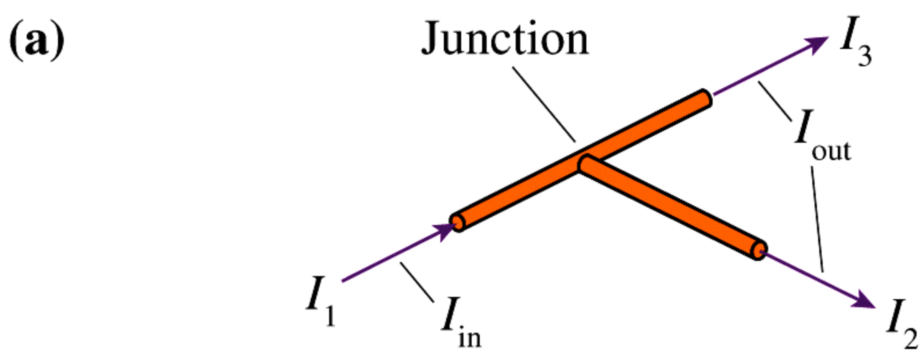
Actual Circuit



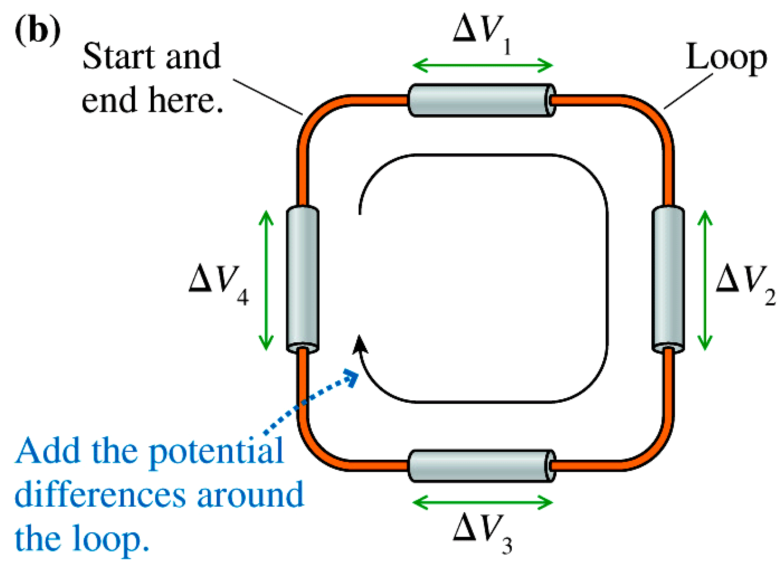
Circuit Diagram



Kirchhoff's Rules for Circuits



Junction law: $I_1 = I_2 + I_3$ -> $I_{in} = I_{out}$



Loop law: $\Delta V_1 + \Delta V_2 + \Delta V_3 + \Delta V_4 = 0$

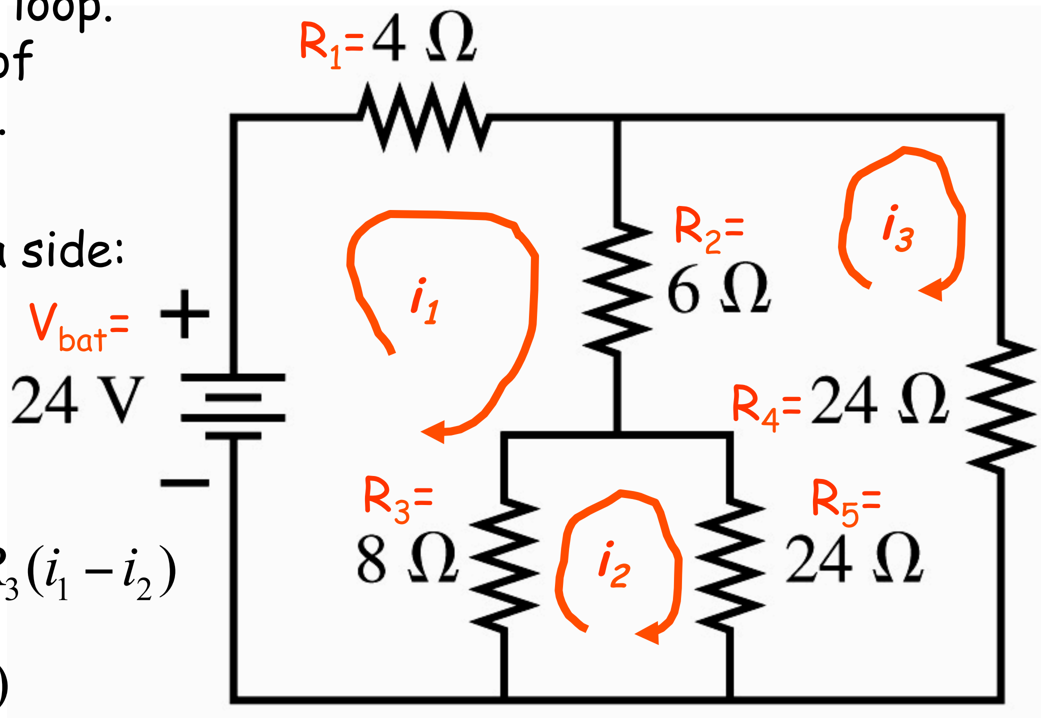
Kirchoff:

1. for any junction in a circuit, current in must equal current out: net current = 0
2. for any closed loop in a circuit, the sum of potential differences must be 0: net ΔV around the loop = 0
   If there is a battery (source of EMF) in the circuit,

$$\Delta V_{loop} = \sum_i (\Delta V)_i = 0 = +\Delta V_{bat} - \sum \Delta V_R$$

Kirchhoff's Laws for Multi-loop Circuits

1. Redraw to make a minimum set of current loops. Label all elements.
2. Write a loop equation for each loop. Take into account orientation of sources : add V's algebraically.
3. Take into account direction of currents where 2 loops share a side: add currents algebraically
4. Solve equations for currents*



Loop equations:

$$\text{Loop 1: } V_{bat} = R_1 i_1 + R_2(i_1 - i_3) + R_3(i_1 - i_2)$$

$$\text{Loop 2: } 0 = R_3(i_2 - i_1) + R_5(i_2 - i_3)$$

$$\text{Loop 3: } 0 = R_2(i_3 - i_1) + R_4 i_3 + R_5(i_3 - i_2)$$

*(If any current comes out *negative*, that means it actually goes opposite to the direction you assumed)

Solving the example circuit:

1. Solve loop equations for currents*

$$24V = 4\Omega \cdot i_1 + 6\Omega \cdot (i_1 - i_3) + 8\Omega \cdot (i_1 - i_2)$$

$$0 = 8(i_2 - i_1) + 24(i_2 - i_3)$$

$$0 = 6(i_3 - i_1) + 24 i_3 + 24(i_3 - i_2)$$

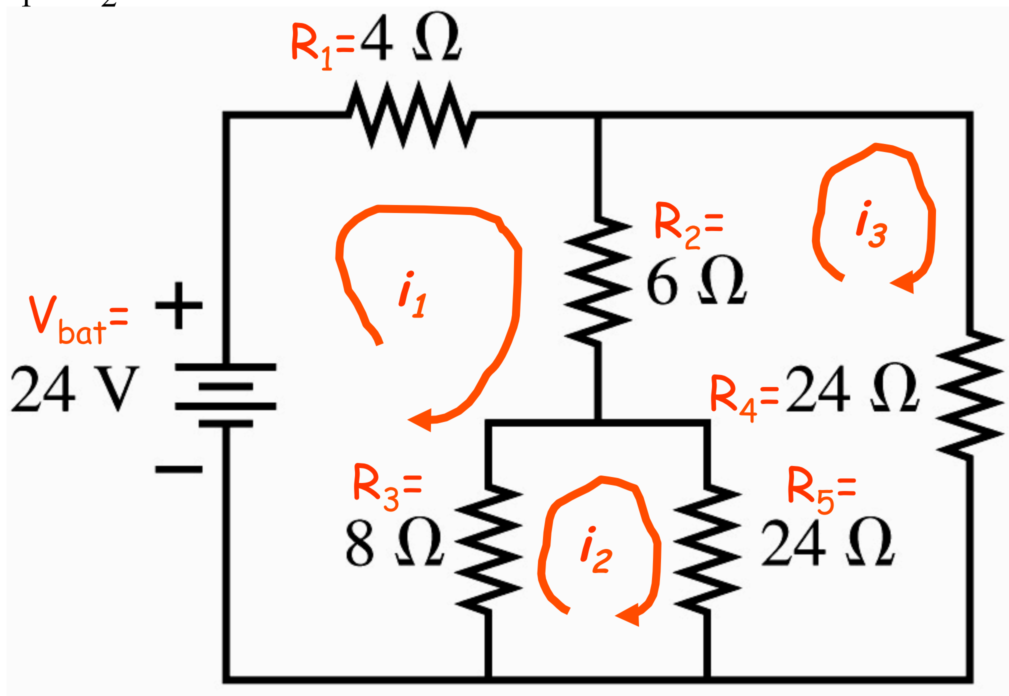
$$8i_1 + 24i_3 = 32i_2 \rightarrow i_2 = \frac{1}{4}i_1 + \frac{3}{4}i_3$$

$$12i_1 = 36i_3 \rightarrow i_3 = \frac{1}{3}i_1 , \quad i_2 = \frac{1}{2}i_1$$

$$24 = 4i_1 + 6(\frac{2}{3}i_1) + 8(\frac{1}{2}i_1) = 12i_1$$

$$i_1 = 2A, \quad i_2 = 1A, \quad i_3 = \frac{2}{3}A$$

Exercise: check that these satisfy the loop equations

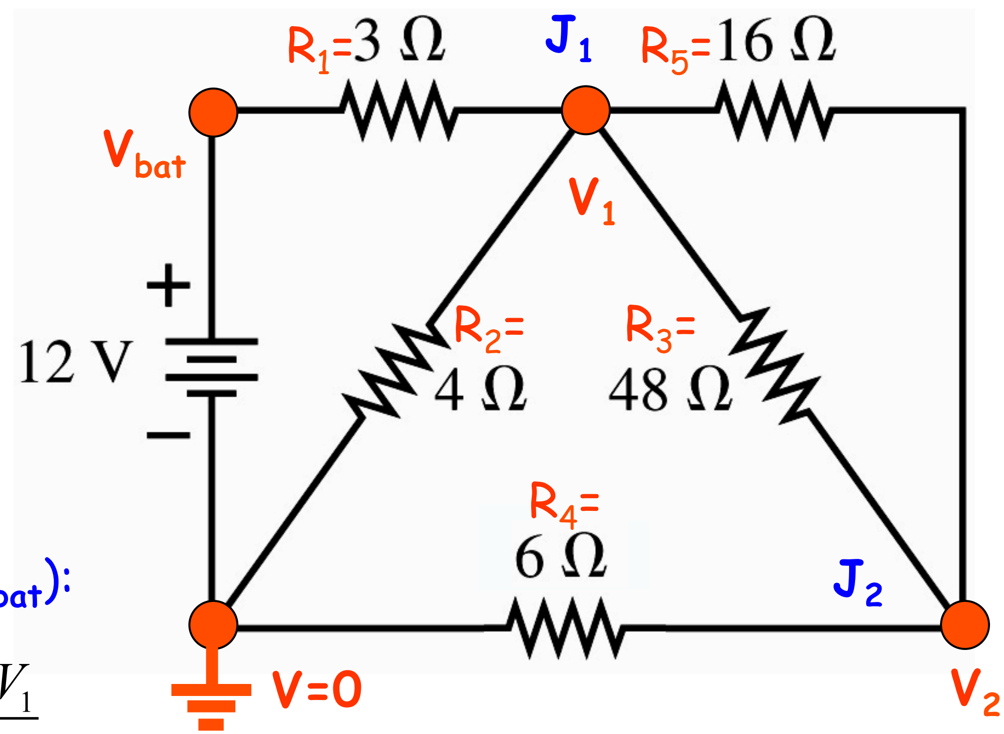


*(If any current comes out *negative*, that means it actually goes opposite to the direction you assumed)
HERE: all currents turned out as shown

Applying Kirchhoff's Junction Law to Multi-Junction circuit

Alternatively, solve a set of junction equations: same number as loop eqns

1. Define a minimum set of junction potentials. You can choose one ground point, **defining** it as 0V. Label all elements.
2. Write a junction equation for each unknown junction: $I_{net} = 0$
3. Solve these equations for the unknown junction potentials.



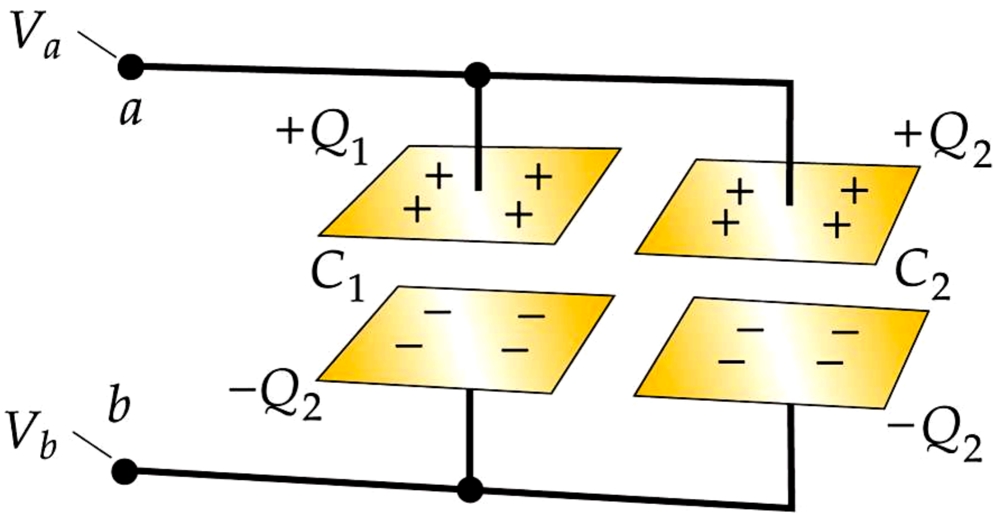
Junction equations
(in this example, we already know V_{bat}):

$$\text{J1:}\quad 0 = \frac{V_{bat} - V_1}{R_1} + \frac{0 - V_1}{R_2} + \frac{V_2 - V_1}{R_3} + \frac{V_2 - V_1}{R_5}$$

$$\text{J2:}\quad 0 = \frac{0 - V_2}{R_4} + \frac{V_1 - V_2}{R_3} + \frac{V_1 - V_2}{R_5}$$

$$\text{For each junction: } \sum_i \frac{V_{i,away} - V_J}{R_i} = 0$$

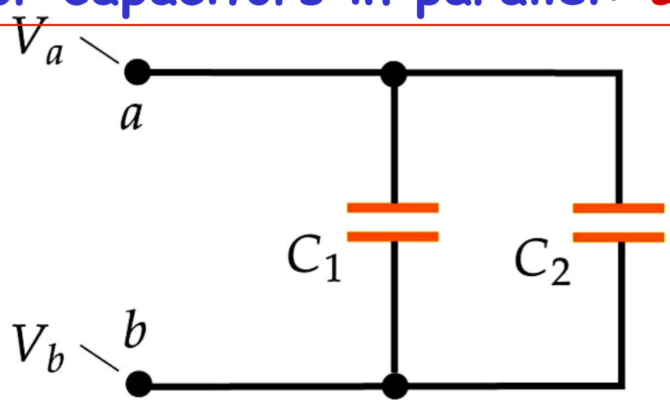
Circuits with Capacitors: C's in Parallel

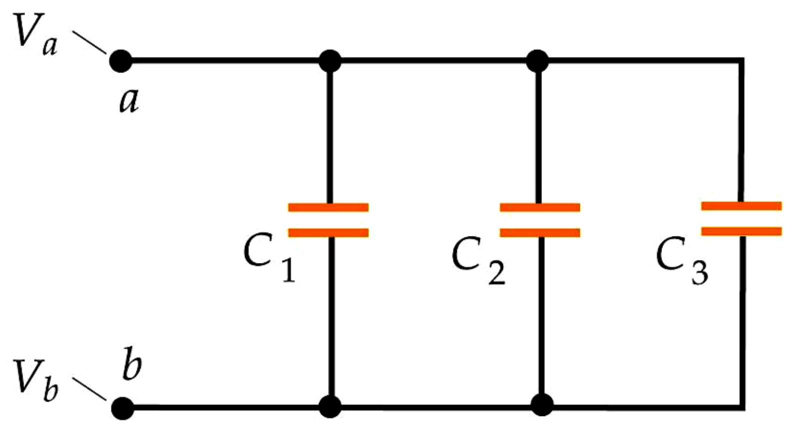


The connected plates are at the same potential, so they form one effective capacitor, with a plate area that is the sum of the two plate areas.

Since $C \sim A$, $C_{eff} = C_1 + C_2$,

For capacitors in parallel: *add* capacitances (like resistors in series).



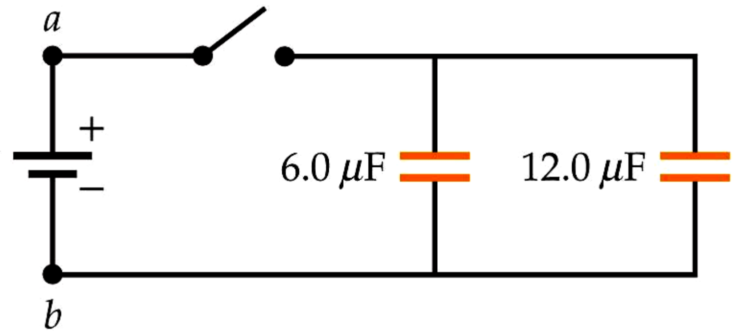


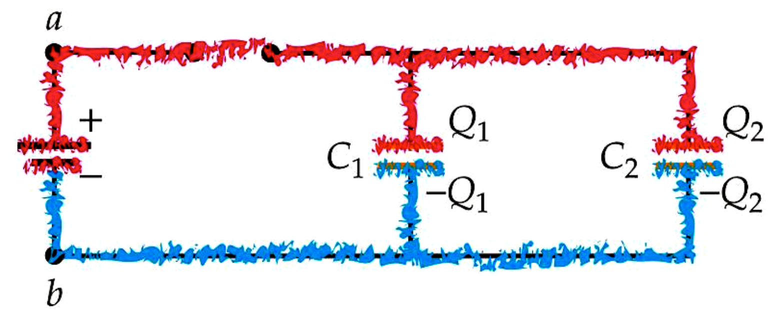
$$C_{eq} = C_1 + C_2 + C_3$$

Example: Capacitors in Parallel

A 6.0 μF and a 12.0 μF capacitor are in parallel (terminals joined), and in series with (terminals end-to-end) a 12 V battery and a switch. Initially, the switch is open and the capacitors are uncharged.
The switch is then closed.

When the capacitors are fully charged
(a) What is voltage across each capacitor in the circuit?
(b) What is the amount of charge on each capacitor plate?
(c) What total charge has passed through the battery?





d) What is effective net capacitance "seen" by the battery?

$V_a = 12\text{ V};\quad V_b = 0\text{ V}.$ Both C's have same ΔV.

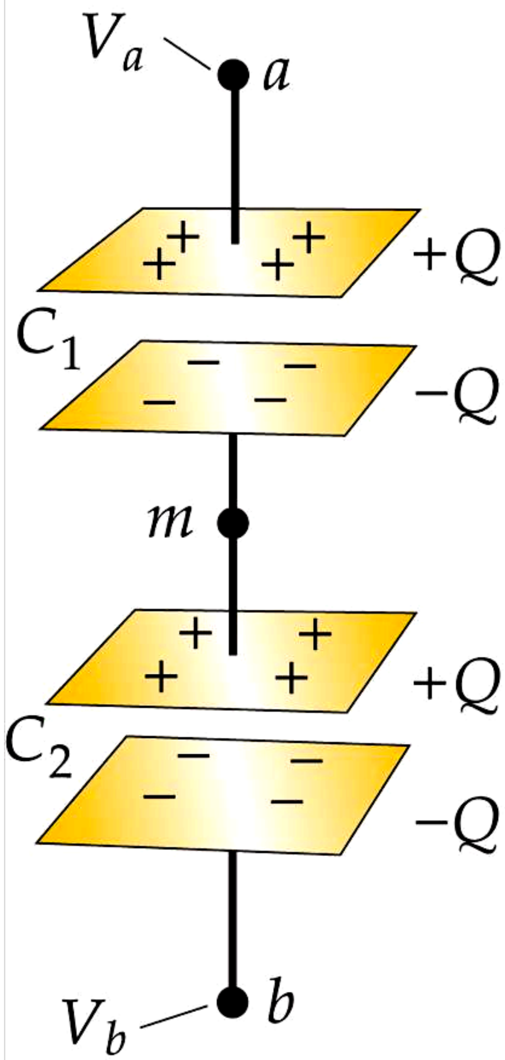
$$Q_1 = C_1 V_a = (6.0\times10^{-6}\text{ F})(12\text{ V}) = 72\times10^{-6}\text{ C} = 72\ \mu\text{C}$$

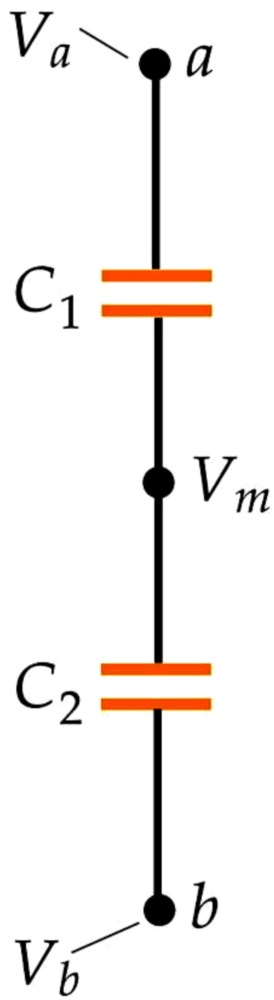
$$Q_2 = C_2 V_a = (12.0\times10^{-6}\text{ F})(12\text{ V}) = 144\times10^{-6}\text{ C} = 144\ \mu\text{C}$$

$$Q_{\text{total}} = Q_1 + Q_2 = (72\ \mu\text{C}) + (144\ \mu\text{C}) = 216\ \mu\text{C}$$

$$C_{eff} = Q_{\text{total}} / V_a = (216\ \mu\text{C})/(12\text{ V}) = 18\ \mu\text{F}$$

Capacitors in Series





The connected string of capacitors all have the same charge **Q** on their plates, and the voltage drops across the invididual capacitors add.

$$Q = C_1 V_1 = C_2 V_2 = C_{eff}(V_1 + V_2)$$

$$V_2 = V_1(C_1 / C_2)$$

$$C_{eff} = \frac{C_1 V_1}{V_1 + V_2} = \frac{C_1 V_1}{V_1\left[1 + (C_1 / C_2)\right]}$$

$$= \frac{C_1 C_2}{C_1 + C_2} = \frac{1}{1/C_1 + 1/C_2}$$

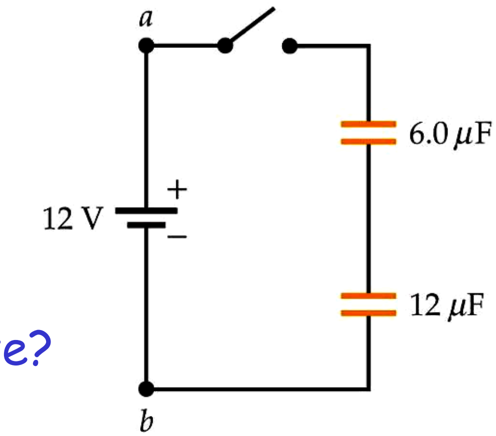
Capacitors in series: *add inversely* (like parallel resistors).

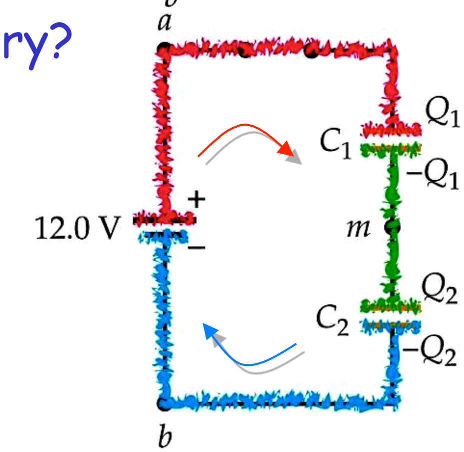
Example: Capacitors in Series

Now the 6.0 μF and 12.0 μF capacitor, 12 V battery and switch are all in series. Initially, the switch is open and the capacitors are uncharged. The switch is then closed.

When the capacitors are fully charged

(a) What is voltage across each device in the circuit?
(b) What is the amount of charge on each capacitor plate?
(c) What total charge has passed through the battery?
(d) What is the effective capacitance across the battery?





$$V_a = 12\text{ V};\quad V_b = 0\text{ V};\quad V_m = ?\qquad V_1 = V_a - V_m;\quad V_2 = V_m - V_b.$$

$$Q_1 = C_1 V_1 = C_1\left(V_a - V_m\right)\qquad Q_2 = C_2 V_2 = C_2\left(V_m - V_b\right)$$

$$Q_1 = Q_2\quad (why\,?)\quad \rightarrow\quad C_1\left(V_a - V_m\right) = C_2\left(V_m - V_b\right)$$

$$V_m = \frac{C_1}{C_1 + C_2}V_a + \frac{C_2}{C_1 + C_2}V_b = \frac{1}{3}\left(12\text{ V}\right) + \frac{2}{3}\left(0\text{ V}\right) = 4\text{ V}$$

$$\mathrm{Q}_1 = Q_2 = Q_{total} = C_1\left(V_a - V_m\right) = (6.0\ \mu\text{F})\left[(12\text{ V}) - (4\text{ V})\right] = 48\ \mu\text{C}$$

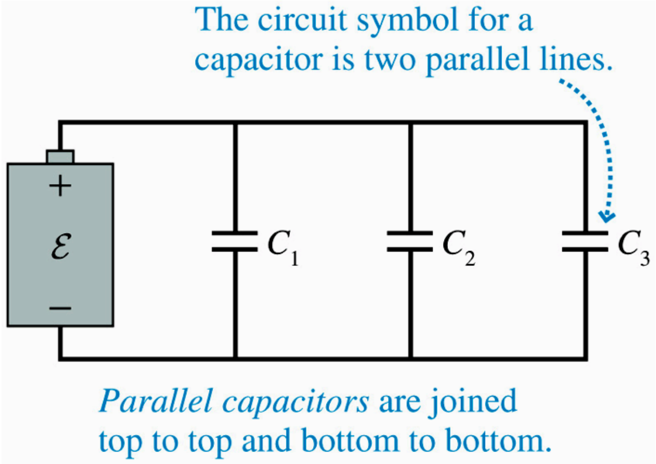
$$C_{eff} = Q_{\text{total}}/(V_a - V_b) = (48\ \mu\text{C})/(12\text{ V}) = 4\ \mu\text{F}$$

Summary: Combining Capacitors

Parallel: Same ΔV, but different Qs.

$$C_{\text{parallel}} = \frac{Q}{\Delta V_{\text{C}}} = \frac{Q_1 + Q_2 + Q_3 + \cdots}{\Delta V_{\text{C}}}$$

$$= C_1 + C_2 + C_3 + \cdots$$

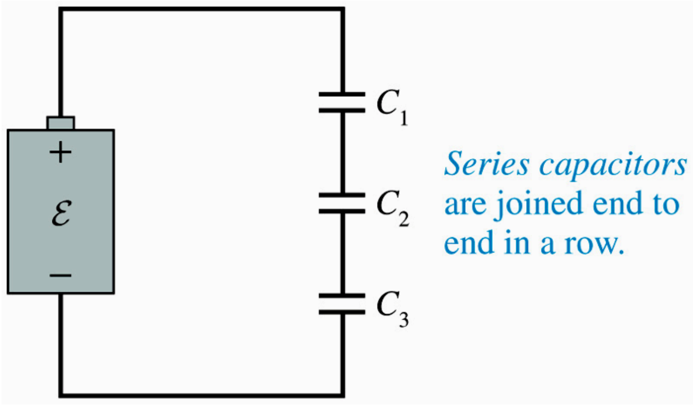


Series: Same Q, but different ΔVs.

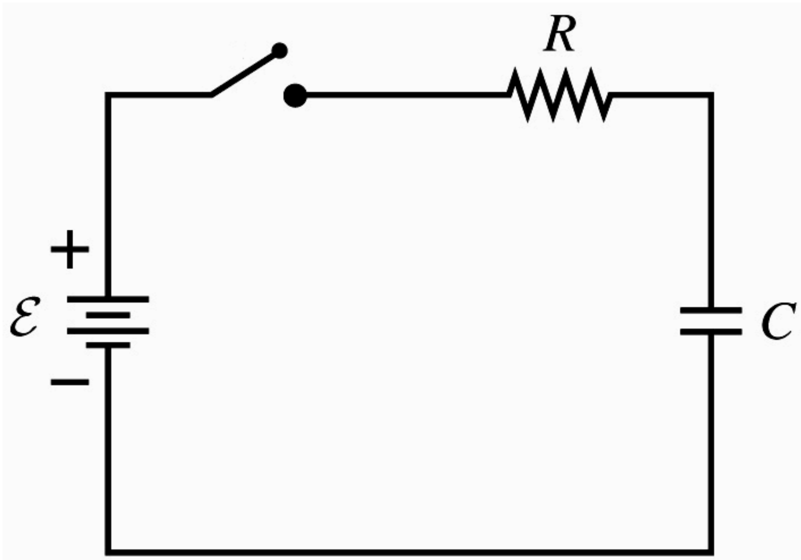
$$C_{\text{series}} = \frac{Q}{\Delta V_{\text{C}}} = \frac{Q}{\Delta V_1 + \Delta V_2 + \Delta V_3 + \cdots}$$

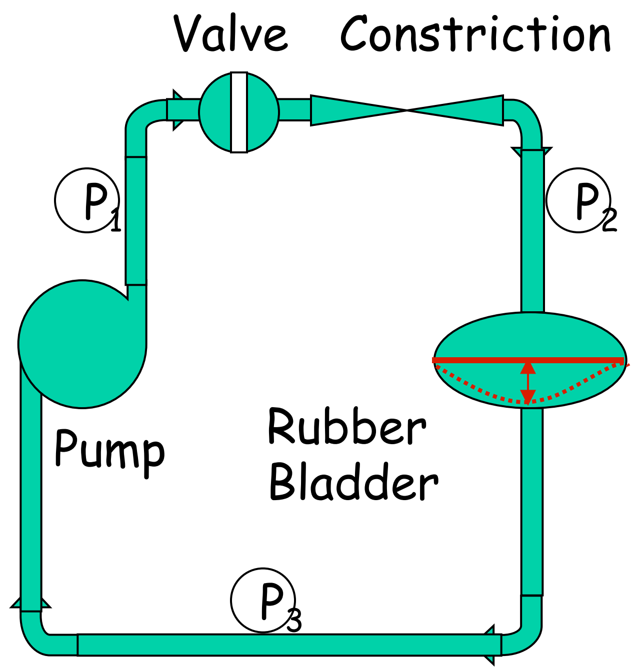
$$= \frac{1}{(\Delta V_1 / Q) + (\Delta V_2 / Q) + (\Delta V_3 / Q) + \cdots}$$

$$= \frac{1}{1/C_1 + 1/C_2 + 1/C_3 + \cdots}$$



"RC circuit" : battery, R, and C in series



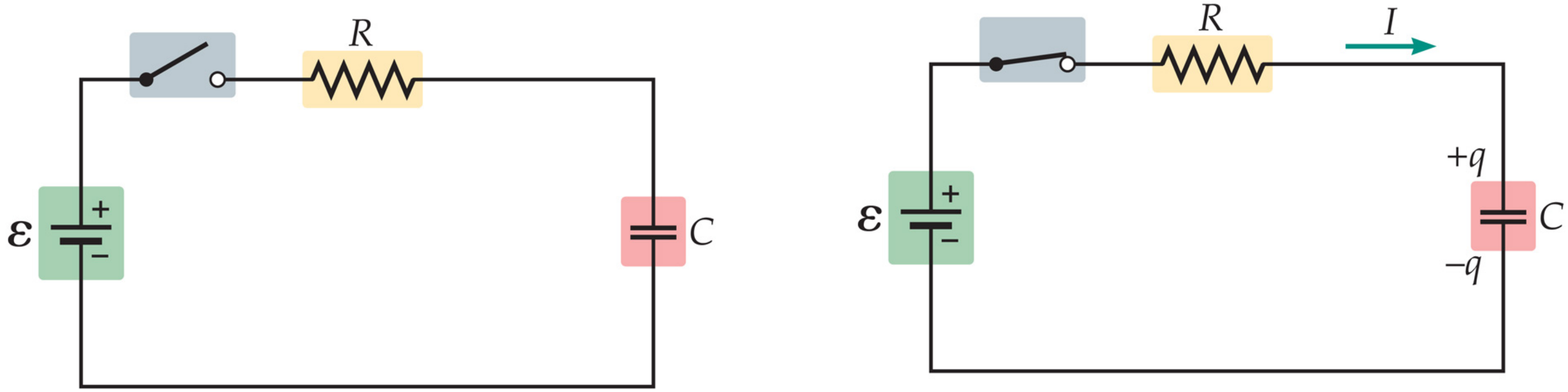


"Plumber's analogy" of an RC circuit: a pump (=battery) pushing water through a closed loop of pipe that includes a valve (=switch), a constriction (=resistor), and a rubber bladder. When the valve starts the flow, the rubber stretches until the pressure difference across the pump (P_1-P_3) equals that across the constriction (P_1-P_2) + bladder (P_2-P_3).

Pump = Battery
Valve = Switch
Constriction = Resistor
Capacitor= Rubber bladder
Pressure = Potential
Water Flow = Current

Capacitors: Time-varying current

When switch closes there is ***a potential difference of 0*** across an uncharged capacitor. After a long time, the capacitor reaches its maximum charge and there is ***no current flow*** through the capacitor. Therefore, at t=0 the capacitor behaves like a *short* circuit (R=0), and at t=∞ the capacitor behaves like an *open* circuit (R=∞).



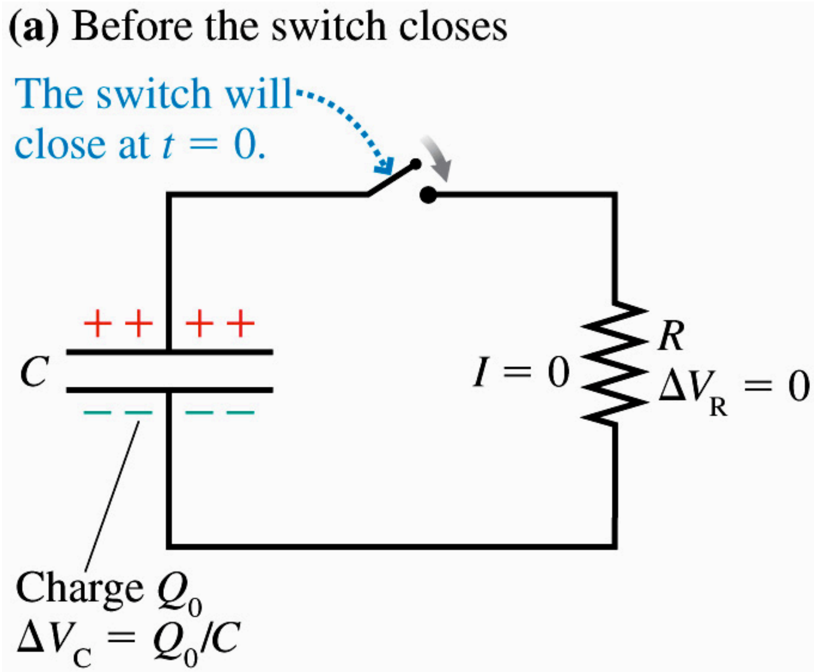
(a) $t < 0$ **(b)** $t > 0$

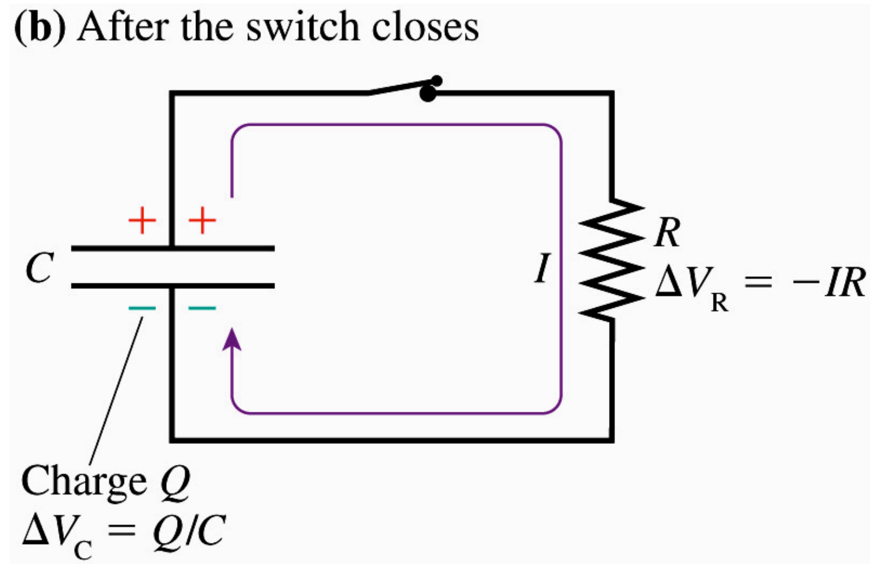
before t=0 (C is uncharged) **At t=0 (C starts charging)**

at t=∞, when C is fully charged: I = 0 again

What's happening? Voltage drop across C at any time must be $V_C = Q/C$.
Current I at any time must be such that $\mathcal{E} = V_C + IR$

Discharging a fully charged capacitor





After switch closes, current flows:

$$\Delta V_C + \Delta V_R = \frac{Q}{C} - IR = \frac{Q}{C} + R\frac{\Delta Q}{\Delta t} = 0$$

Through the magic of calculus...

$$\frac{dQ}{Q} = -\frac{1}{RC}dt \qquad \int_{Q_0}^{Q_f}\frac{dQ}{Q} = -\frac{1}{RC}\int_0^t dt$$

$$Q_f = Q_0 e^{-t/RC}$$

We find :

$$\frac{Q(t)}{Q_0} = \exp\left[-\frac{t}{RC}\right]$$

Exponential behavior !

RC Exponential Decay

$$Q(t) = Q_0 e^{-t/RC} = Q_0 e^{-t/\tau}$$

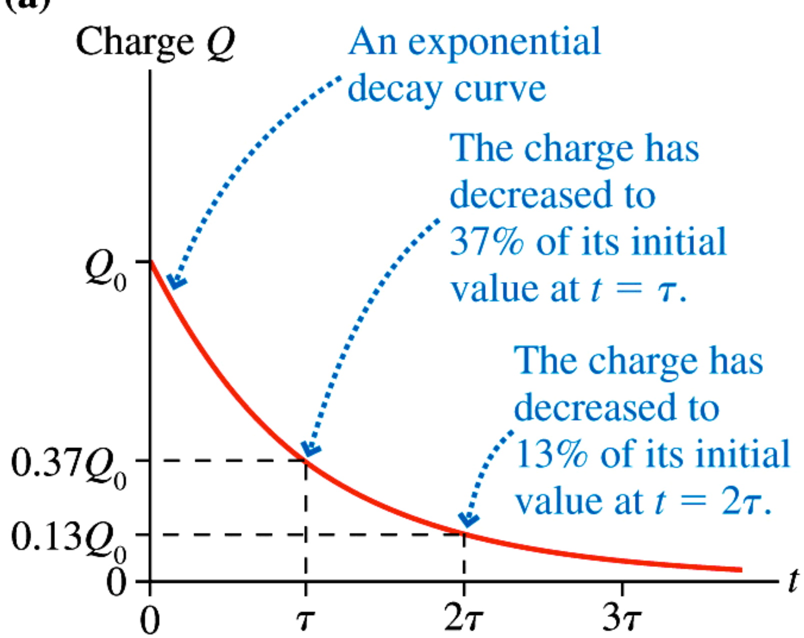
Define RC time constant: $\tau \equiv RC$

$$1/e = 1/2.71828 = 0.367879$$

$$I(t) = \frac{dQ(t)}{dt} = \frac{Q_0}{RC} e^{-\frac{t}{RC}} = I_0 e^{-t/RC} = I_0 e^{-t/\tau}$$

$$I_0 = Q_0/RC$$

(a)



(b)