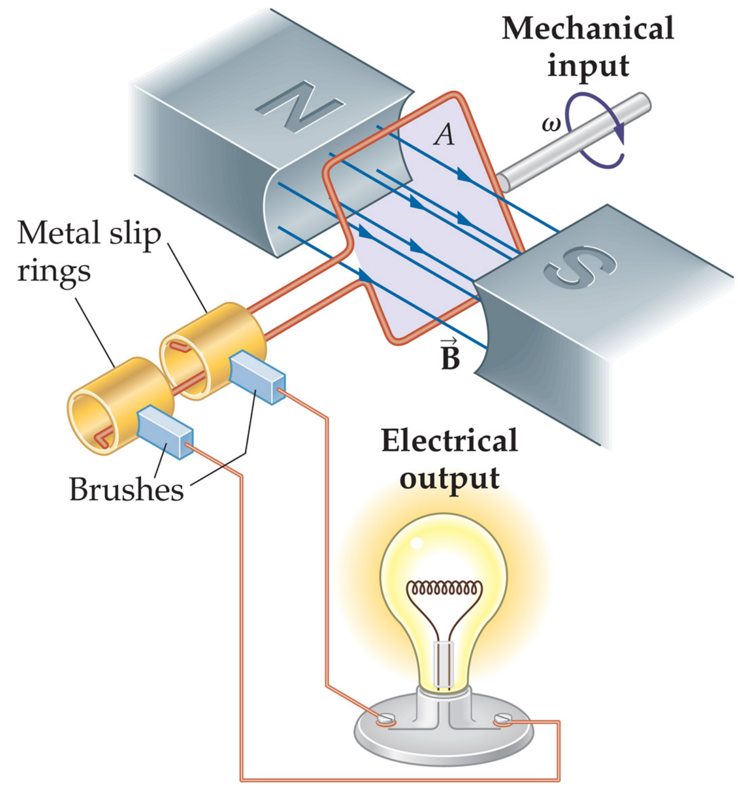
Physics 115
General Physics II



Session 31

Induced currents
Inductance
Generators and motors

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Lecture Schedule

12-May	Mon	23	DC Circuits & Meters	21.5-21.8
13-May	Tues	24	DC Circuits	21.5-21.8
15-May	Thurs	25	RC circuits	21.6-21.7
16-May	Fri	26	Circuits - Neurons	
19-May	Mon	27	Magnetism	22.1
20-May	Tues	28	Magnetic Force	22.2-22.5
22-May	Thurs	29	Magnetic Fields	22.6-22.7
22-May	Fri	30	Induced EMF, Applications	23.1-23.3
26-May	***holiday***		NO CLASS	
27-May	Tues	31	Energy, RL circuits	23.4-23.8
29-May	Thurs	32	Transformer	23.9-23.10
30-May	**Fri**		**EXAM 3 - Chapters 21,22,23**	
2-Jun	Mon	33	AC circuits	24.1-24.3
3-Jun	Tues	34	AC circuits	24.4-24.5
5-Jun	Thurs	35	Resonance, Applications	24.6
6-Jun	Fri	36	Last class - review	
June 9	**FINAL EXAM**		**2:30-4:20 p.m.**	**Comprehensive**
	Mon		**Monday, June 9, 2014**	

Today

Announcements

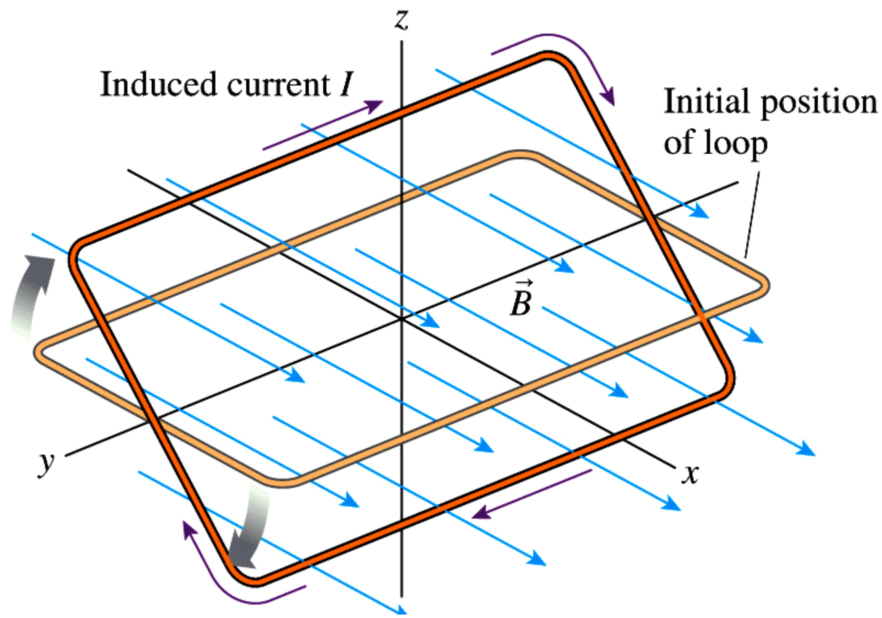
- Exam 3 , Friday 5/30
  - Same format and procedures as previous exams
  - YOU must bring bubble sheet, pencil, calculator
  - Covers material discussed in class from Chs. 21, 22, 23
    - we will skip section 22-8, magnetism in matter
  - Practice questions will be posted tonight, reviewed Thursday.

Last time

Induced current in a **rotating** loop

A loop of wire is initially in the xy plane in a uniform magnetic field in the x direction. It is suddenly rotated 90^0 about the y axis, until it is in the yz plane. Flux changes due to changing area presented to B field.

In what direction will be the induced current in the loop?

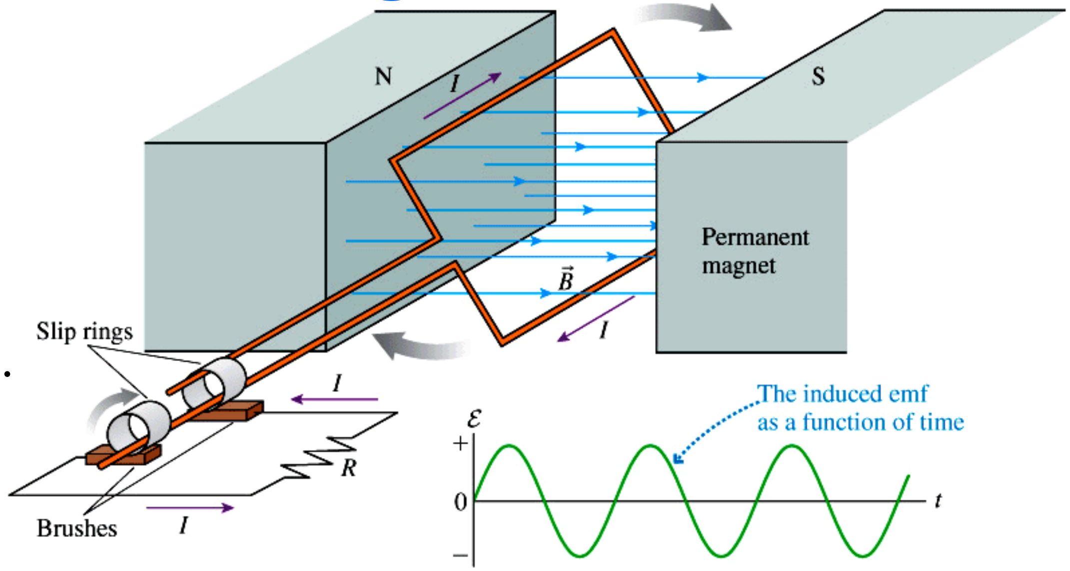


Initially: no flux through the coil.
During rotation: increasing flux, pointing in the +x direction.

Induced current in the coil opposes this change by creating flux in the –x direction.
Therefore, the induced current must be clockwise, as shown in the figure.
If rotation stops, current stops.

Rotating loops = electric **generators**

- A coil rotating in a B field sees constantly changing flux.
- I in coil increases and decreases with every half-turn
- The induced current changes direction after every half-turn.
- The flux vs time through a coil with N turns is (for angular velocity ω radians/sec):



$$\Phi_m = \vec{A}\cdot\vec{B} = AB\cos\theta$$

$$= AB\cos\omega t \quad \left(\omega = \Delta\theta/\Delta t \rightarrow \theta = \omega t\right)$$

$$\mathcal{E}_{\text{coil}} = N\frac{\Delta\Phi_m}{\Delta t} = NAB\frac{\Delta\left(\cos\omega t\right)}{\Delta t} = -\omega ABN\sin\omega t$$

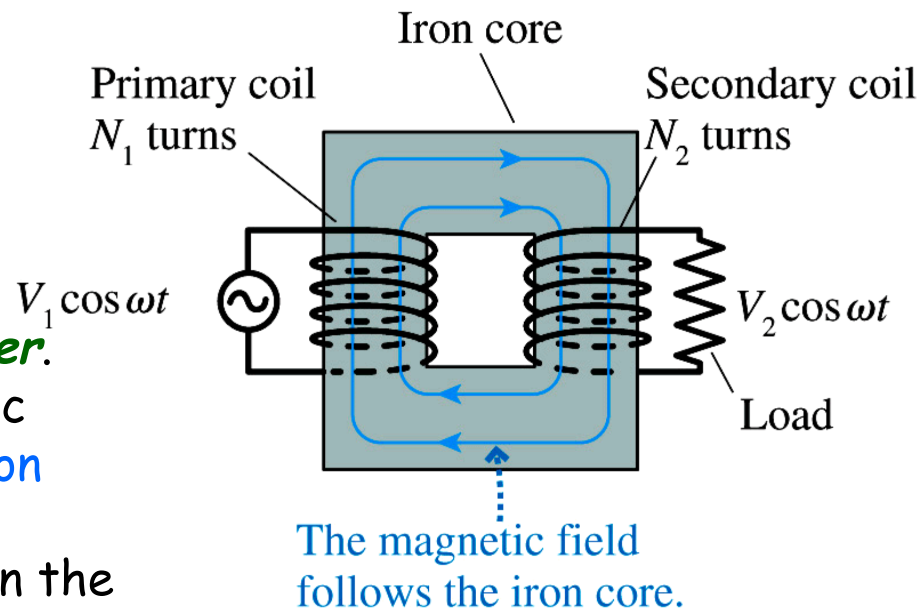
Math fact: $\frac{\Delta\left(\cos\omega t\right)}{\Delta t} = -\omega\sin\omega t$

(Clever device: connect the coil to an external circuit via slip rings and brushes that transmit current regardless of rotational motion.)

The coil produces emf and I that vary like a sine function, alternately positive and negative. This is called an alternating current generator, producing what we call AC power.

Applied induction: transformers

When a coil is driven by AC voltage **$V_1\cos\omega t$**, it produces an oscillating B field that will induce an emf **$V_2\cos\omega t$** in a secondary coil nearby.



This arrangement is called a ***transformer***. Iron is a good "conductor" of magnetic flux, so coils are often wound on an iron core.

- The input emf **V_1** drives a current **I_1** in the primary coil proportional to $1/R \sim$ **$1/N_1$**.
- The flux in the iron is proportional to **I_1**
- The induced emf **V_2** in the secondary coil is proportional to **N_2**.
- Therefore, $V_2 = V_1(N_2/N_1)$.
- From conservation of energy, assuming no losses in the core, $V_1I_1 = V_2I_2$.
- So, currents in the primary and secondary are related by $I_1 = I_2(N_2/N_1)$.

A transformer with **$N_2 >> N_1$** is called a ***step-up transformer***, which boosts the secondary voltage. A transformer with **$N_2 << N_1$** is called a ***step-down transformer***, and it drops the secondary voltage.

Example

- Transformer has 800 turns in its primary coil and 100 turns in its secondary coil
  - What is the secondary voltage, if primary V is 120 volts AC ?

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2}$$

$$V_2 = V_1 \frac{N_2}{N_1} = 120V\left(\frac{100}{800}\right) = 15V$$

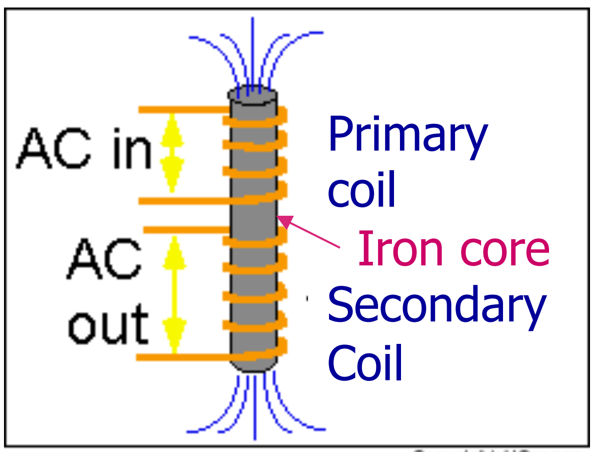
  - The secondary coil is connected to a battery charger that draws 2 amperes – what current flows in the primary coil?

$$I_1 = I_2 \frac{N_2}{N_1} = 2A\left(\frac{100}{800}\right) = 0.25A$$

  - Notice:
    - Power in secondary = VI = 15V(2A) =30W
    - Power in primary = 120V(0.25A) = 30W

Transformers

- Critical development for our modern world! Transformers allow convenient transmission of electric power
  - Step up voltage for transmission; step it down at user's home
- Principle:
  - convert changing electric current into changing magnetic field
  - Use iron core to link magnetic field from input to output coil
  - convert magnetic field back into electric current at a *different* voltage
    - *Requires* alternating current (AC): can't work with DC



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Small transformers for audio signals or computer power supplies

Power line transformer:
10 kV to 240V for your home

Photo credit:
Oak Ridge National Laboratory

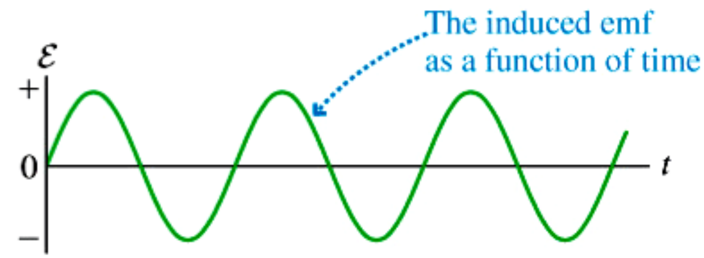
AC Power

- AC generator is a source of emf that varies sinusoidally:
  - That means: either sine or cosine function can describe $\mathcal{E}(t)$
  - The only difference is where we choose t=0

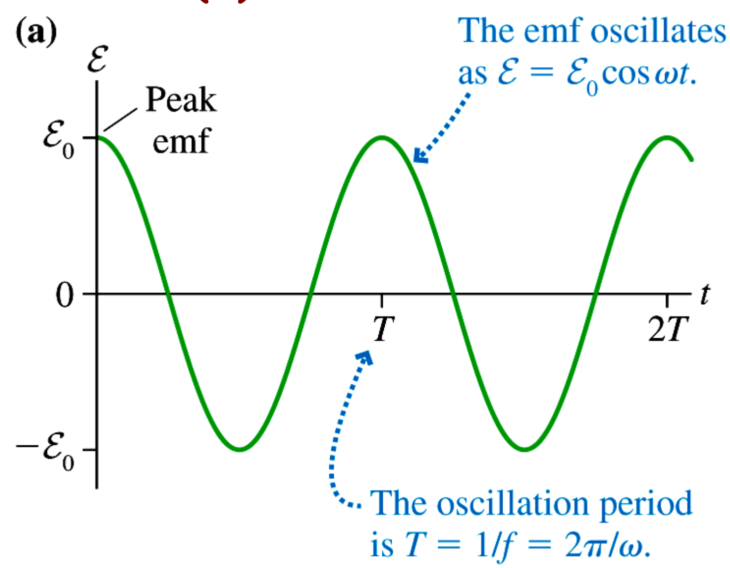
If we choose t=0 when $\mathcal{E}$ =max:
$\mathcal{E}(t) = \mathcal{E}_0 \cos \omega t$
$\mathcal{E}_0$ is the peak emf
ω is the angular frequency,
$\omega = 2\pi f$, where f is the frequency in Hz (cycles/sec).
Period T = time for one full cycle.
$\mathcal{E}$ pattern repeats itself every T.
T = 1 / f : so f = 2 Hz → T=0.5sec

$$2\pi\ rad = 1 \text{ full cycle of sin/cos}$$
$$T = \text{time for 1 cycle}$$
$$= (1s)/f(cycles/s) = (2\pi\ rad)/\omega(rad/s)$$
$$\frac{1}{f} = \frac{2\pi}{\omega} \rightarrow 2\pi f = \omega: \text{ relation between } f \text{ and } \omega$$

sin(t) = 0 when t=0



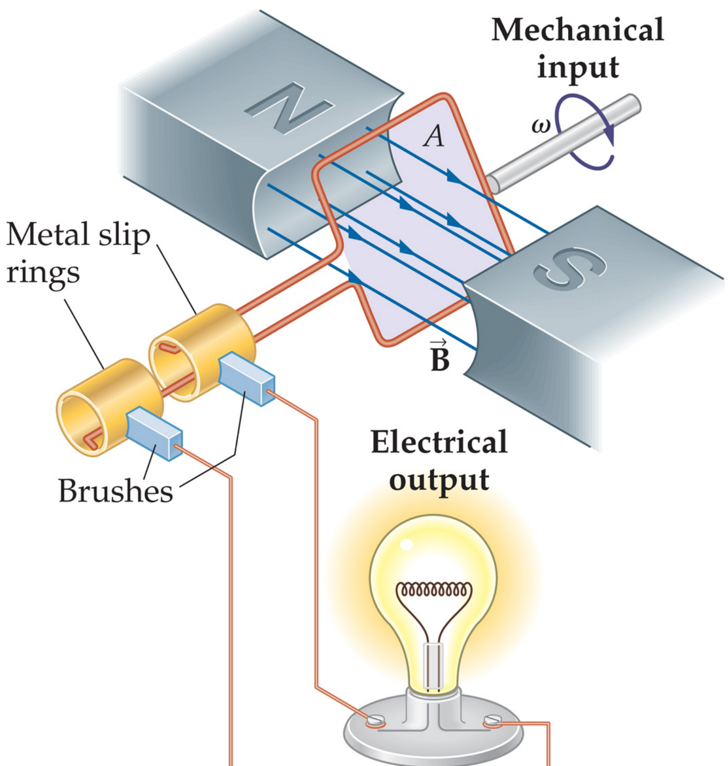
cos(t) = 1 when t=0

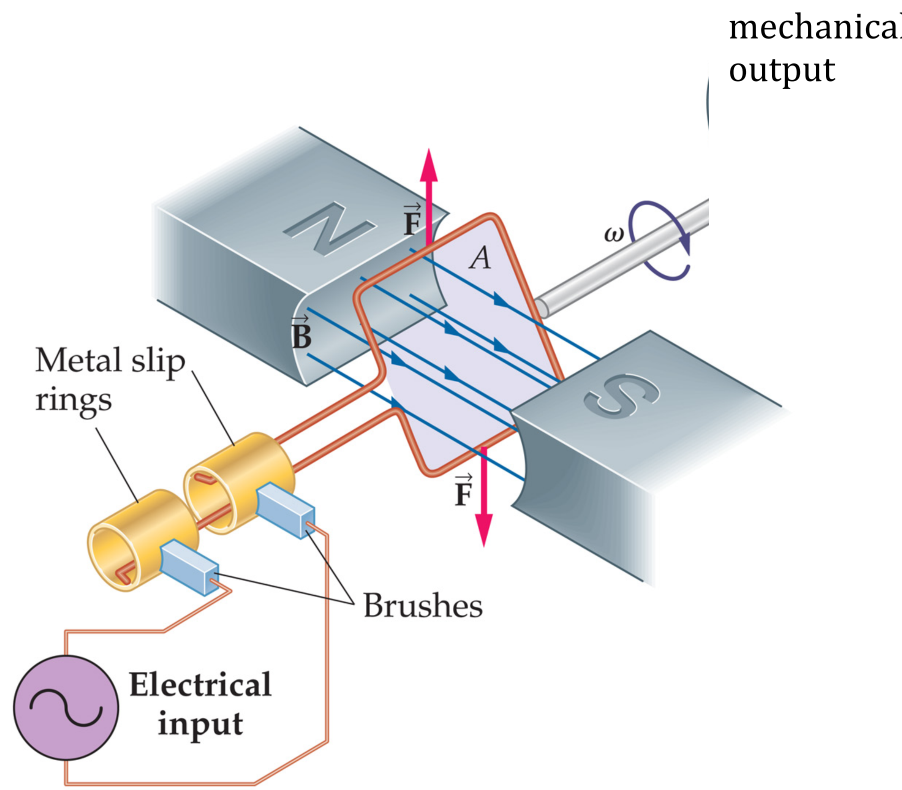


Note: sin/cos argument must be pure number (radians), no units!

Reverse the process: electric **motor**

- AC generator takes mechanical power and makes electrical power via induction
- If we supply electrical power to the same setup, we get mechanical power out – we have an AC motor





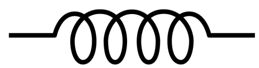
Inductance

$$\mathcal{E}_{\text{coil}} = N\frac{\Delta\Phi_m}{\Delta t}; \quad \Phi_m \propto B \propto I \rightarrow \frac{\Delta\Phi_m}{\Delta t} \propto \frac{\Delta I}{\Delta t} \Rightarrow \mathcal{E}_{\text{coil}} = (const)\frac{\Delta I}{\Delta t}$$

We define the ***inductance*** L of a coil of wire producing flux ϕ_m as:

$$\mathcal{E}_{\text{coil}} = N\frac{\Delta\Phi_m}{\Delta t} = L\frac{\Delta I}{\Delta t} \quad \rightarrow \quad \boxed{L = \frac{\Delta\Phi_m}{\Delta I}}$$

Unit of inductance is the ***henry*** : 1 henry = 1 H = 1 T m²/A = 1 Wb/A=1 V s /A

The circuit diagram symbol used to represent inductance is: 

Example: The inductance of a long solenoid with **N** turns of cross sectional area **A** and length ℓ is:

$$B = \frac{\mu_0 N I}{\ell} \qquad \phi_{\text{per turn}} = BA$$

$$\phi_m = NBA = N\left(\frac{\mu_0 N\, I}{\ell}\right)A = \frac{\mu_0 N^2 A}{\ell} I$$

If we change current from 0 to I :

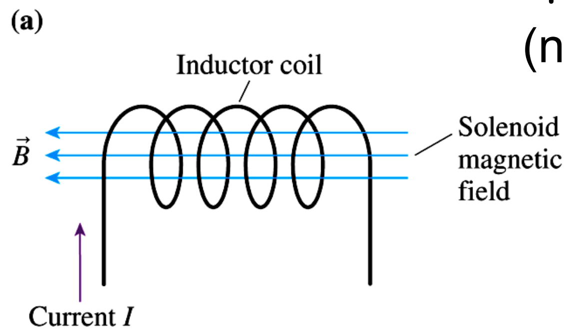
$$for\ \Delta I = (I-0),\ \Delta\Phi_m = \left(\frac{\mu_0 N^2 A}{\ell} I - 0\right) \rightarrow \frac{\Delta\Phi_m}{\Delta I} = \frac{\Phi_m}{I}$$

$$\boxed{L_{\text{solenoid}} = \frac{\phi_m}{I} = \frac{\mu_0 N^2 A}{\ell}}$$

EMF Across an Inductor

When I through a coil changes, the flux changes.
Changing flux -> induced current opposing the change -> back-EMF !

(notice: only while current is changing)



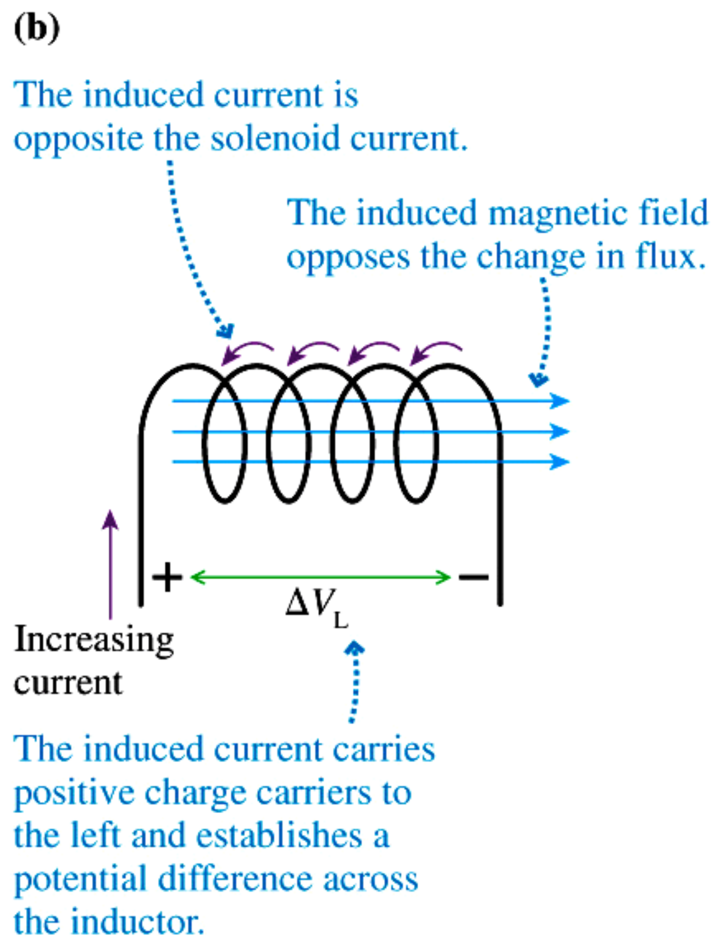
Neglecting direction of current (magnitudes only), we find

$$\left|\mathcal{E}_{\text{coil}}\right| = N\left|\frac{\Delta\Phi_{\text{per turn}}}{\Delta t}\right| = L\left|\frac{\Delta I}{\Delta t}\right|$$

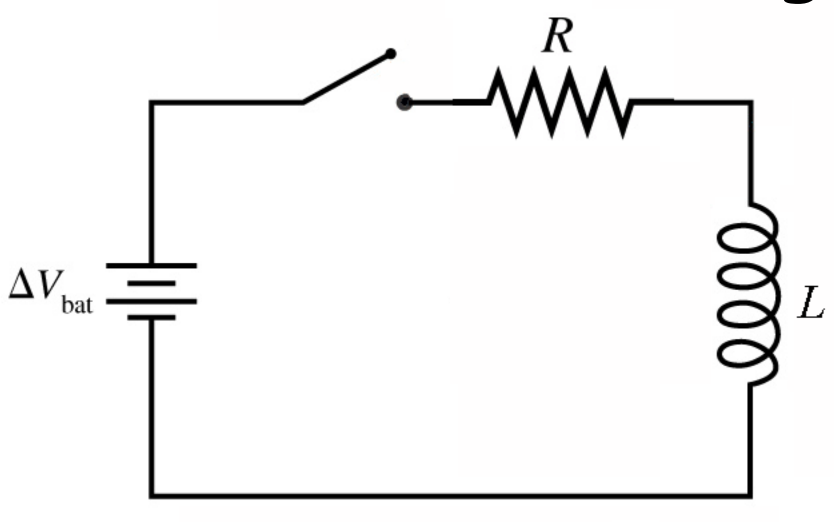
Coil geometry doesn't change, only I:

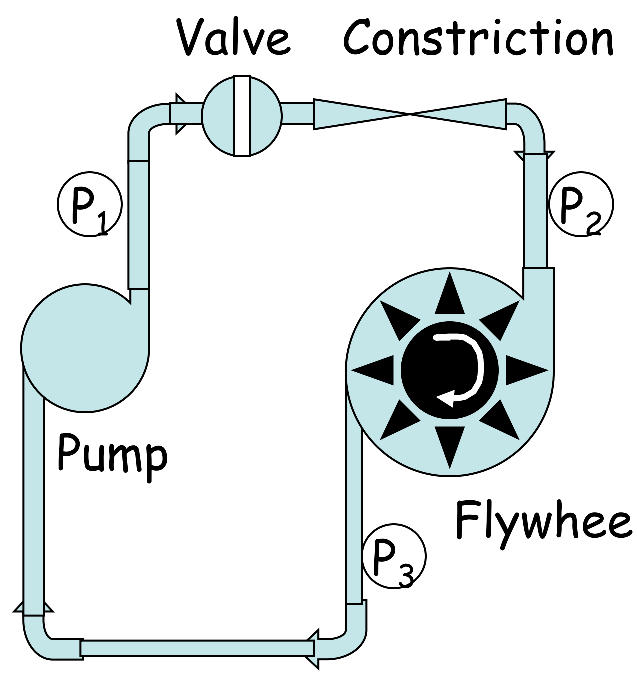
$$\left|\mathcal{E}_{\text{coil}}\right| = L\left|\frac{\Delta I}{\Delta t}\right|$$

L contains all info about coil geometry



Water flow analogy for RL circuit





Pump = Battery
Valve = Switch
Constriction = Resistor
Flywheel = Inductor
Pressure = Potential
Water Flow = Current

The "plumber's analogy" of an RL circuit is a pump (=battery) pumping water in a closed loop of pipe that includes a valve (=switch), a constriction (=resistor), and rotating flywheel (inductor) turned by the flow.

When the valve is opened, the flywheel **starts to turn (inertia!)** until a steady flow is reached, and the pressure difference across the flywheel ($P_2 - P_3$) goes to zero.

What happens when we turn off the pump?

Does flow stop right away?

Inductors: Currents initially, and 'after a long time'

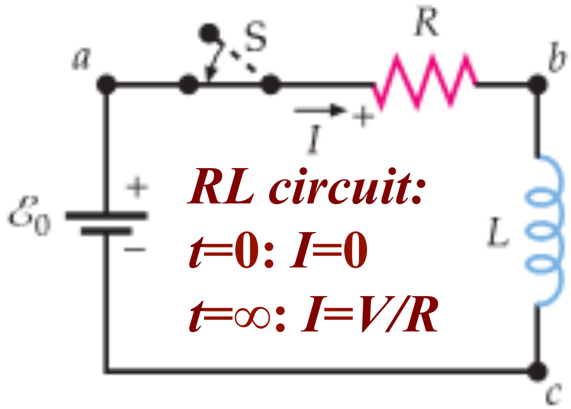
When an inductor is in series with a battery and R, if the switch is closed after a long time open, current flow is not immediate and constant (as in a resistor):

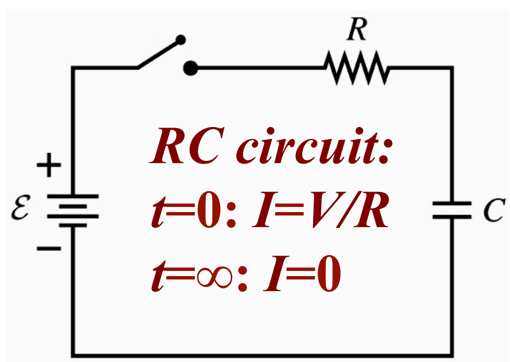
- at $t=0$ the inductor behaves like an open circuit ($R=\infty$), and
- at $t=\infty$ the inductor behaves like a short circuit ($R=0$).

The inductor provides a form of electrical inertia – doesn't allow I to change instantly

This behavior is ***opposite*** that of a capacitor.

- at $t=0$ the capacitor behaves like a short circuit ($R=0$), and
- at $t=\infty$ the capacitor behaves like an open circuit ($R=\infty$).

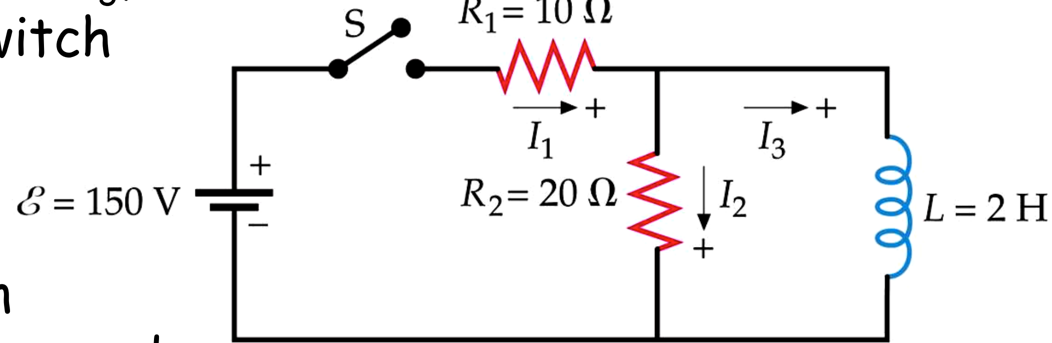




Example: Initial and Final Currents for RL circuit

Find the currents I_1, I_2, and I_3,
(a) Immediately after the switch closes;
(b) A long time after the switch closes;

After the switch has been closed for a long time and is opened.



(c) Find the currents immediately after the switch opens;
(d) Find the currents a long time after the switch opens.

$(a)\ I_3 = 0; \quad I_1 = I_2 = \mathcal{E}/(R_1 + R_2) = 5.0\ \text{A}$ L is an open circuit (**R**=∞).

$(b)\ I_2 = 0; \quad I_1 = I_3 = \mathcal{E}/R_1 = 15.0\ \text{A}$ L is a short circuit (**R**=**0**)

$(c)\ I_1 = 0; \quad I_2 = I_3 = 15.0\ \text{A}$ Takes time for L's B field to collapse, instant change impossible ("inertia")

$(d)\ I_1 = I_2 = I_3 = 0$ R_2 long ago dissipated back current from L. Left loop is an open circuit. so I=0

RL Circuits: exponential decay of I

The switch has been in position a for a long time. For this loop we get

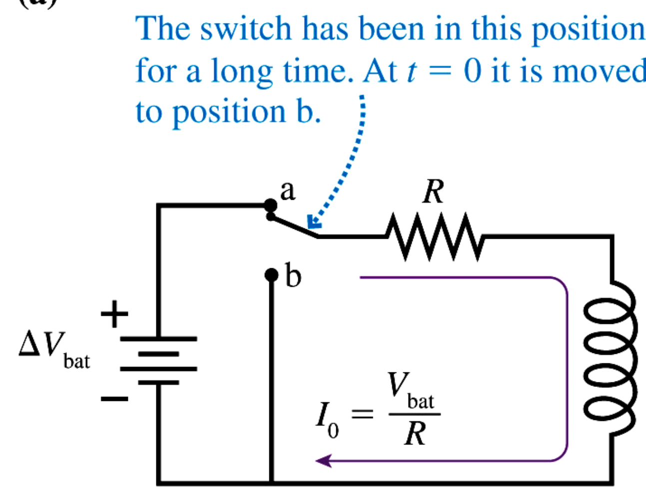
$$\Delta V_{BAT} + \Delta V_R + \Delta V_L = 0$$

$$\Delta V_L = 0 \quad \text{(for constant I, inductor is just a wire!)}$$

$$\Delta V_{BAT} + \Delta V_R = 0$$

$$\left|\Delta V_{BAT}\right| = \left|\Delta V_R\right| \rightarrow \left|\mathcal{E}\right| = I_0 R \rightarrow I_0 = \frac{\left|\mathcal{E}\right|}{R}$$

(a)



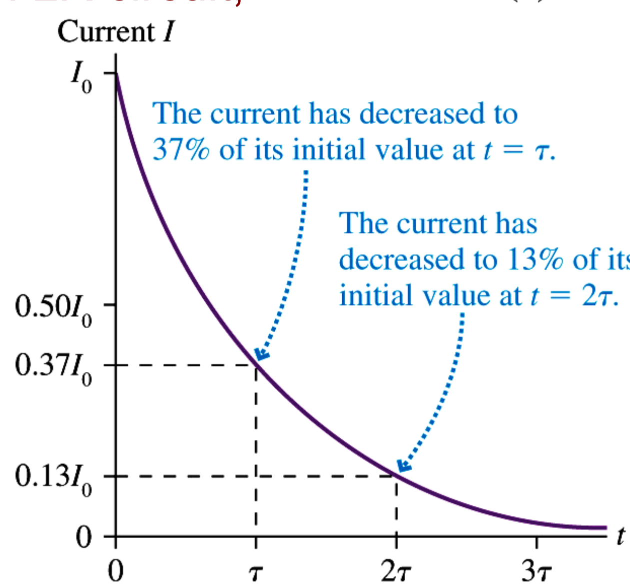
Flip the switch to b at t=0: now loop is just L + R

"It can be shown" that for an LR circuit,

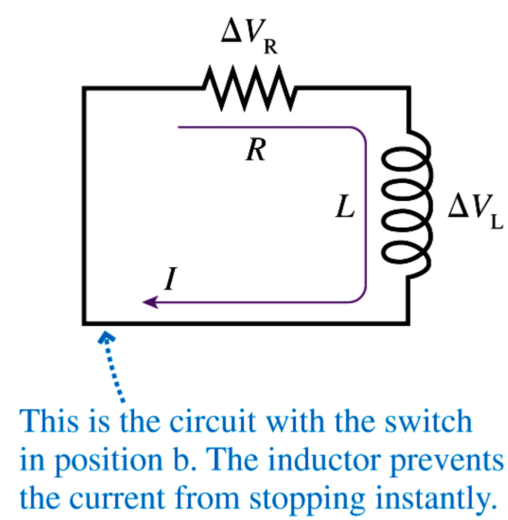
$$\boxed{I(t) = I_0 e^{-t/\tau}}$$

$$\boxed{\tau \equiv L/R}$$

$$I(t) = \frac{\mathcal{E}}{R} e^{-t/(L/R)}$$



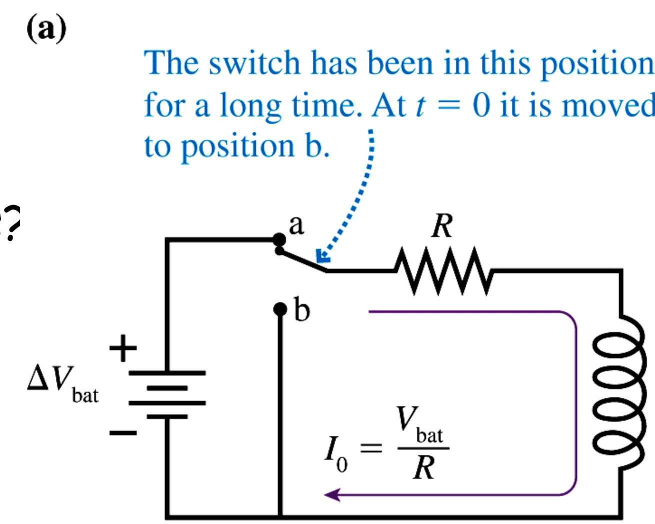
(b)



RL Circuits: exponential buildup of I

Now suppose the switch has been in position **b** "for a long time."
Flip it to position **a** at t=0.
Example: 10V battery, R=100 ohms, L= 1.0 mH

a. What is the current in the circuit at **t** = 5 μs?
b. What is the current after "a long time"?



Answer to (b) is easy:

$$I_{MAX} = \mathcal{E} / R = (10\ \text{V}) / (100\ \Omega) = 100\ \text{mA}$$

For (a): $\tau = L/R = 0.001\text{H}/100\Omega = 10\ \mu s$

I goes from 0 to I_{MAX} exponentially:

For RL circuits, buildup of current behaves like decay of current in RC circuits:

$$I(t) = I_0\left(1 - e^{-t/\tau}\right) = \frac{\mathcal{E}}{R}\left(1 - e^{-tR/L}\right)$$

$$I(t) = (100\ \text{mA})e^{-(5\ \mu s)/(10\ \mu s)} = 61\ \text{mA}$$