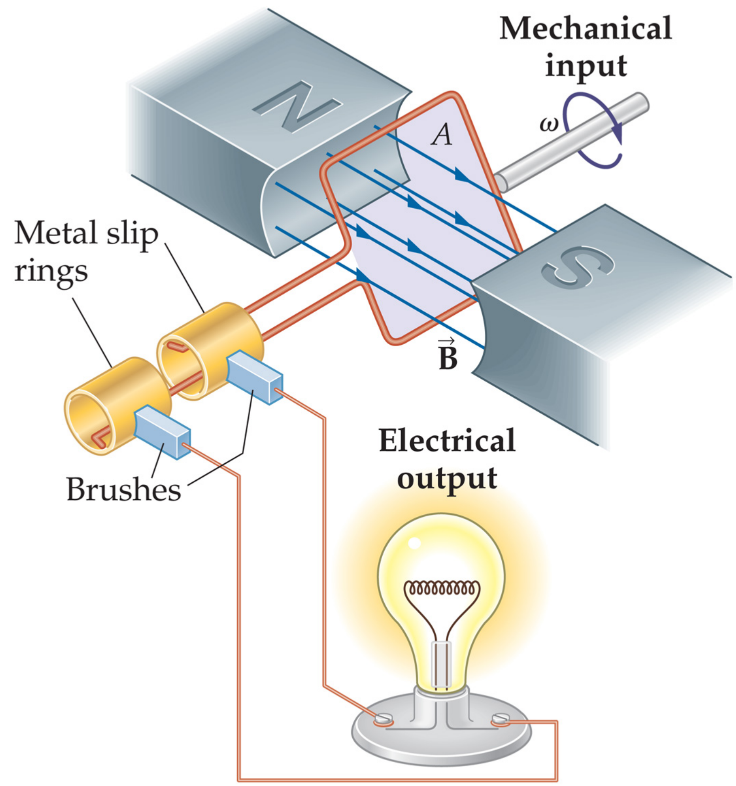
Physics 115
General Physics II



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Session 32

Induced currents
Work and power
Generators and motors

- **R. J. Wilkes**
- Email: phy115a@u.washington.edu

- Home page: http://courses.washington.edu/phy115a/

Lecture Schedule

12-May	Mon	23	DC Circuits & Meters	21.5-21.8
13-May	Tues	24	DC Circuits	21.5-21.8
15-May	Thurs	25	RC circuits	21.6-21.7
16-May	Fri	26	Circuits - Neurons	
19-May	Mon	27	Magnetism	22.1
20-May	Tues	28	Magnetic Force	22.2-22.5
22-May	Thurs	29	Magnetic Fields	22.6-22.7
22-May	Fri	30	Induced EMF, Applications	23.1-23.3
26-May	***holiday***		NO CLASS	
27-May	Tues	31	Energy, RL circuits	23.4-23.8
29-May	Thurs	32	Transformer	23.9-23.10
30-May	**Fri**		**EXAM 3 - Chapters 21,22,23**	
2-Jun	Mon	33	AC circuits	24.1-24.3
3-Jun	Tues	34	AC circuits	24.4-24.5
5-Jun	Thurs	35	Resonance, Applications	24.6
6-Jun	Fri	36	Last class - review	
June 9	**FINAL EXAM**		**2:30-4:20 p.m.**	**Comprehensive**
	Mon		**Monday, June 9, 2014**	

Today

Announcements

- Exam 3 tomorrow, Friday 5/30
  - Same format and procedures as previous exams
  - YOU must bring bubble sheet, pencil, calculator
  - Covers material *discussed in class* from Chs. 21, 22, 23
  - Practice questions will be reviewed in class today.
- Homework set 9 is due **Friday** 6/6, 11:59pm
  - **Not** due Weds night as usual

Formula sheet (final)

Electric Current $I = \Delta Q/\Delta t$ = Rate of flow of electric charge

Ohm's Law $V = IR$

$R = \rho L/A$, ρ resistivity

Series $R = R_1 + R_2 + \ldots\ldots\ldots$ Parallel $R^{-1} = R_1^{-1} + R_2^{-1} + \ldots\ldots$
$Q = CV$
Series $C^{-1} = C_1^{-1} + C_2^{-1} + \ldots$ Parallel $C = C_1 + C_2 + \ldots\ldots\ldots$
Charging a capacitor in an RC circuit
$Q(t) = Q_{max}(1 - e^{-t/\tau})$ $\tau = RC$, Q_{max} = maximum charge on C (at t=infinity)

$F_B = q v B \sin(\theta)$, $F_E = q E$ (on a charge q)

Work $= q V$

$F_B = I l B \sin(\theta)$ (on wire with length l)

Torque on coil of N loops $= N I B A \sin(\theta)$

Force per unit length between parallel currents $= \mu_0 I_1 I_2 / 2\pi D$
D is distance between wires

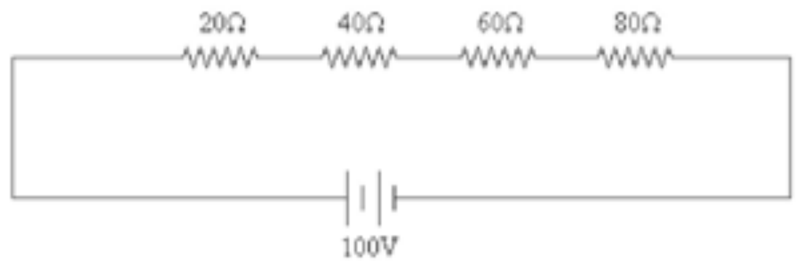
Magnetic Permeability of Vacuum $\mu_0 = 4\pi \times 10^{-7}$

Power $= V I$
Loop Rule Sum of Voltage Drops around any Loop = Zero
Junction Rule Sum of Currents In = Sum of Currents Out at any junction
Magnetic field at distance R from a long straight wire with current I
$B = 2 \times 10^{-7} I/R$
Cyclotron formula for charged particle moving perpendicular to uniform field B
$R = mv/(qB)$, R radius of the circular trajectory
Solenoid field $B = \mu_0 N I / l$ (N turns over length l)
Transformers: $(V_2 / V_1) = (N_2 / N_1) = (I_1 / I_2)$
Inductance $L = \Delta\Phi_m / \Delta I$ Inductance of solenoid (N turns, length l): $L = \mu_0 N^2 A / l$
Kinetic energy for mass m, speed v = $\frac{1}{2} mv^2$

Practice question solutions

1) The length of a certain wire is kept same while its radius is doubled. What is the change in the resistance of this wire?
A) It is doubled.
B) It is quadrupled.
C) It is reduced by a factor of 2.
D) It is reduced by a factor of 4.
E) It is reduced by a factor of 8.
Answer: D

$$R = \rho\frac{L}{A} = \rho\frac{L}{\pi r^2} \quad R' = \rho\frac{L}{\pi(2r)^2} = \rho\frac{L}{\pi 4r^2} = \frac{R}{4}$$



2) A 100 V EMF is applied to four resistors as shown above. The values of the resistors are 20 Ω, 40 Ω, 60 Ω, and 80 Ω What is the voltage drop across the 40 Ω resistor?
A) 20 V
B) 40 V
C) 60 V
D) 80 V
E) 100 V
Answer: A

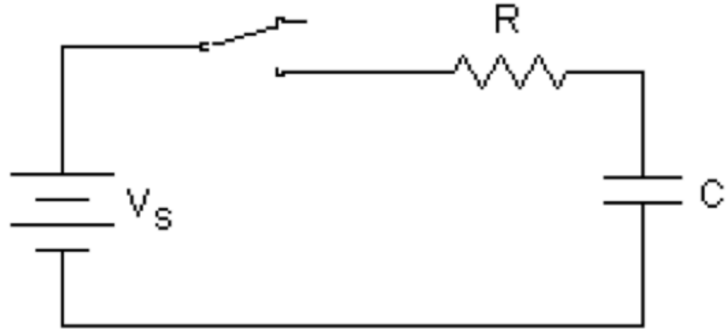
$$I = \frac{V}{R_{eq}} = \frac{100\ \text{V}}{20\ \Omega + 40\ \Omega + 60\ \Omega + 80\ \Omega} = \frac{100\ \text{V}}{200\ \Omega} = 0.50\ \text{A}$$

$$V_{40\Omega} = I(40\ \Omega) = 20\ \text{V}$$

$$P = I^2 R \Rightarrow I_{MAX}^2 = \frac{P}{R} \rightarrow I = \sqrt{\frac{0.80W}{400\Omega}} = 0.0447A = 45mA$$

3) A 400 Ω resistor dissipates 0.80 W. What is the current in this resistor?

A) 45 mA
B) 18 mA
C) 4.4 mA
D) 2.0 mA
E) 320 mA   Answer: A



4) . A flash unit for a camera consists of an RC circuit with R = 10 kilohms . What capacitance C is needed here if the unit is to charge to 50 % of full charge in 10 seconds ?

(a) 728 microfarads
(b) 137
(c) 1443
(d) 1529
(e) 988

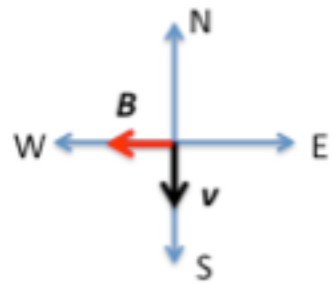
Ans: C

We want $q(t = 10s) = q_{MAX}\left(1 - e^{-(10s)/\tau}\right) = 0.5 q_{MAX}$

$$\left(1 - e^{-(10s)/\tau}\right) = 0.5 \rightarrow e^{-(10s)/\tau} = 0.5 \rightarrow -10s/\tau = \ln(0.5) = -0.693$$

$$\tau = 10s/0.693 = 14.43s = RC \rightarrow C = 14.43s/10^4\Omega = 1.443 \times 10^{-3}F = 1443\mu F$$

5) An electron is accelerated from rest through a potential difference of 3750 V.
After acceleration its velocity vector points south.
Then it enters a region where the uniform magnetic field is 0.004 T, pointing west.



a) In what direction will the electron's path be deflected initially, as it first enters the B field region?

A) downward
B) towards the east
C) upward
D) towards the west
E) towards the north

Answer: C

RHR: v x B = down,
but electron is negative

b) Calculate the radius of the path this electron will follow. (Electron mass is 9.11×10^{-31} kg)

A) 1.2 cm
B) 2.2 cm
C) 3.2 cm
D) 4.2 cm
E) 5.2 cm

Answer: E

$$e\Delta V = KE = mv^2/2$$

$$v = \sqrt{2e\Delta V/m} = \sqrt{2\left(1.60\times10^{-19}\text{ C}\right)\left(3750\text{ V}\right)/\left(9.11\times10^{-31}\text{ kg}\right)}$$

$$= 3.63\times10^{7}\text{ m/s}$$

$$r = \frac{mv}{eB} = \frac{\left(9.11\times10^{-31}\text{ kg}\right)\left(3.63\times10^{7}\text{ m/s}\right)}{\left(1.60\times10^{-19}\text{ C}\right)\left(0.004\text{ T}\right)} = 5.16\times10^{-2}\text{ m} = 5.2\text{ cm}$$

6) Two wires, each 300 m in length, run parallel to each other with a separation of 10 cm . If the lines carry currents of 100 A in the same direction, what is the magnitude of the force between them ?

(a) 3 N
(b) 11
(c) 17
(d) 6
(e) 13
Answer: D

$$|F| = \frac{\mu_0 I_1 I_2}{2\pi d} L$$

$$= \frac{(4\pi \times 10^{-7})\,(100A)^2\,(300m)}{2\pi(0.1m)} = 2\times 10^{-7} \times 10^{4} \times 3 \times 10^{2} \times 10 = 6\text{N}$$

Last time

RL Circuits: exponential decay of I

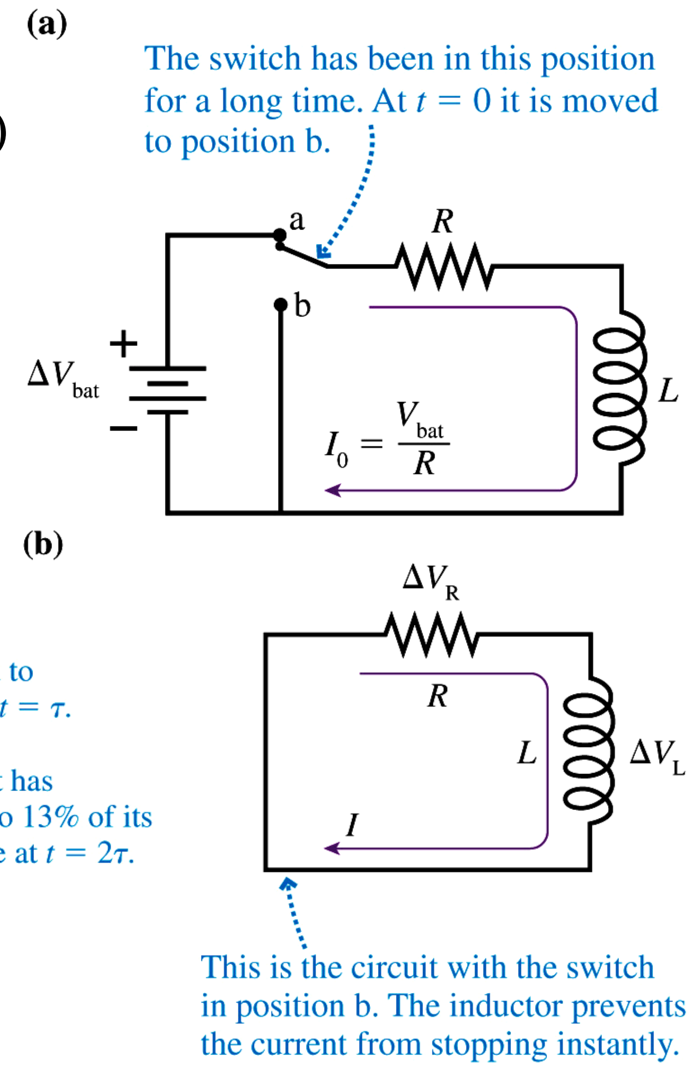
The switch has been in position a for a long time. For this loop we get

$$\Delta V_{BAT} + \Delta V_R + \Delta V_L = 0$$

$$\Delta V_L = 0 \quad \text{(for constant I, inductor is just a wire!)}$$

$$\Delta V_{BAT} + \Delta V_R = 0$$

$$\left|\Delta V_{BAT}\right| = \left|\Delta V_R\right| \rightarrow \left|\mathcal{E}\right| = I_0 R \rightarrow I_0 = \frac{\left|\mathcal{E}\right|}{R}$$



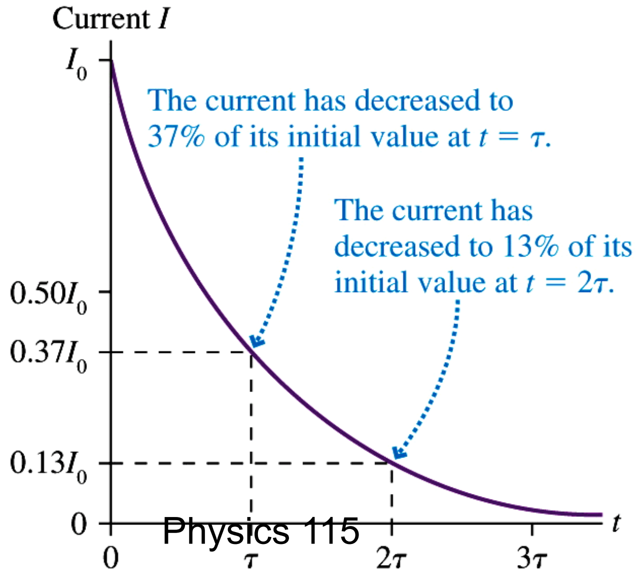
Flip the switch to b at t=0: now loop is just L + R

"It can be shown" that for an LR circuit,

$$\boxed{I(t) = I_0 e^{-t/\tau}}$$

$$\boxed{\tau \equiv L/R}$$

$$I(t) = \frac{\mathcal{E}}{R} e^{-t/(L/R)}$$

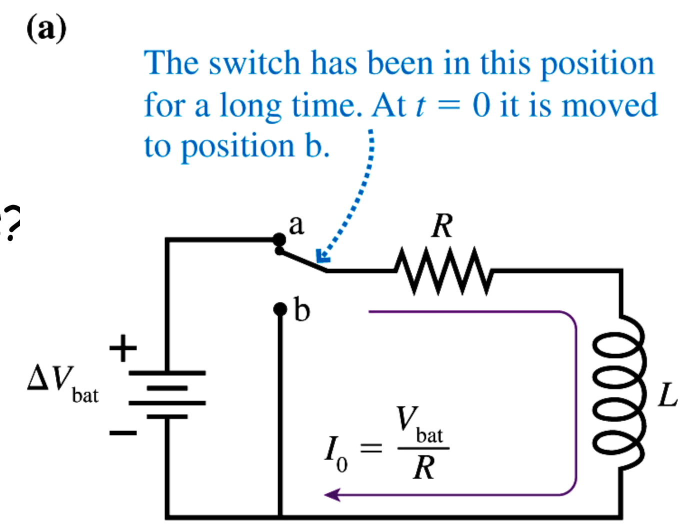


Last time

RL Circuits: exponential buildup of I

Now suppose the switch has been in position **b** "for a long time."
Flip it to position **a** at t=0.
Example: 10V battery, R=100 ohms, L= 1.0 mH

a. What is the current in the circuit at **t** = 5 μs?
b. What is the current after "a long time"?



Answer to (b) is easy:

$$I_{MAX} = \mathcal{E}/R = (10\text{ V})/(100\ \Omega) = 100\text{ mA}$$

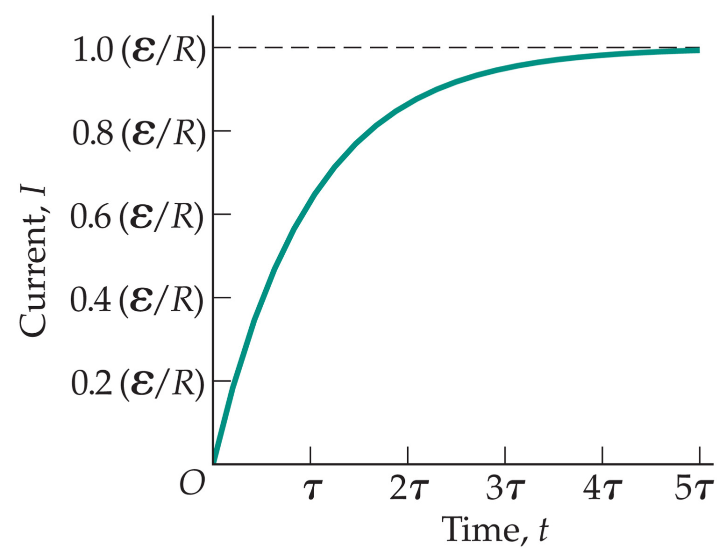
For (a): $\tau = L/R = 0.001\text{H}/100\Omega = 10\ \mu\text{s}$

I goes from 0 to I_{MAX} exponentially:

For RL circuits, buildup of current behaves like decay of current in RC circuits:

$$I(t) = I_0\left(1 - e^{-t/\tau}\right) = \frac{\mathcal{E}}{R}\left(1 - e^{-tR/L}\right)$$

$$I(t) = (100\text{ mA})e^{-(5\ \mu\text{s})/(10\ \mu\text{s})} = 61\text{ mA}$$



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Energy in Inductors and Magnetic Fields

Like an E field, a magnetic field stores energy.
So, inductors, which create B fields, store energy.
How much energy U_L is stored in an inductor **L** carrying current **I** ?

$$P = I\Delta V = I\left|-L\frac{\Delta I}{\Delta t}\right| = LI\frac{\Delta I}{\Delta t} = \frac{\Delta U_L}{\Delta t}$$

$$\text{for change } \Delta I = 0 \rightarrow I \text{ in time } \Delta t = T,\ \frac{\Delta I}{\Delta t} = \frac{I}{T}$$

$$P(t) = I(t)\frac{L\Delta I}{T};\ \text{ here, } \Delta I = (I - 0) = I,\ \text{but I(t) varies - average} = \frac{I}{2}$$

$$P_{AVG} = \frac{LI^2}{2T} \rightarrow \quad U_L = P_{AVG}T = \frac{LI^2}{2}$$

$$\boxed{U_L = \tfrac{1}{2}LI^2}$$

Energy density in Inductors and B Fields

As with E fields, it is useful to define the B field's energy density = energy/per unit volume

Example:

A solenoid of length l, area A, and N turns has $L = \mu_0 N^2 A/l$, so:

$$U_L = \tfrac{1}{2} L I^2 = \frac{\mu_0 N^2 A}{2l} I^2 = \frac{1}{2\mu_0} Al \left(\frac{\mu_0 N I}{l} \right)^2$$

$$= \frac{1}{2\mu_0} AlB^2$$

Solenoid's B field is $B = \mu_0 NI/l$ so:

$$u_B = U_L / \left(\text{volume occupied by B field} \right)$$

$$u_B = \frac{U_L}{Al} = \frac{1}{2\mu_0} B^2$$

$$\boxed{u_B = \frac{1}{2\mu_0} B^2}$$

"It turns out" this is true for any B field, not just inside a solenoid

Electromagnetic Energy Density

SO: both B and E fields store energy in the space they occupy.

Example: A region of space has a uniform magnetic field of 0.020 T and a uniform electric field of 2.50×10^6 N/C (i.e., 2.50 MV/m). What is

(a) the total *electromagnetic* energy density in the region?

(b) the energy contained in a cubic volume, 12 cm on a side?

$$u_e = \tfrac{1}{2}\varepsilon_0 E^2$$
$$= \tfrac{1}{2}(8.85\times 10^{-12}\ \mathrm{C^2/Nm^2})(2.50\times 10^{6}\ \mathrm{N/C})^2$$
$$= 27.7\ \mathrm{J/m^3}$$

$$u_m = \tfrac{1}{2}B^2/\mu_0$$
$$= \tfrac{1}{2}\frac{(0.0200\ \mathrm{T})^2}{(4\pi\times 10^{-7}\ \mathrm{N/A^2})}$$
$$= 159\ \mathrm{J/m^3}$$

$$u = u_e + u_m = (27.7\ \mathrm{J/m^3}) + (159\ \mathrm{J/m^3}) = 187\ \mathrm{J/m^3}$$

$$U = uV = u\mathrm{l}^3 = (187\ \mathrm{J/m^3})(0.120\ \mathrm{m})^3 = 0.323\ \mathrm{J}$$

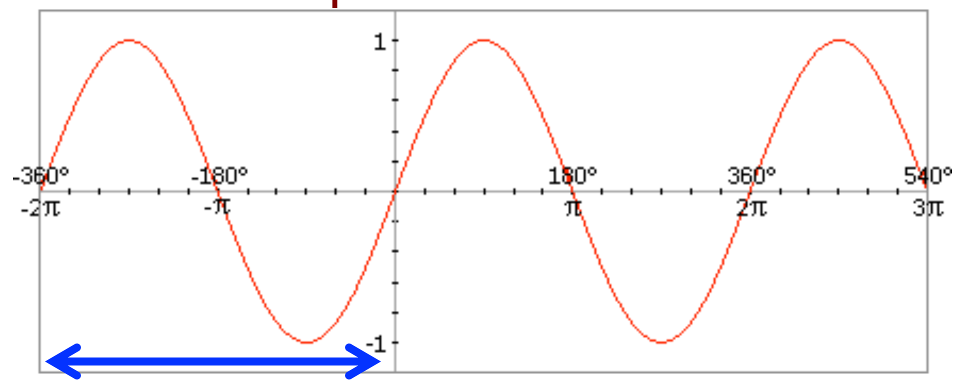
Notice: a ***rather weak*** magnetic field stores **more** energy than a ***very intense*** electric field.

Reminder : about sin and cos functions

As we saw, the EMF of a rotating loop in a magnetic field is a sin (or cos) function:

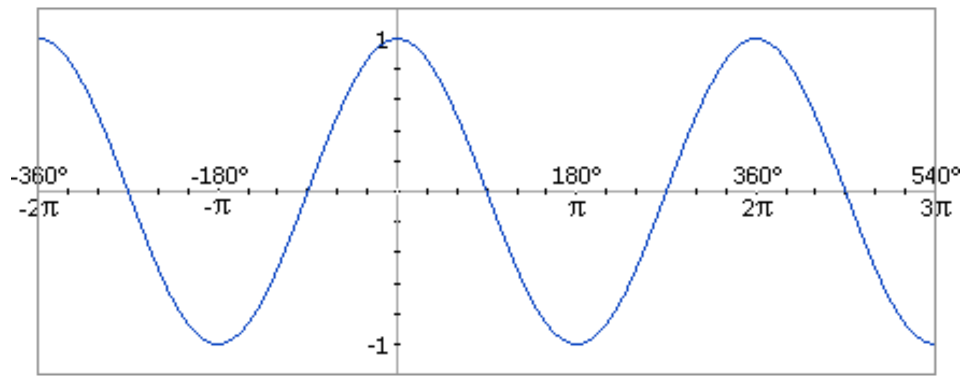
The voltage on an AC power line has this shape

- Sine function: $\sin(\theta) \rightarrow$



Period T = 1/f seconds

- Cosine function: $\cos(\theta) \rightarrow$



Both have max value ± 1

Only difference:

Sin(x=0) starts at 0, rising

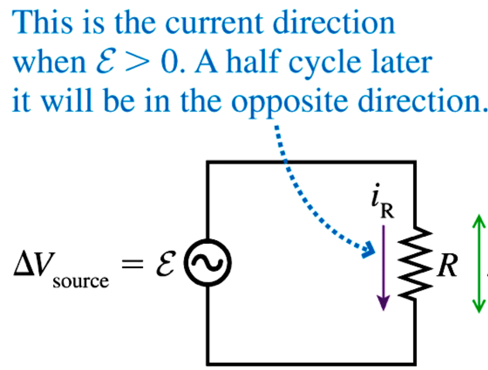
Cos(x=0) starts at 1, falling - same shape, cos is just shifted by π radians

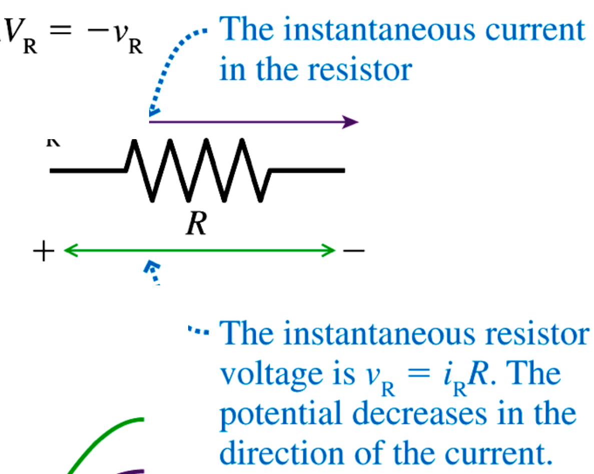
AC Circuits: Resistors

So far we have only discussed behavior of circuits with DC (one-way) currents

For an AC current i_R through a resistor, Ohm's Law gives the potential drop, or *resistor voltage* v_R.

$$v_R = i_R R$$



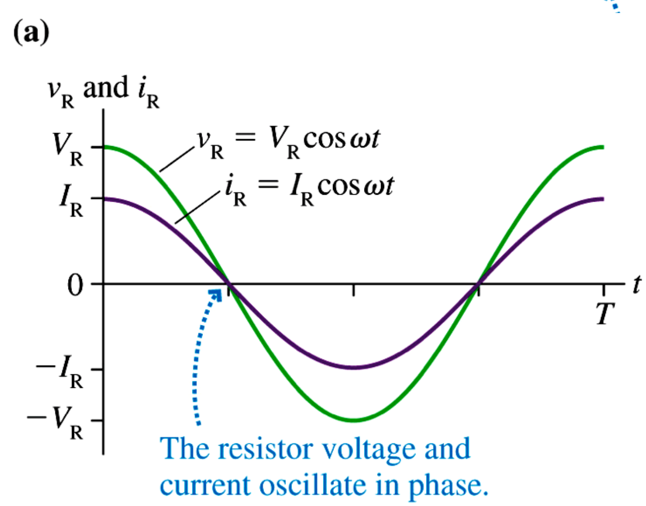


If the resistor is connected in a circuit as shown, then Kirchhoff's loop law tells us:

$$\Delta V_{soruce} + \Delta V_R = \mathcal{E} - v_R = 0$$

$$\mathcal{E}(t) = \mathcal{E}_0 \cos\omega t = v_R$$

$$i_R = \frac{v_R}{R} = \frac{\mathcal{E}_0}{R}\cos\omega t = I_R \cos\omega t$$



Key points here:

1. Ohm's Law applies, but I is time varying, same as $\mathcal{E}$.
2. I(t) has same time dependence as $\mathcal{E}$: I is **in phase with** $\mathcal{E}$: aligned in t

Power in AC Circuits

The instantaneous power supplied by the generator in an AC circuit is $p_{source} = iE$ where i and E are the instantaneous current and emf, respectively.

The power dissipated in a resistor is
$p_R = i_R v_R = i_R^2 R$, with $i_R = I_R \cos \omega t$.
So,
$p_R = I_R^2 R \cos^2 \omega t = \frac{1}{2} I_R^2 R(1+\cos 2\omega t)$.
(math identity: $1+\cos 2x = 2\cos^2 x$)

The average power $P = \langle p_R \rangle$ is the total energy dissipated per second (or over one full cycle, if f < 1 Hz).
In the expression above, the first term is constant, while the second term is alternately positive and negative and will average to zero. So: $P = \frac{1}{2} I_R^2 R$

I and V are in phase in a resistor