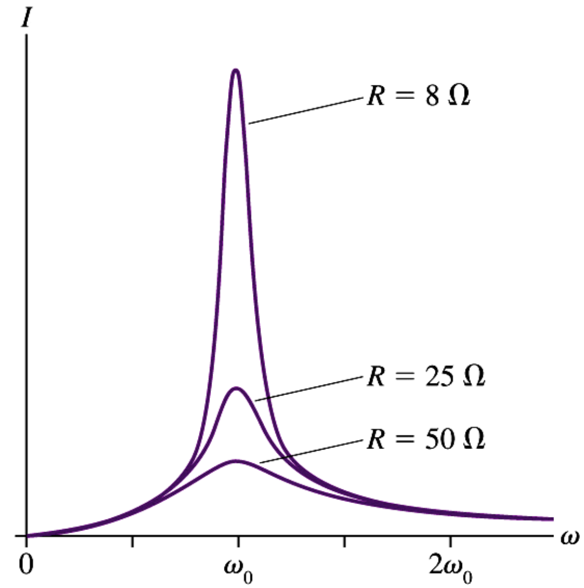
Physics 115
General Physics II



Session 34

Inductors, Capacitors, and RLC circuits

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Lecture Schedule

12-May	Mon	23	DC Circuits & Meters	21.5-21.8
13-May	Tues	24	DC Circuits	21.5-21.8
15-May	Thurs	25	RC circuits	21.6-21.7
16-May	Fri	26	Circuits - Neurons	
19-May	Mon	27	Magnetism	22.1
20-May	Tues	28	Magnetic Force	22.2-22.5
22-May	Thurs	29	Magnetic Fields	22.6-22.7
22-May	Fri	30	Induced EMF, Applications	23.1-23.3
26-May	***holiday***		NO CLASS	
27-May	Tues	31	Energy, RL circuits	23.4-23.8
29-May	Thurs	32	Transformer	23.9-23.10
30-May	**Fri**		**EXAM 3 - Chapters 21,22,23**	
2-Jun	Mon	33	AC circuits	24.1-24.3
3-Jun	Tues	34	AC circuits	24.4-24.5
5-Jun	Thurs	35	Resonance, Applications	24.6
6-Jun	Fri	36	Last class - review	
June 9	**FINAL EXAM**		**2:30-4:20 p.m.**	**Comprehensive**
	Mon		**Monday, June 9, 2014**	

Today

Announcements

Final exam is **2:30 pm, Monday 6/9, here**

- 2 hrs allowed for exam (really: 1 to 1.5 hr), comprehensive, but with extra items on material covered after exam 3
- Usual arrangements
- If you took midterms with section B please do **NOT** do that for final – everyone takes it with our group on Monday

- Final exam will contain ONLY Ch. 24 topics covered in class
- I will be away all next week
  - Final exam will be hosted by Dr. Scott Davis
  - If you need to see me, do so this week...
  - Exam scores and grade data will be posted before the end of next week, final grades before Tuesday 6/17
- TA Songci Li will have office hour **MONDAY** 12:30-1:30, B-442 PAB
- Homework set 9 is due **Friday** 6/6 11:59pm

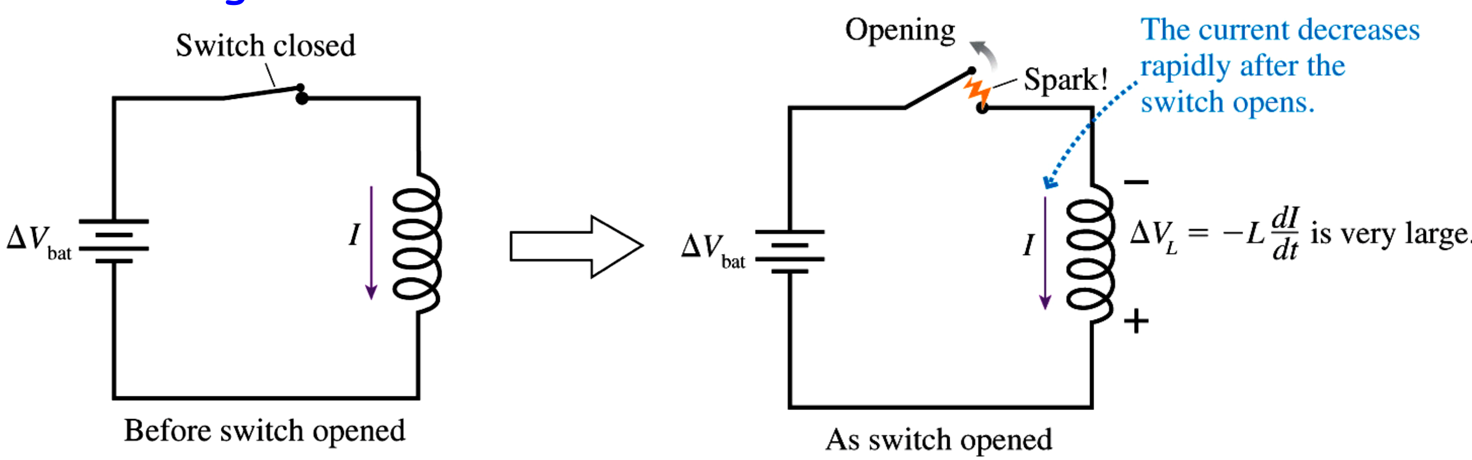
Announcements

"How best to study for final? "

- Review and work to understand what you did not get right when you did HW problems, quizzes, or mid-term exam questions.
- Final Exam will not go into tricky details or fine points! Focus on main ideas
- A few practice questions for ch. 24 will be posted tonight, reviewed Friday in class

Inductors can make sparks

Electromagnet circuit

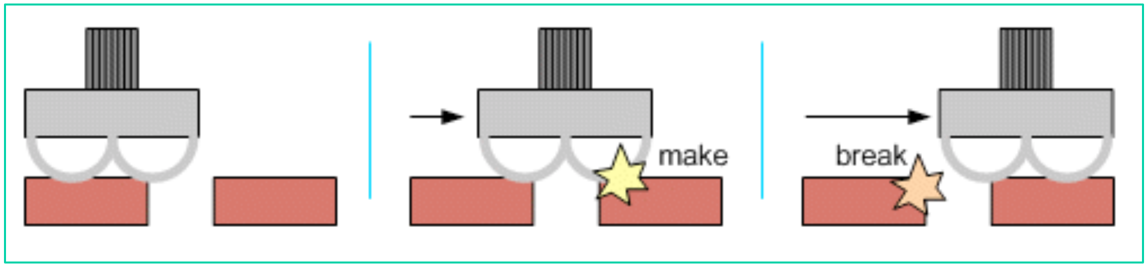


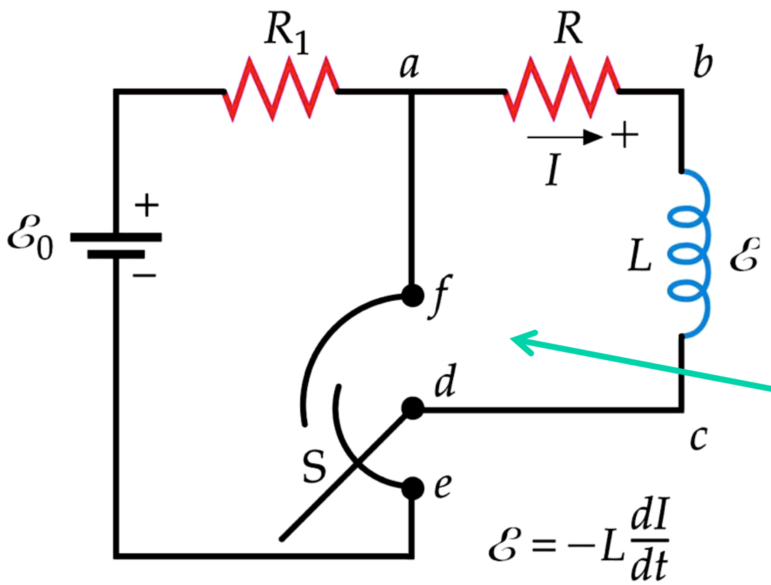
If we quickly interrupt DC current flow through an inductor, the back-EMF may cause a very large voltage ($L\, dI/dt$) across its terminals. The induced V typically causes an arc (spark) across the switch or broken wire that is breaking the current.

Example: large electric motors act like inductors – a simple on/off switch would pull a spark when opened

Sparks can damage switches or cause fires, so we use special switch arrangements in such circuits

Make-Before-Break Switches





Sliding contacts: d-f is closed before d-e opens

Special "make-before-break" switches are used for inductive circuits: the inductor is shorted across a resistor before the switch actually opens the circuit. R dissipates the current generated by back-EMF, and R_1 keeps the EMF source from being shorted out.

Example:
Large Voltage across an Inductor

A 1.0 A current passes through a 10 mH inductor coil. What potential difference is induced across the coil if the current drops to zero in 5 μs?

$$\frac{dI}{dt} = \frac{\Delta I}{\Delta t} = \frac{(\text{-10. A})}{(5.0\times10^{-6}\ \text{s})} = -2.0\times10^{5}\ \text{A/s}$$

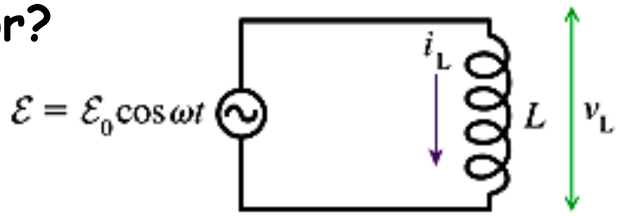
$$\Delta V_{\text{L}} = -L\frac{dI}{dt} == -(0.010\ \text{H})(-2.0\times10^{5}\ \text{A/s}) = 2000\ \text{V}$$

Big jolt from a small current and inductance! Where does the energy for this come from…?

Example: Inductive reactance

A 10 H inductor is connected to a 1000 Hz oscillator with a peak emf of 5.0 V.

What is the RMS current in the inductor?



$$X_L(1000\ \text{Hz}) = 2\pi f L = 2\pi(1000\ \text{s}^{-1})(10\ \text{H}) = 6.28 \text{ x } 10^4\ \Omega$$

$$I_{L(PEAK)} = \frac{V_{PEAK}}{X_L} = \frac{(5.0\ \text{V})}{(6.28 \text{ x } 10^4\ \Omega)} = 8 \text{ x } 10^{-5}\ \text{A} \quad (80\ \mu\text{A})$$

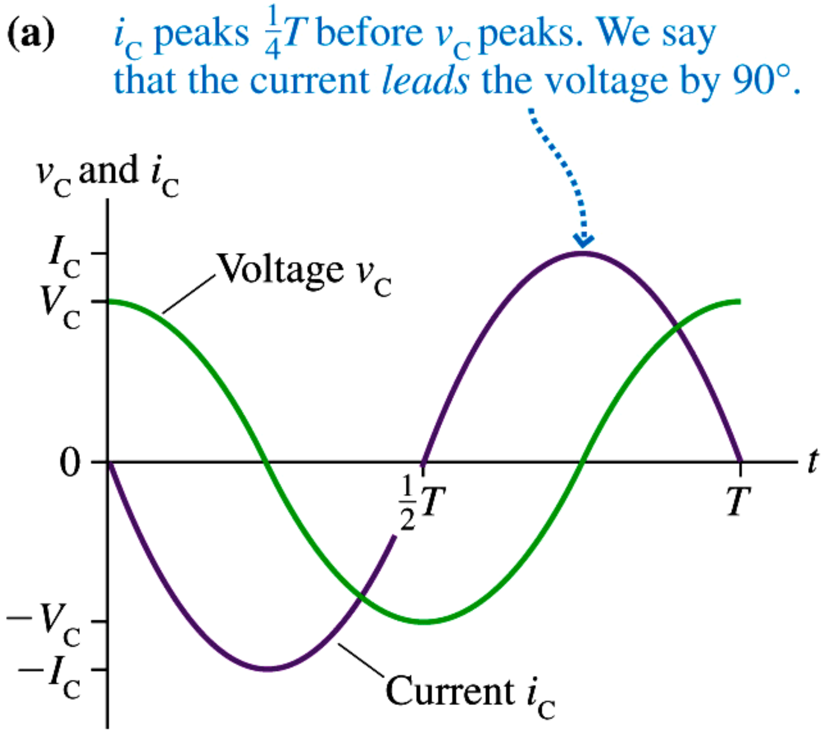
$$I_{RMS} = \frac{I_{PEAK}}{\sqrt{2}} = 0.707 I_{PEAK} = 56\ \mu\text{A}$$

Remember:
Reactance does not *dissipate* energy like a resistor: energy is *stored* in electromagnetic fields

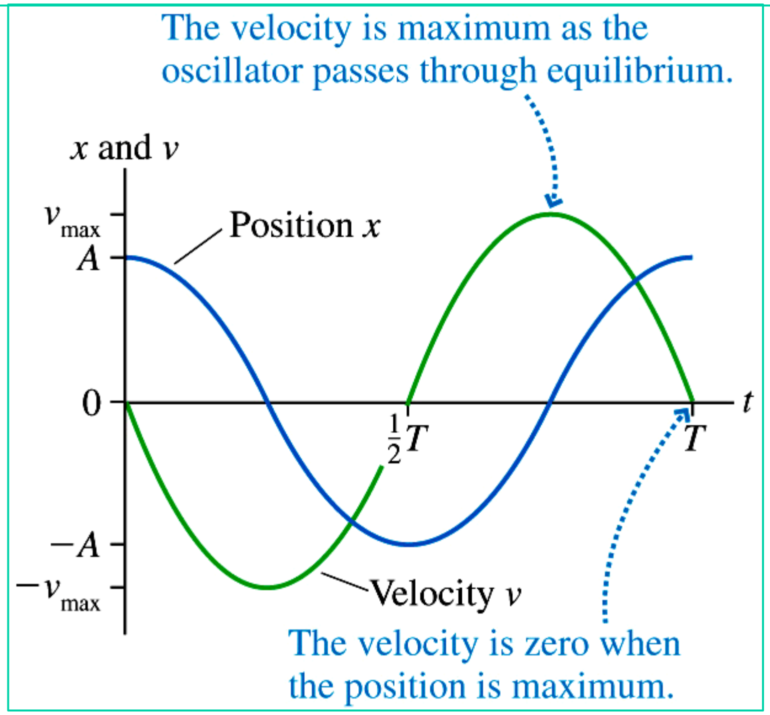
Capacitors and springs

AC voltage and current in reactance are related like position and velocity in a spring+mass system: when one is max the other is zero

AC current through a capacitor *leads* the capacitor voltage by π/2 rad or 90⁰.



This is just like the relationship of the position and velocity for a mass + spring, or a pendulum.



LC circuits - resonance

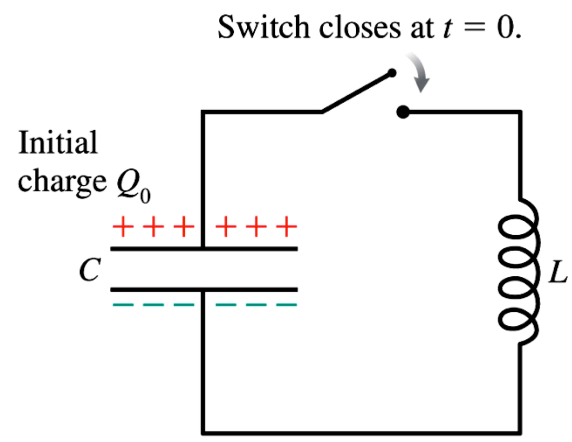
For an LC circuit, suppose we put charge Q on the capacitor initially.

Once the switch closes,

charge flows from C through L (E field decreases, B field increases) and back again: oscillation of current flow (AC).

(if we really had no R, it would go on forever)

"It can be shown" that for this situation,



Q varies sinusoidally:

$$Q(t) = Q_{PEAK}\cos(\omega t)$$

Calculus fact: for this $Q(t)$,

$$I(t) = -\omega Q_{PEAK}\sin(\omega t)$$

$$where\ \omega = \frac{1}{\sqrt{LC}} = 2\pi f \rightarrow f = \frac{1}{2\pi\sqrt{LC}}$$

$(\omega = rad/s)$ $(f = 1/s = Hz)$

Oscillation frequency depends only on L and C

This is called the resonant frequency for the LC combination

Example:
An AM Radio tuning circuit

You have a 10mH inductor. What capacitor do you need with it to make resonant circuit with a frequency of 920 kHz? (This frequency is near the center of the AM radio band.)

$$\omega = 2\pi f = 2\pi(9.20\times10^5 \text{ s}^{-1}) = 5.78\times10^6 \text{ s}^{-1}$$

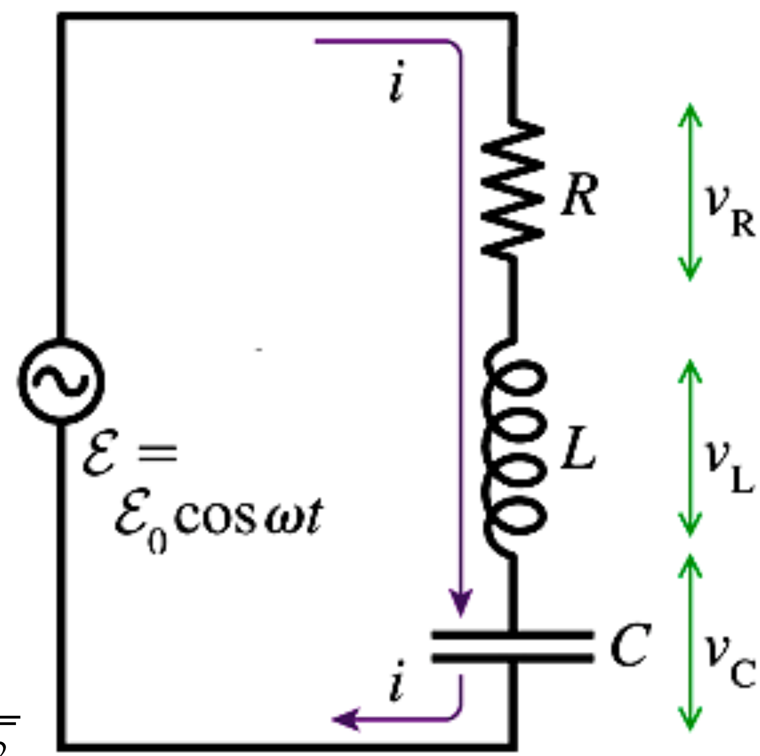
$$C = \frac{1}{\omega^2 L} = \frac{1}{(5.78\times10^6 \text{ s}^{-1})^2(1.0\times10^{-2} \text{ H})} = 3.0\times10^{-11} \text{ F} = 30 \text{ pF}$$

Such circuits were used to tune in on desired stations in old radios: now tuners are built into complex microchips (integrated circuits) for radio receivers

The Series RLC Circuit

Now add a resistor in series with the inductor and capacitor.
The same current i passes through all of the components.

Fact: The C and L reactances create currents with $\pm 90^o$ phase shifts, so their contributions end up 180^o out of phase – tending to cancel each other.
So the net reactance is $X = (X_L - X_C)$



$$I = \frac{\mathcal{E}_0}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{\mathcal{E}_0}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

$$\sqrt{R^2 + (X_L - X_C)^2} = Z$$

Z = "**Impedance**" : resistance and/or reactance

$$\mathcal{E}_0^2 = V_R^2 + (V_L - V_C)^2 = \left[R^2 + (X_L - X_C)^2\right] I^2$$

Impedance and resonance for RLC

We define the *impedance* Z of the circuit as:

$$Z \equiv \sqrt{R^2 + (X_L - X_C)^2}$$
$$= \sqrt{R^2 + (\omega L - 1/\omega C)^2}$$

Then $I = \mathcal{E} / Z$ (Peak, or RMS – here we mean peak values)

If circuit includes no C or L, then Z is just the resistance.

If t frequency f is just such that $X_L = X_C$, we get resonance: minimum possible Z.
Then the circuit "looks like" only the resistor. Current is maximum.

Notice: if there *are* reactances in addition to R, they do not contribute to RMS power dissipation – **but** the circuit has to handle the reactive currents they produce (eg, wire sizes may need to be be larger)

Series RLC Resonance

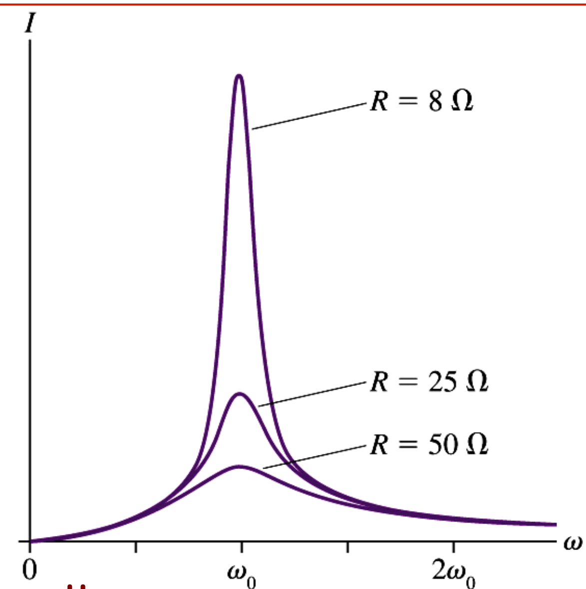
$$I = \frac{\mathcal{E}_0}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} = \frac{\mathcal{E}_0}{Z(\omega)}$$

The current I will be a maximum when ωL=1/ωC. This defines the *resonant frequency* ω_0:

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Note ("cultural comment, not on test"):
Resonance is an important phenomenon in physics! (Example: Tacoma Narrows Bridge*)
Off-resonance, the current is given by

$$I = \frac{E_0}{\sqrt{R^2 + (L\omega)^2 \left[1 - \left(\frac{\omega_0}{\omega}\right)^2\right]^2}}$$



The resonance is sharper if the resistance is smaller.
(analogy: mass + spring *with friction*: greater friction diminishes the amplitude of motion rapidly.

* https://archive.org/details/CEP176

quiz

- Which of the following is **TRUE** when a circuit with R, L, C in series is at its **resonant frequency**?

A. Net impedance = 0
B. Capacitive reactance = Inductive reactance
C. EMF source "sees" only reactance, not R
D. The capacitor explodes
E. None of the above