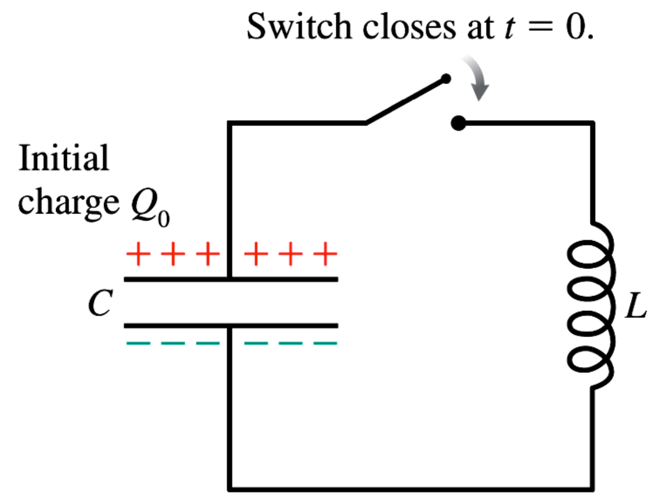
Physics 115
General Physics II

Session 33

AC:
RL vs RC circuits
Phase relationships
RLC circuits



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Lecture Schedule

12-May	Mon	23	DC Circuits & Meters	21.5-21.8
13-May	Tues	24	DC Circuits	21.5-21.8
15-May	Thurs	25	RC circuits	21.6-21.7
16-May	Fri	26	Circuits - Neurons	
19-May	Mon	27	Magnetism	22.1
20-May	Tues	28	Magnetic Force	22.2-22.5
22-May	Thurs	29	Magnetic Fields	22.6-22.7
22-May	Fri	30	Induced EMF, Applications	23.1-23.3
26-May	***holiday***		NO CLASS	
27-May	Tues	31	Energy, RL circuits	23.4-23.8
29-May	Thurs	32	Transformer	23.9-23.10
30-May	**Fri**		**EXAM 3 - Chapters 21,22,23**	
2-Jun	Mon	33	AC circuits	24.1-24.3
3-Jun	Tues	34	AC circuits	24.4-24.5
5-Jun	Thurs	35	Resonance, Applications	24.6
6-Jun	Fri	36	Last class - review	
June 9	**FINAL EXAM**		**2:30-4:20 p.m.**	**Comprehensive**
	Mon		**Monday, June 9, 2014**	

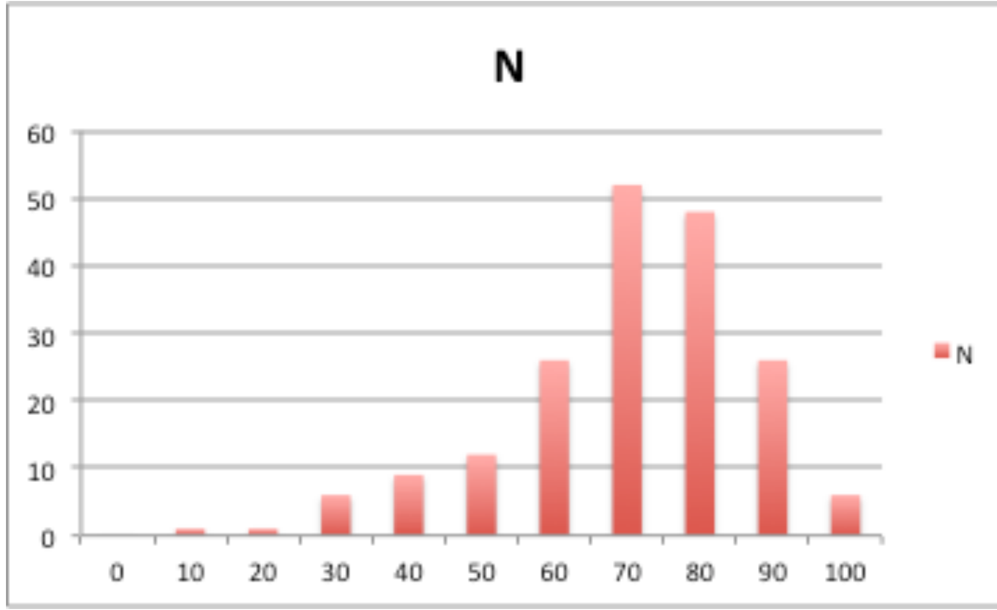
Today

Announcements

- Final exam is one week from today! **2:30 pm, Monday 6/9, here**
    - 2 hrs allowed, will probably take you about 1 hr
    - Comprehensive, but with extra items on material covered after exam 3
    - Final exam will contain ONLY Ch. 24 topics covered in class
    - Usual arrangements, procedures
- Homework set 9 is NOT due Weds night, but **Friday** 6/6, 11:59pm

EXAM 3: Scores back from scan shop, should be posted soon

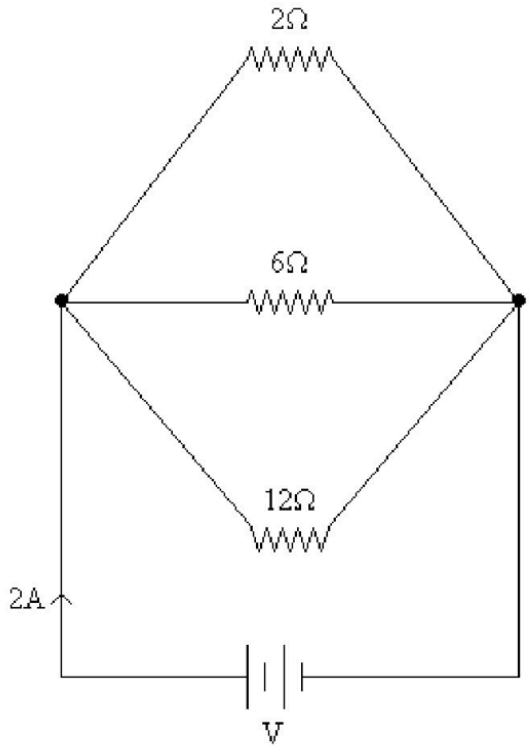
AVG: 66
STD. DEV: 17



1) [4 pts] The length of a certain wire is doubled and at the same time its radius is reduced by a factor of 2. How does the resistance of this wire change ?

A) It is doubled.
B) It is quadrupled.
C) It increases by a factor of 6.
D) It increases by a factor of 8.
E) It is reduced by a factor of 2.
Answer: D

$$R = \rho \frac{L}{A} = \rho \frac{L}{\pi r^2} \quad R' = \rho \frac{2L}{\pi \left(r/2 \right)^2} = \rho \frac{8L}{\pi r^2} = 8R$$



2) [6 pts] Three resistors of values 2 Ω, 6 Ω and 12 Ω are connected across a DC voltage source as shown above.
If the total current from the source is 2.0 A, what is the applied voltage V?
A) 6.0 V
B) 3.0 V
C) 2.0 V
D) 2.7 V
E) 1.5 V

$$\frac{1}{R_{eq}} = \frac{1}{2\Omega} + \frac{1}{6\Omega} + \frac{1}{12\Omega} \rightarrow R_{eq} = 1.33\Omega$$

Answer: D

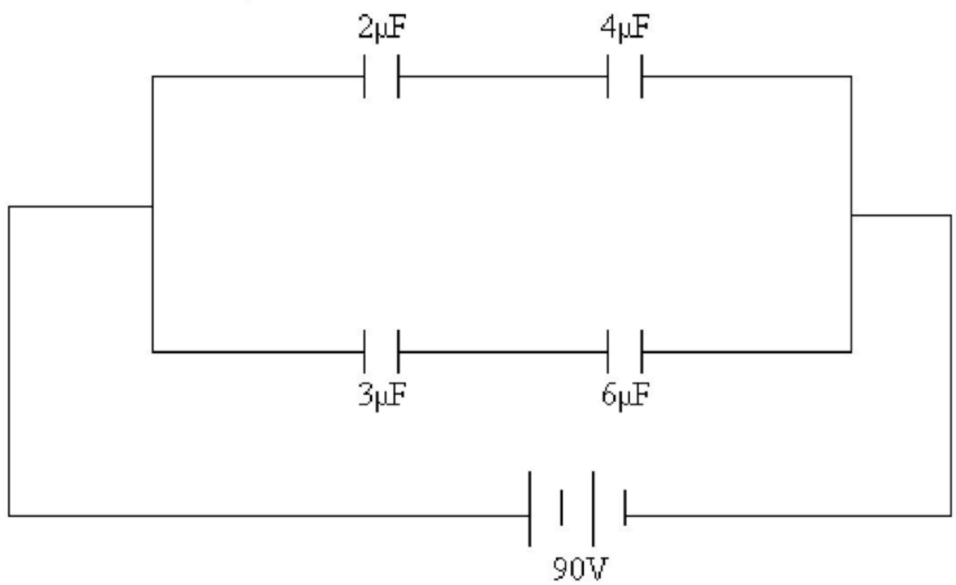
$$V = I\,R_{eq} = 2A\left(1.33\Omega\right) = 2.66V$$

3) (4 pts) In the circuit above, what is the current in the 6 ohm resistor?
A) 0.44 A
B) 0.56 A
C) 0.30 A
D) 1.33 A
E) None of the above

Ans: A $I = V / R = 2.66V / 6\Omega = 0.443A$

4) [6 pts] The power rating of a resistor is 0.80 W. (This means it will be damaged if it is required to dissipate more than 0.80 W) The value of the resistor is 400 Ω.
What is the maximum voltage drop V this resistor can handle?
A) 110 V
B) 17.9 V
C) 170 V
D) 1.80 V
Answer: B

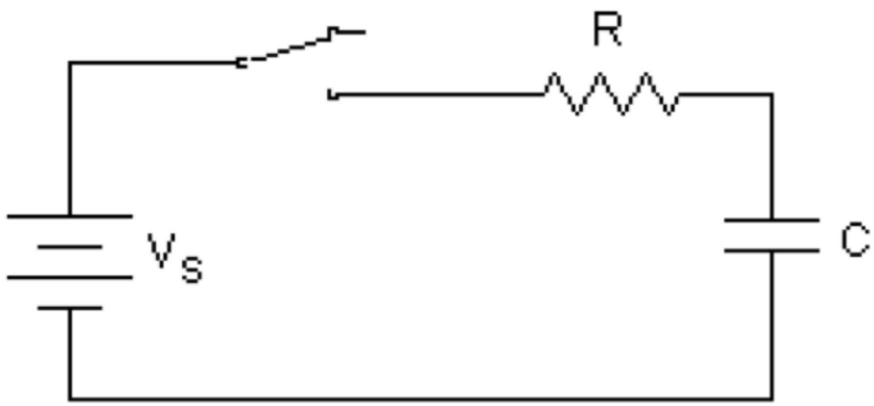
$$P_{MAX} = V^2 / R \Rightarrow V_{MAX}^2 = P_{MAX} R \rightarrow V_{MAX} = \sqrt{0.80W(400\Omega)} = 17.88V$$



5) [6 pts] In the circuit shown above, what is the voltage across the 6-μF capacitor?
A) 10 V
B) 20 V
C) 30 V
D) 60 V
E) 90 V
Answer: C

$$90V = V_6 + V_3 = \frac{Q_6}{6\mu F} + \frac{Q_3}{3\mu F};$$

$$Q_6 = Q_3 \rightarrow \frac{V_6}{V_3} = \frac{3\mu F}{6\mu F} \rightarrow V_3 = 2V_6 \rightarrow 90V = 3V_6 \rightarrow V_6 = 30V$$



(For 6, 7): A capacitor with C=8.0 μF is connected in series with a resistor R=6.0 kΩ, across a DC source with V_S=20 V, and an open switch, as shown above.

6) [6 pts] The switch is closed at t=0. After a long time has passed, approximately how much charge does the capacitor hold?
A) 160 C
B) 1.6 x 10^{-4} C
C) 48 C
D) 4.8 x 10^{-5} C
E) none of the above
Answer: B

$$q(t) = C\mathcal{E}\left(1 - e^{-t/\tau}\right); \quad q_{MAX} = q(t = \infty) = C\mathcal{E} = \left(8.0\times10^{-6}\,\mathrm{F}\right)20V = 1.6\times10^{-4}C$$

7. [6 pts] How long after the switch is closed will the charge on the capacitor be 63% of its maximum charge?

A) 160 microseconds
B) 3.3 milliseconds
C) 48 milliseconds
D) 48 seconds
E) None of the above.

Answer: C

$$q(t) = q_{MAX}\left(1 - e^{-t/\tau}\right) \rightarrow \frac{q(t)}{q_{MAX}} = 0.63 = \left(1 - e^{-t/\tau}\right)$$

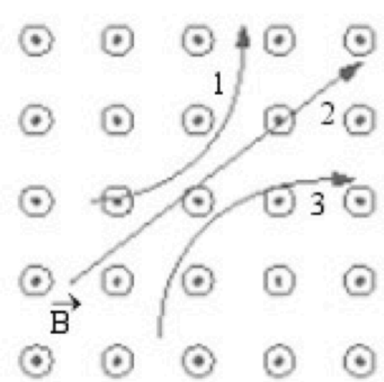
$$\rightarrow e^{-t/\tau} = 0.37 \rightarrow \left(-\frac{t}{\tau}\right) = \ln(0.37) = -1 \rightarrow t = \tau$$

$$\tau = RC = \left(6.0\times10^{3}\ \Omega\right)\left(8.0\times10^{-6}\ \mathrm{F}\right) = 48.0\times10^{-3}\ \mathrm{s} \quad (48\ \mathrm{ms})$$

8) [6 pts] An electron moves at right angles to a magnetic field of 1 tesla . What is its speed if the force exerted on it is 10^{-15} N ? Charge on the electron is -1.6 X 10^{-19} C

A) 4527 m/s
B) 1758
C) 8992
D) 6250
E) 1449
Answer: D

Right angle between v and B, so $F_B = q v B$
$\rightarrow v = F_B/(qB) = 10^{-15}\,\text{N} / \{(1.6 \times 10^{-19}\,\text{C})(1\,\text{T})\} = 0.625 \times 10^4$ m/s



9) [6 pts] Three particles travel through a region of space where the magnetic field is out of the page, as shown above. Which statement correctly describes the electric charge of each of the three particles?
A) 1 is neutral, 2 is negative, and 3 is positive. *No: 1 bends, so it cannot be neutral.*
B) 1 is neutral, 2 is positive, and 3 is negative. *No: 1 bends, so it cannot be neutral.*
C) 1 is positive, 2 is neutral, and 3 is negative. *No: 2 and 3 bend in directions* opposite *to RHR.*
D) 1 is positive, 2 is negative, and 3 is neutral. *No: 3 bends, so it cannot be neutral.*
E) 1 is negative, 2 is neutral, and 3 is positive. *CORRECT: 1 and 3 bend according to RHR*

10) [8 pts] An electron is accelerated from rest through a potential difference of 100 V, and then enters a uniform magnetic field. The electron follows a circular path of radius 1 m . What is the strength of the magnetic field ? Mass of the electron is 9.1 X 10^{-31} kg

A) 34 micro Tesla
B) 19
C) 118
D) 67
E) 351

Answer: A

$$e\Delta V = KE = mv^2/2$$

$$v = \sqrt{2e\Delta V/m} = \sqrt{2\left(1.60\times10^{-19}\,\mathrm{C}\right)\left(100\ \mathrm{V}\right)/\left(9.11\times10^{-31}\,\mathrm{kg}\right)} = 5.92\times10^{6}\ \mathrm{m/s}$$

$$r = \frac{mv}{eB} \rightarrow B = \frac{mv}{er} = \frac{\left(9.11\times10^{-31}\,\mathrm{kg}\right)\left(5.92\times10^{6}\ \mathrm{m/s}\right)}{\left(1.60\times10^{-19}\,\mathrm{C}\right)\left(1m\right)} = 33.7\times10^{-6}\,\mathrm{T}$$

11) [6 pts] A 2-m long wire is carrying a current of 2 A. The wire is placed at an angle of 60° with respect to a magnetic field. If the wire experiences a force of 0.2 N, what is the strength of the magnetic field?
A) 0.02 T
B) 0.03 T
C) 0.04 T
D) 0.05 T
E) 0.06 T
Answer: E

F_B = I l B Sin (θ) (on wire with length l) → B = F_B / (I l Sin (60deg))
B= (0.2N)/{ (2A) (2m) (0.866) = 0.057T

12) [6 pts] A circular loop of wire of radius 0.50 m is in a uniform magnetic field of 0.30 T. The current in the loop is 2.0 A. What is the magnetic torque when the direction of the magnetic field lies in plane of the loop ?

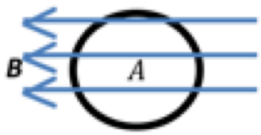
A) zero
B) 0.41 m · N
C) 0.47 m · N
D) 0.52 m · N
E) 0.58 m · N

Answer: C

$\tau = NIAB\sin\theta, \quad N =$ turns, A=area of loop

$\theta =$ angle between *normal* to plane of loop and $\vec{B}$, here θ=90° $\rightarrow \sin\theta = 1$.

$$\tau = (A)IB = (\pi r^2)IB = 3.14(0.5m)^2(2A)(0.3T) = 0.47Nm$$



13) [8 pts] Two long parallel wires carry currents of 20 A and 5.0 A in opposite directions. The wires are separated by 0.20 m. What is the magnitude of the magnetic field midway between the two wires?

A) 1.0×10^{-5} T
B) 2.0×10^{-5} T
C) 3.0×10^{-5} T
D) 4.0×10^{-5} T
E) 5.0×10^{-5} T

Answer: E

Contributions from the 2 wires point in the same direction (RHR: same direction for B between wires, for opposite directions of I), so they add: total field magnitude is (4.0+1.0) × 10^{-5} T.

$$B_1 = \frac{\mu_0 I_1}{2\pi r_1}, \quad B_2 = \frac{\mu_0 I_2}{2\pi r_2}$$

$$r_2 = r_1 = 0.10m,$$

$$|B_1| = \frac{\mu_0 I}{2\pi r} = \frac{(1.26\times10^{-6}T\cdot m/A)20A}{2\pi(0.1m)} = 4.0\times10^{-5}T$$

$$|B_2| = \frac{(1.26\times10^{-6}T\cdot m/A)5A}{2\pi(0.1m)} = 1.0\times10^{-5}T$$

14) [6 pts] The current in a 110 V reading lamp is 2 A . If the cost of electricity is 10 cents per kilowatt-hour , how much does it cost to operate the lamp for 7 hours ?

A) 17.3 cents
B) 9.2
C) 23.2
D) 5.8
E) 15.4

P=IV=220W; energy = 0.220kW (7hr)=1.54 kw-hr → cost = 1.54*(10¢)=15.4 cents

15) [4 pts] How many turns should a 10-cm long solenoid have, if it is to generate a magnetic field of 1.5×10^{-3} T when carrying 1.0 A of current?

A) 12
B) 15
C) 119
D) 1194
E) 3183

Answer: C

$$B_{SOLENOID} = \mu_0 n I \rightarrow n = \frac{B}{\mu_0 I} = \frac{\left(1.5\times10^{-3}T\right)}{\left(1.26\times10^{-6}T\cdot m/A\right)1.0A} = 1190/m$$

$$N = nL = 1190/m\left(0.1m\right) = 119$$

16. [6 pts] How would the self-inductance, L, of a solenoid coil change if you increase its radius by a factor of two and increase its length by a factor of two? (Number of turns remains the same),

A) L would not change.
B) L would be doubled.
C) L would be quadrupled.
D) L would be halved.
E) L would be reduced by 4

Ans: B

$$L_{\text{solenoid}} = \frac{\mu_0 N^2 A}{\ell}$$

$$r \to 2r, \quad \ell \to 2\ell : L'_{\text{solenoid}} = \frac{\mu_0 N^2 4A}{2\ell} = 2L_{\text{solenoid}}$$

17. [6 pts] The primary coil of a transformer has 100 turns and its secondary coil has 400 turns. If the ac current in the secondary coil is 2 A, what is the current in its primary coil?

A) 2 A
B) 8 A
C) 1/2 A
D) 1/4 A
E) 4 A

Answer: B

$(N_2 / N_1) = (I_1 / I_2) \rightarrow I_1 = I_2 \ (N_2 / N_1) = 2A\,(400/100) = 8A$

Root-Mean-Square V and I

"RMS" stands for "root-mean-square"
1. square the quantity (gets rid of polarity)
2. average the squared values over time,
3. take the square root of the result.

P is proportional to I^2 , so RMS gives us the equivalent DC voltage in terms of power:
DC voltage that would dissipate the same power in the resistor (same joules/sec heating effect)

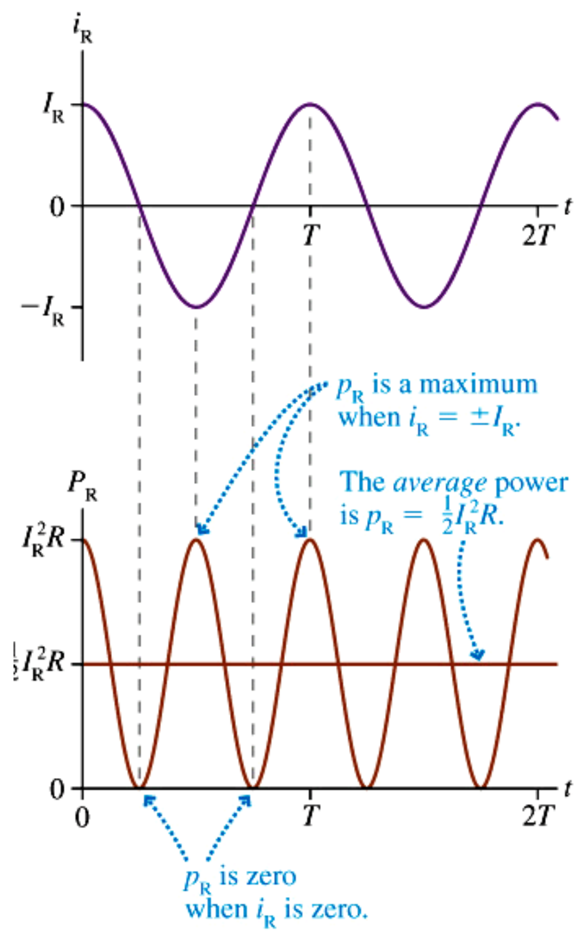
$$P = \tfrac{1}{2} I_R^2 R = \left(\frac{I_R}{\sqrt{2}}\right)^2 R = (I_{\mathrm{RMS}})^2 R$$

I_R and V_R = peak values

$$I_{\mathrm{RMS}} \equiv \frac{I_R}{\sqrt{2}} \qquad V_{\mathrm{RMS}} \equiv \frac{V_R}{\sqrt{2}} \qquad \mathrm{E}_{\mathrm{RMS}} \equiv \frac{\mathrm{E}_R}{\sqrt{2}}$$

$$P_R = (I_{\mathrm{RMS}})^2 R = \frac{(V_{\mathrm{RMS}})^2}{R} = I_{\mathrm{RMS}} V_{\mathrm{RMS}}$$

$$\boxed{P_{\mathrm{source}} = I_{\mathrm{RMS}} \mathrm{E}_{\mathrm{RMS}}}$$

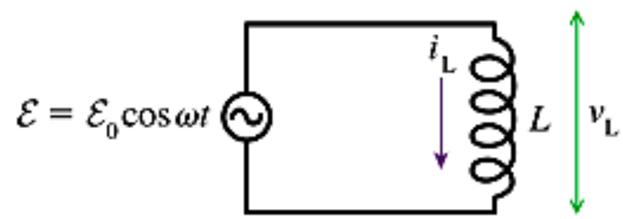


Inductors in AC Circuits

For AC current i_R through an inductor:
The changing current produces an *induced EMF = voltage* v_L.

$$v_L = L\frac{\Delta i_L}{\Delta t}$$

For the AC circuit as shown, Kirchhoff's loop law tells us: $\Delta V_{source} + \Delta V_L = \mathcal{E} - v_L = 0$



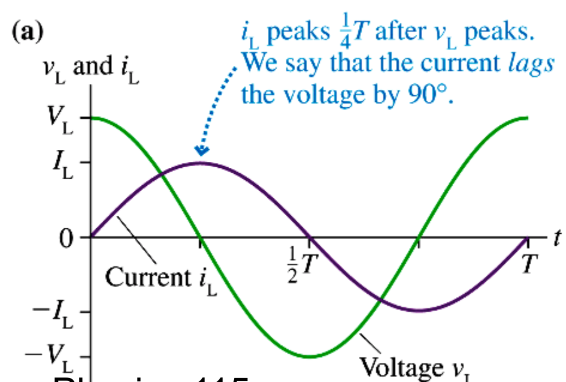
$$\mathcal{E}(t) = \mathcal{E}_0 \cos\omega t = v_L$$

"It can be shown" (calculus) that:

$$\Delta i_L = \left(\frac{1}{L}\right) v_L \Delta t \rightarrow i_L = \frac{V_L}{\omega L}\sin\omega t = \frac{V_L}{\omega L}\cos\left(\omega t - \frac{\pi}{2}\right) = I_L \cos\left(\omega t - \frac{\pi}{2}\right)$$

For inductors, ωL acts like R in Ohm's Law:
I = V/ R →
$I_L = V/(\omega L)$

Notice: the inductor current i_L *lags* the voltage v_L by $\pi/2$ radians $= 90^0$, so that i_L peaks T/4 later than v_L.



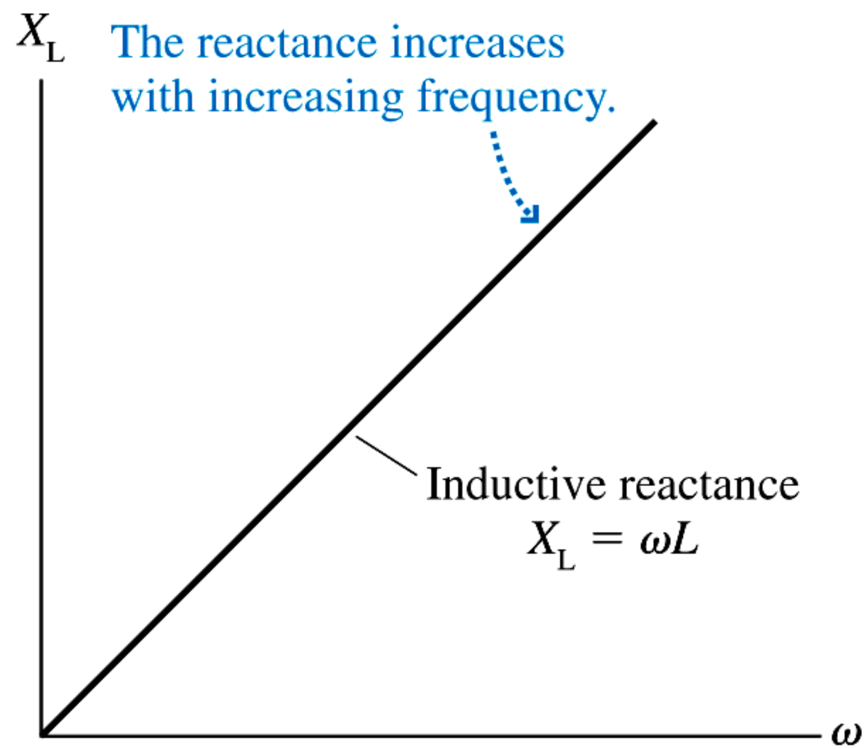
Inductive Reactance

For AC circuits we define a resistance-like quantity, measured in ohms, for inductance. It is called the *inductive reactance* X_L:

$$\boxed{X_L \equiv \omega L = 2\pi f L}$$

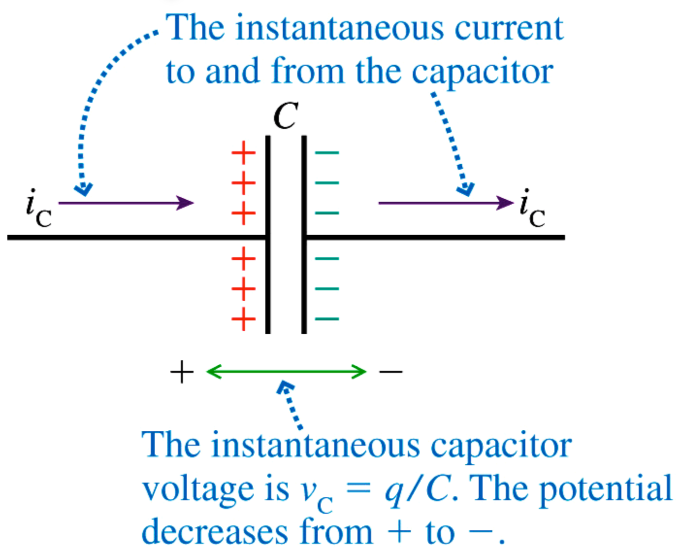
We can then use a form of Ohm's Law to relate the *peak* voltage V_L, the peak current I_L, and the inductive reactance X_L in an AC circuit:

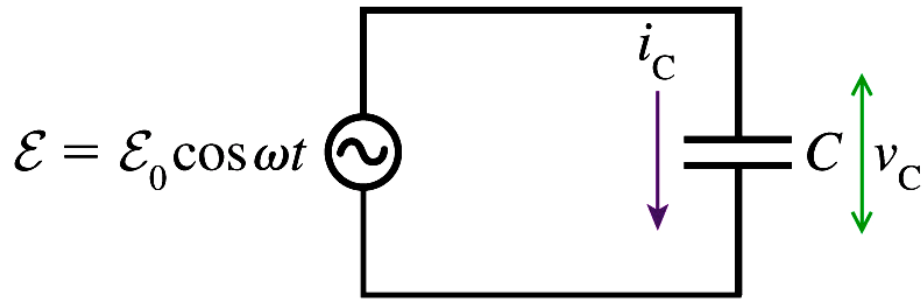
$$I_L = \frac{V_L}{X_L} \quad \text{and} \quad V_L = I_L X_L$$



Reactance is not the same as resistance: it depends on f (time variation), and current is not in phase with voltage!

Capacitors in AC Circuits





For AC current i_C through a capacitor as shown, the capacitor voltage $v_C = E = E_0 \cos \omega t = V_C \cos \omega t$.

The charge on the capacitor will be $q = C\, v_C = C\, V_C \cos \omega t$.

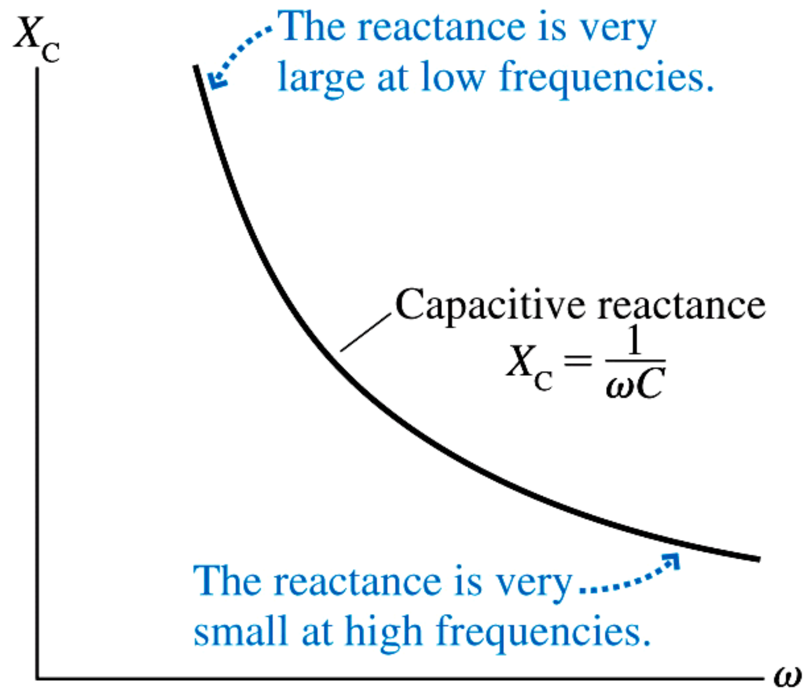
$$i_C = \frac{dq}{dt} = \frac{d}{dt}\left(CV_C \cos \omega t\right) = -\omega C V_C \sin \omega t \qquad i_C = \omega C V_C \cos(\omega t + \pi/2)$$

So: AC current through a capacitor leads the capacitor voltage by $\pi/2$ rad or 90^0.

Capacitive Reactance

For AC circuits we also define a resistance-like quantity, measured in ohms, for capacitance. It is called the *capacitive reactance* X_C :

$$\boxed{X_C \equiv \frac{1}{\omega C} = \frac{1}{2\pi f C}}$$



Again, we use a form of Ohm's Law to relate the *peak* voltage V_C, the peak current I_C, and the capacitive reactance X_C in an AC circuit:

$$I_C = \frac{V_C}{X_C} \quad \text{and} \quad V_C = I_C X_C$$

Capacitive vs inductive reactance:
X_L is proportional to f and L
X_C is proportional to 1/f and 1/C
Current leads in capacitors, lags in inductors

Example: Capacitive Current

A 10 μF capacitor is connected to a 1000 Hz oscillator with a peak emf of 5.0 V.

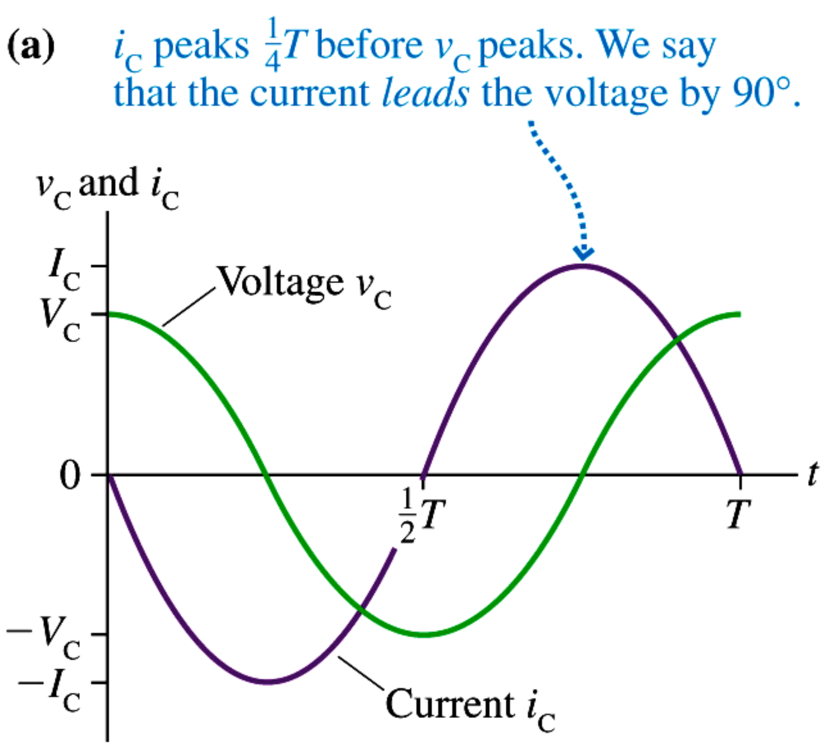
What is the peak current in the capacitor?

$$X_C(1000\ \text{Hz}) = \frac{1}{2\pi(1000\ \text{s}^{-1})(1.0\times10^{-5}\ \text{F})} = 15.9\ \Omega$$

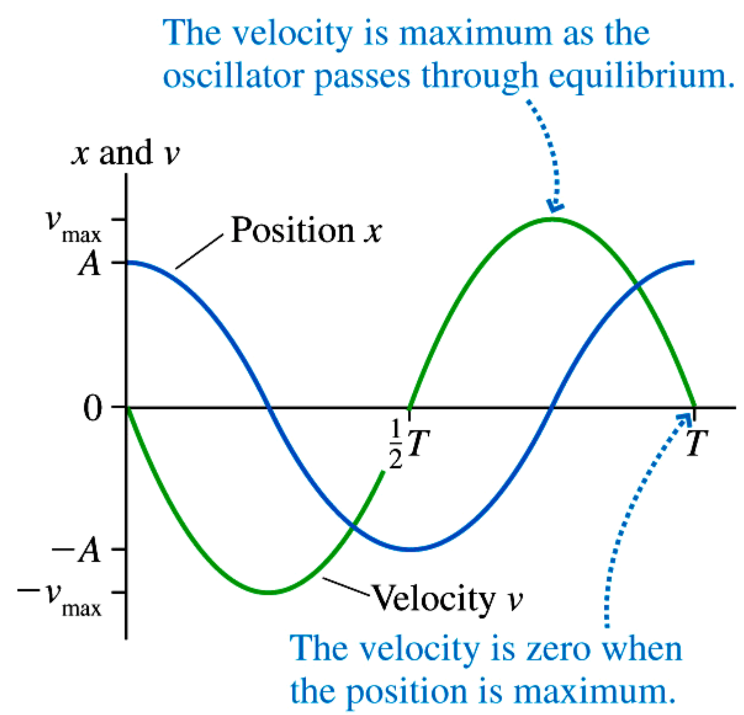
$$I_C = \frac{V_C}{X_C} = \frac{(5.0\ \text{V})}{(15.9\ \Omega)} = 0.314\ \text{A}$$

Capacitors and springs

AC current through a capacitor *leads* the capacitor voltage by π/2 rad or 90^0.

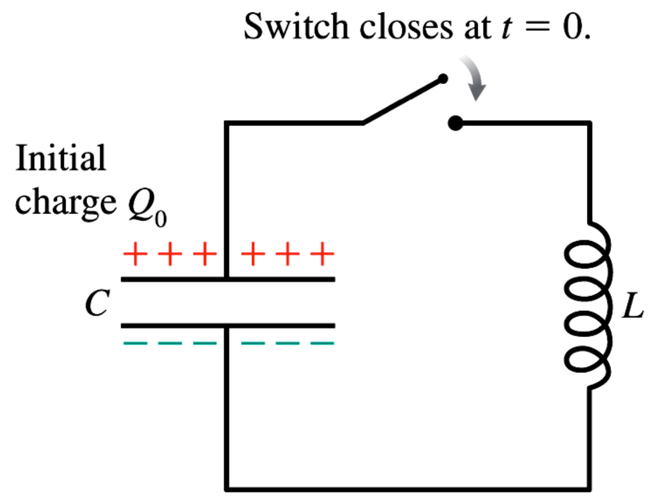


This is analogous to the behavior of the position and velocity of a mass-and-spring harmonic oscillator.



LC Circuits

A charged capacitor is analogous to a stretched spring :
stores energy even when the charge is not moving.
An inductor resembles a moving mass (remember the flywheel), stores energy only when charge is in motion.

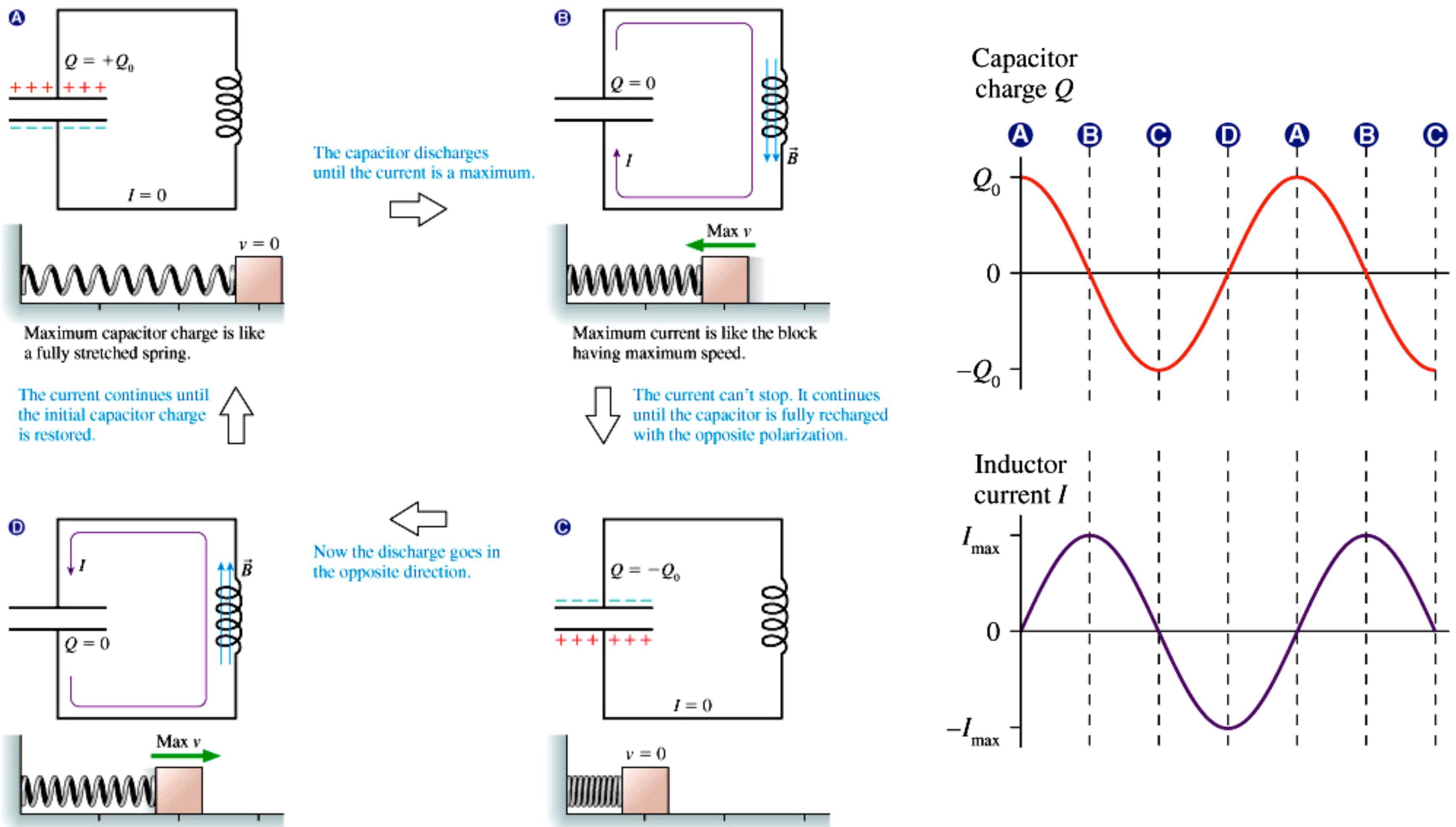


Mass + spring = an oscillator.
What about a capacitor + inductor?
When the switch is closed in the circuit shown in the diagram:

1. The capacitor discharges, creating a current in the inductor.
2. There is no dissipative element (resistor = friction) in this system, so its energy is conserved.
3. So, when the capacitor charge reaches 0, all of its stored (E field) energy must now be stored in the inductor's B field.
4. Then the current in the inductor falls as it charges the capacitor in the opposite direction.

And so on …

The Oscillation Cycle



Clicker Question

In the circuit above, the frequency is initially f Hz

If the generator's frequency is doubled, to $2f$, what happens to the inductor's reactance X_L ?

A. It doubles X_L is proportional to f
B. It quadruples
C. It is unchanged
D. It is 50% smaller
E. It is $\frac{1}{4}$ the original value