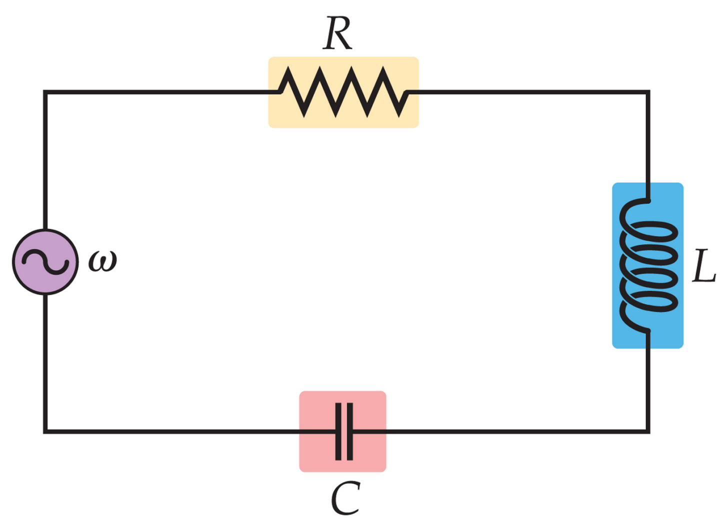
Physics 115
General Physics II

Session 35

AC circuits
Reactances
Phase relationships
Evaluation



- **R. J. Wilkes**
- Email: phy115a@u.washington.edu

Lecture Schedule

12-May	Mon	23	DC Circuits & Meters	21.5-21.8
13-May	Tues	24	DC Circuits	21.5-21.8
15-May	Thurs	25	RC circuits	21.6-21.7
16-May	Fri	26	Circuits - Neurons	
19-May	Mon	27	Magnetism	22.1
20-May	Tues	28	Magnetic Force	22.2-22.5
22-May	Thurs	29	Magnetic Fields	22.6-22.7
22-May	Fri	30	Induced EMF, Applications	23.1-23.3
26-May	***holiday***		NO CLASS	
27-May	Tues	31	Energy, RL circuits	23.4-23.8
29-May	Thurs	32	Transformer	23.9-23.10
30-May	**Fri**		**EXAM 3 - Chapters 21,22,23**	
2-Jun	Mon	33	AC circuits	24.1-24.3
3-Jun	Tues	34	AC circuits	24.4-24.5
5-Jun	Thurs	35	Resonance, Applications	24.6
6-Jun	Fri	36	Last class - review	
June 9	**FINAL EXAM**		**2:30-4:20 p.m.**	**Comprehensive**
	Mon		**Monday, June 9, 2014**	

Today

Announcements

Please pick up class evaluation forms at front of room – pencils available if needd

Formula sheet(s) for final exam are posted in slides directory

- Final exam is **2:30 pm, Monday 6/9, here**
  - 2 hrs allowed, (really, 1.5 hr needed),
  - Comprehensive, but with extra items on material covered after exam 3
  - Usual arrangements
  - I will be away all next week, Dr. Scott Davis will be your host
- Homework set 9 is due tomorrow, **Friday** 6/6, 11:59pm

Reminder: Grading scheme

1. Midterm Exams: sum of best 2 out of 3 midterm exams, max = 200
   - We must rescale midterm 2 scores (exams 1 and 3 had very similar averages and standard deviations):

     100s remain 100s, all other scores will be scaled:

     Z=(your score – average)/std.dev, [original avg was 77, SD=19]

     new exam 2 score = adjusted avg + SD*Z = 67 + 18*Z

     (So new exam scores will have average 67 and SD=18)
2. Clicker Quizzes: sum of **best** 10 out of 21 quizzes, max = 30
   - 0 pts if no entry, 1 if wrong answer, 3 if correct
3. Webassign Homework sets: sum of best 7 out of 9, max = 700
4. Final sum (max 100 pts) = 100pts*

[0.4*(exams/200) + 0.3*(final/150) + 0.15*quiz/30 + 0.15* HW/700]

Course grade is based on this sum. Class average will be 3.0

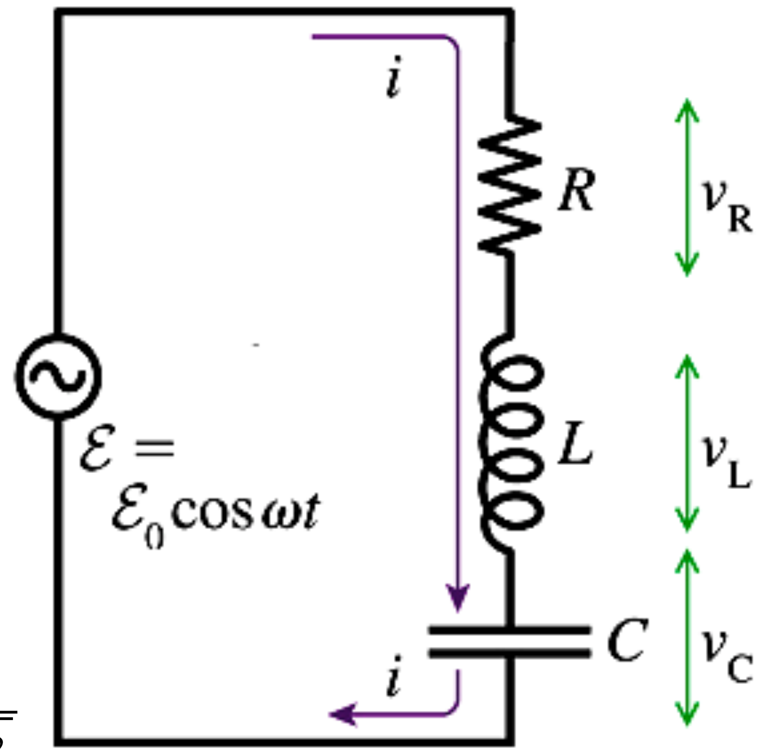
Sums and grades will be posted on Catalyst gradebook next week

Last time

The Series RLC Circuit

Now add a resistor in series with the inductor and capacitor.
The same current i passes through all of the components.

Fact: The C and L reactances create currents with $\pm 90^\circ$ phase shifts, so their contributions end up 180° out of phase – tending to cancel each other.
So the net reactance is $X = (X_L - X_C)$



$$I = \frac{\mathcal{E}_0}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{\mathcal{E}_0}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

$$\sqrt{R^2 + (X_L - X_C)^2} = Z$$

Z = "**Impedance**" : resistance and/or reactance

$$\mathcal{E}_0^2 = V_R^2 + (V_L - V_C)^2 = \left[R^2 + (X_L - X_C)^2\right] I^2$$

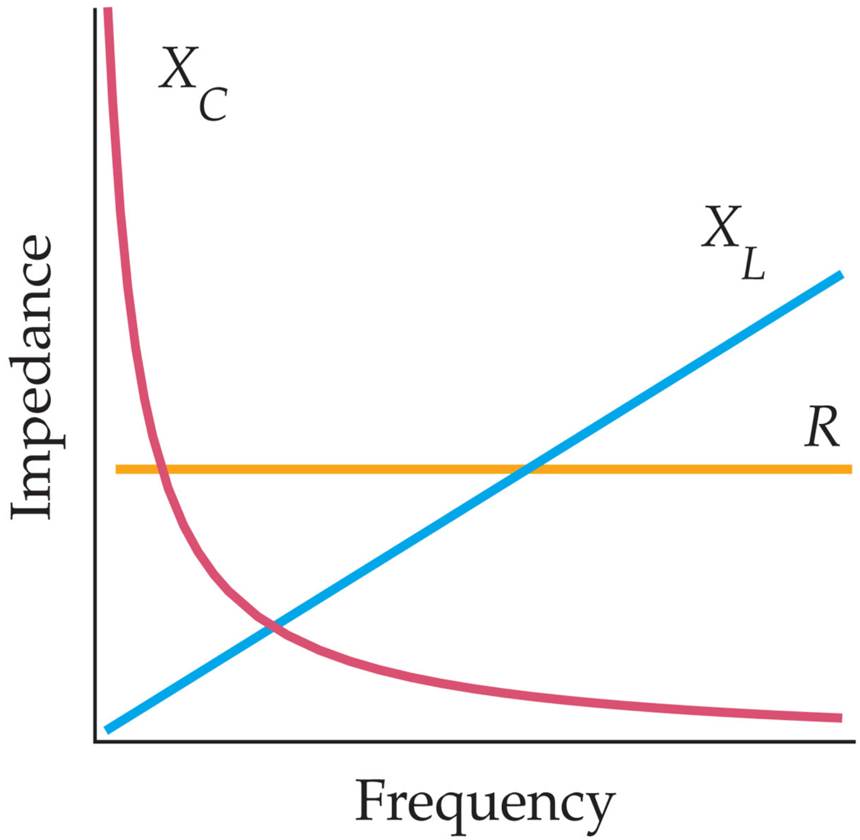
At resonant $\omega = 1/\sqrt{[LC]}$:
$X_L = X_C \rightarrow$ Z=minimum = R

Reactance and resistance: f dependence

Resistance R does not depend on frequency: R = constant

Capacitive reactance is inversely proportional to frequency: $X_C = 1/(\omega C)$

Inductive reactance is proportional to frequency: $X_L = \omega L$



Phase relationships in AC circuits

Useful picture to help understand phase relationships:

For AC circuit with only R,

$V(t)=V_{max} \sin(\omega t)$, where $\omega=2\pi f$

(ω = radians/s, f=cycles/s)

Imagine V (or I) as a vector of length V_{max} that rotates (convention: CCW direction) around the z, axis with angular speed ω .

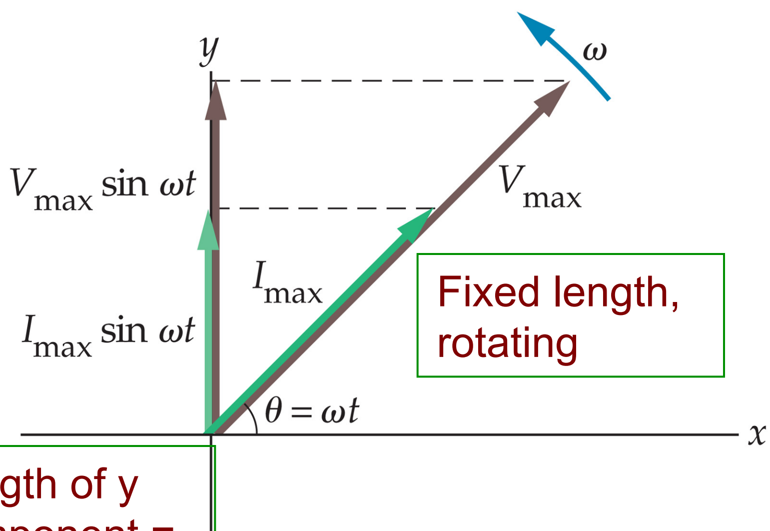
Then the instantaneous V(t) at any time t is the projection of this vector on the y-axis:

$V(t)=V_{max} \sin(\omega t)$

For R only, I is *in phase* with V, so

$I(t) = I_{max} \sin(\omega t)$,

where $I_{max} = V_{max} / R$



Length of y component = V(t) or I(t)
V and I are aligned

http://www.kwantlen.ca/science/physics/faculty/mcoombes/P2421_Notes/Phasors/sine.gif

7

Phasor diagrams

We call these rotating vectors phasors:
Phasor diagrams help us sort out phase relationships

For AC circuit with only C,

V(t) is not in phase with I(t),

It lags I(t) by $\pi/2 = 90^o$ in phase

Picture the pair of phasors rotating around the z axis:

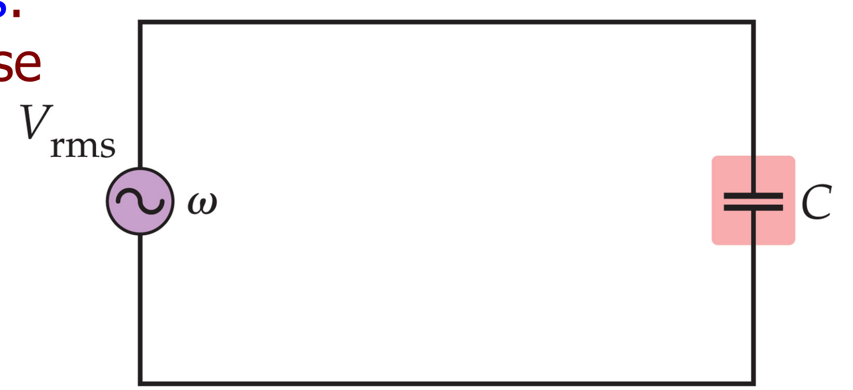
Whatever angle I phasor makes wrt y axis, V phasor is 90 deg behind:

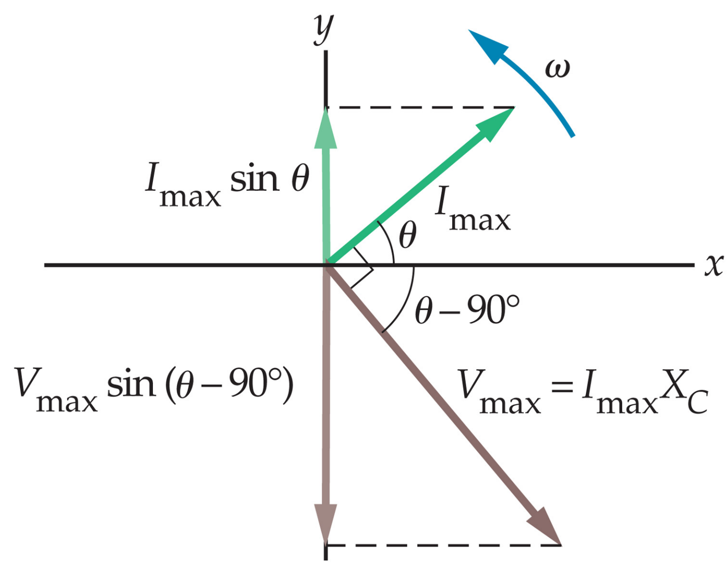
$\theta = \omega t$ = angle of I phasor wrt x axis

$I(t)$=y component = $I_{max} \sin(\omega t)$,

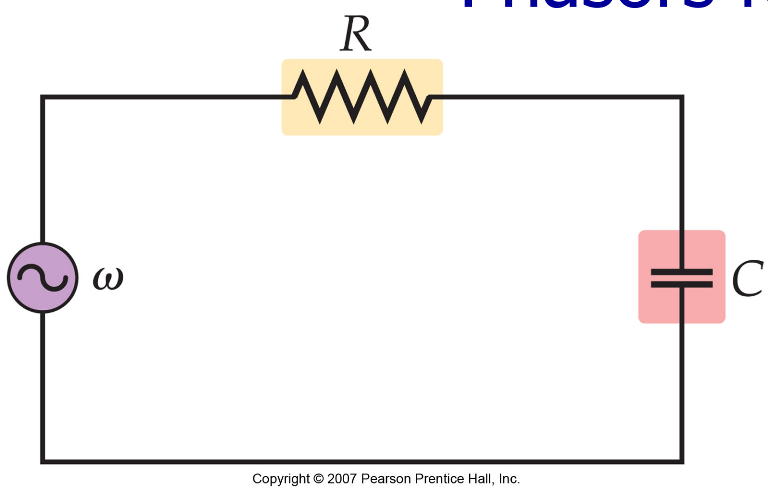
$V(t)$=y component = $V_{max} \sin(\omega t - 90^o)$

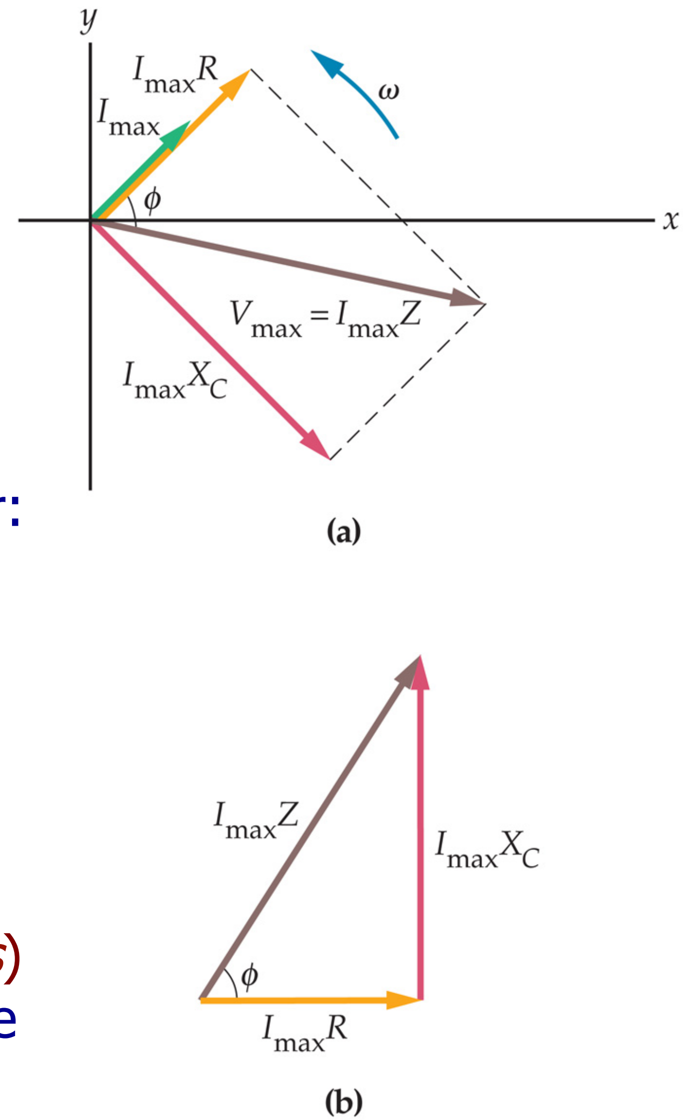
Now R=0, so $V_{max} = I_{max} X_C$





Phasors for RC circuit





Now we have 2 voltage phasors to consider:

1. Voltage across R = I R (no lag with I)
2. Voltage across C = I X_C (90deg lag)

Recall: impedance $Z=\sqrt{[R^2 + X_C^2]}$

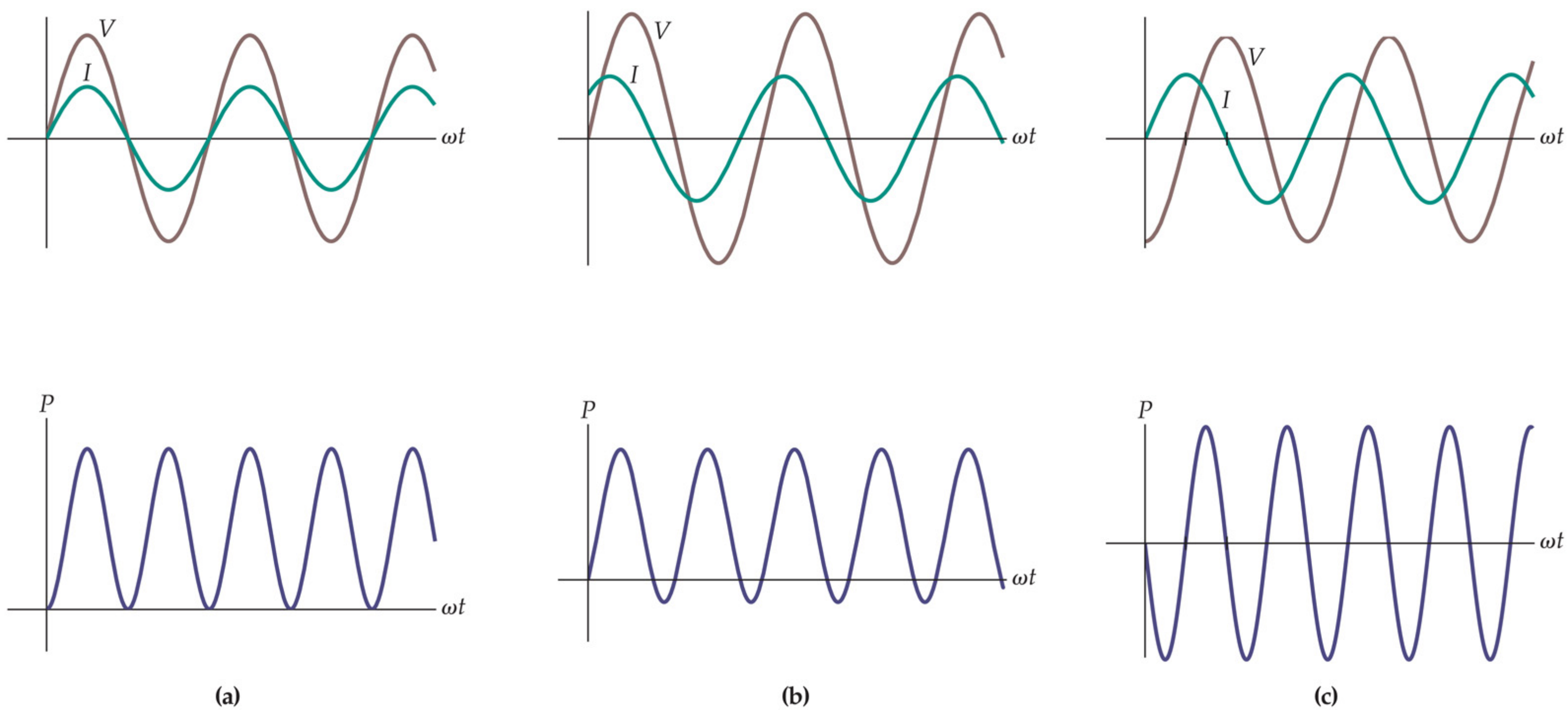
The voltage across the source is V(t)=I(t)Z - but what is its phase?

To find phase we combine V_C and V_R taking into account phase (add phasors as *vectors*)

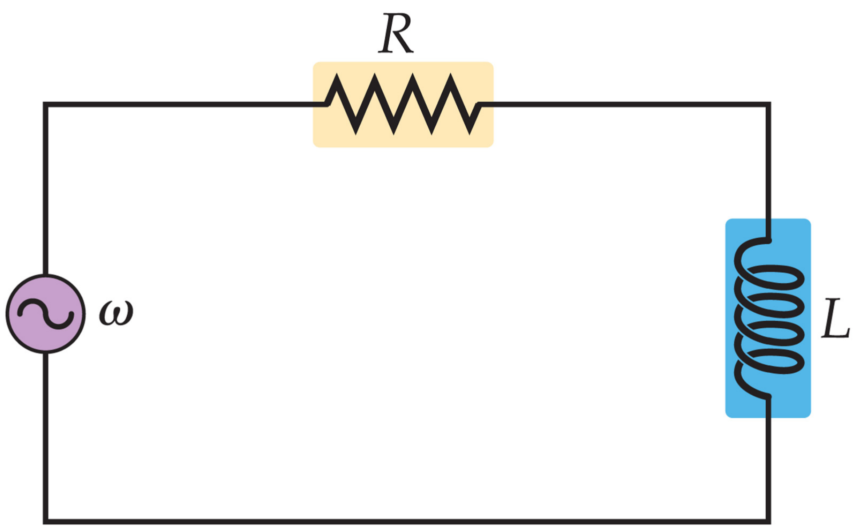
Result: we find V(t) lags I(t) by phase angle ϕ where $\tan \phi = (X_C / R)$ OR: $\cos \phi = (R/Z)$

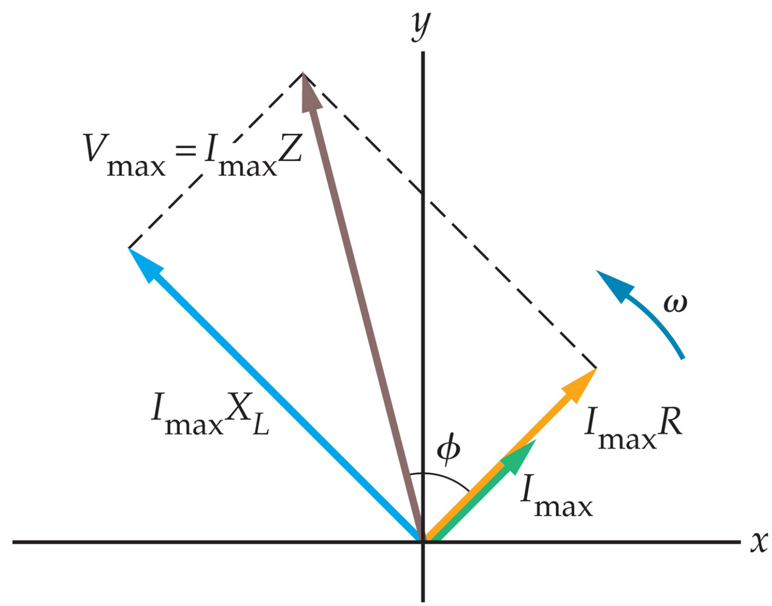
Recall: what phase lag means

(a) I and V inn phase, (b) V lags I by 45deg, (c) by 90 deg



Phasors for RL circuit





Again we have 2 voltage phasors to consider, but:

1. Voltage across R = I R (in phase with I)
2. Voltage across L = I X_L (90deg lead wrt I)

impedance $Z=\sqrt{[R^2 + X_L^2]}$

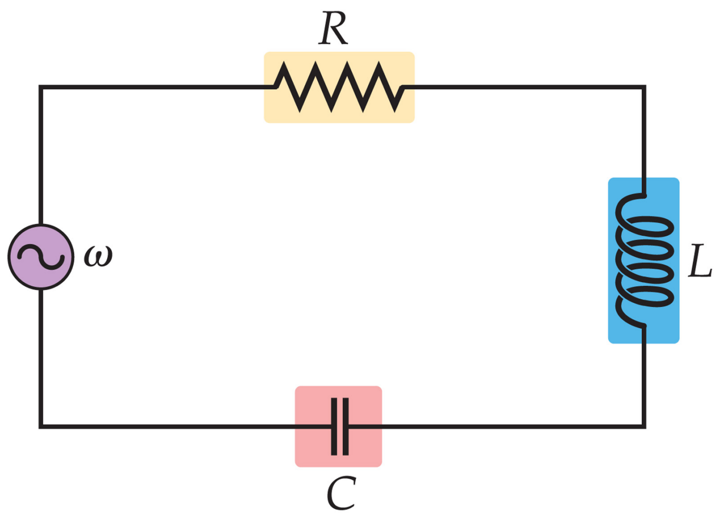
The voltage across the source is V(t)=I(t)Z

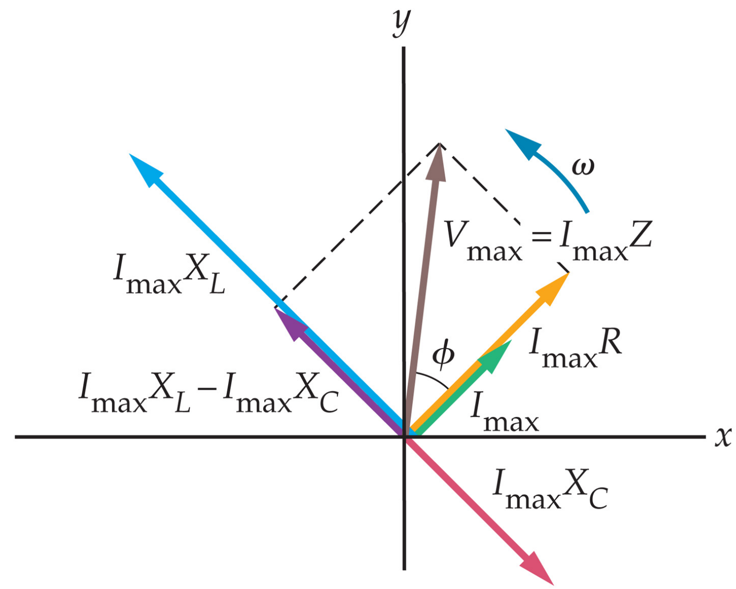
Combine V_L and V_R (add phasors as *vectors*)

Result: we find V(t) leads I(t) by phase angle ϕ where once again

$\tan\phi$=(reactance/resistance) = (X_L / R) OR: $\cos\phi$ = (R/Z)

Most general case: RLC circuit





Now we have 3 voltage phasors to consider:

1. Voltage across R = I R (in phase with I)
2. Voltage across C = I X_C (90deg lag wrt I)
3. Voltage across L = I X_L (90deg lead wrt I)

Now impedance $Z=\sqrt{[R^2 + (X_L - X_C)^2]}$

The voltage across the source is V(t)=I(t)Z

Combine V_C, V_L, and V_R (add phasors as *vectors*)

Result: we find V(t) leads or lags I(t) by phase angle ϕ where

tanϕ=(reactance/resistance) = $(X_L - X_C)$ / R,   OR: cos ϕ = (R/Z)

Leads if $X_L > X_C$
Lags if $X_L < X_C$
In phase if $X_L = X_C$
(resonance)

Why do this? Power factor

Phasor diagrams are useful for analyzing power in AC circuits:

Recall: $P(t) = I(t)\,V(t)$ and $P_{avg} = I^2_{RMS}\,R$
(*Reactances* do not dissipate energy, only R does)
Distinguish between *dissipated power* in watts, and "volt-amperes" = effective *energy delivered to circuit* even if energy is not used.

$P_{avg} = I_{RMS}\,(V_{RMS}/Z)\,R = I_{RMS}\,V_{RMS}\,(R/Z) = I_{RMS}\,V_{RMS}\cos\phi$

→ Observing the phase lead or lag of V vs I tells us the fraction of Z that is resistive.

$\cos\phi$ = the power factor (PF)

PF= 0 → R=0, Z is only reactance; no power consumed (but current and voltage must be supplied)
PF= 1 → R=Z, purely resistive (either no reactances, or $X_L = X_C$)
PF in between: circuit 'seen' by EMF source is partially reactive