HW #1 Solutions

Note Title 4/3/2005

Chap 19, #6

$f = 493\text{ Hz}, \quad v = 353\text{ m/s}$

a) Two points, $\Delta\phi = 55°$. How far apart?

$y(x,t) = y_m \sin(\underbrace{kx} - \omega t)$ $\hookrightarrow |k(x_1 - x_2)| = \Delta\phi$

$\Delta x = \dfrac{\Delta\phi}{k} \qquad k = \dfrac{2\pi}{\lambda} \qquad vT = \lambda = \dfrac{v}{f}$

$= \dfrac{\Delta\phi\,\lambda}{2\pi} = \dfrac{1}{2\pi}\dfrac{v\Delta\phi}{f}$

$\Delta\phi$ – radians!

$\boxed{\Delta x = 0.109393\text{ m}}$

b) Find $\Delta\phi$ for single point, vary by $1.12\text{ ms} = \Delta t$

$y = y_m \sin(kx - \underbrace{\omega t})$ $\hookrightarrow |\omega\Delta t| = \Delta\phi$

$\omega = \dfrac{2\pi}{T} = 2\pi f$

$\Rightarrow \Delta\phi = 2\pi f\Delta t = $ 3.469324 (rad)
198.7776 (deg)

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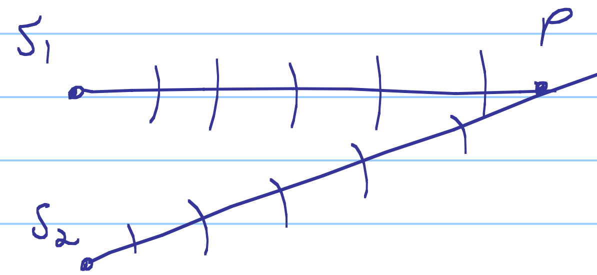

$$y_1 = y_m \sin(kr_1 - \omega t)$$

$$y_2 = y_m \sin(kr_2 - \omega t)$$

r_1 and r_2 are dist along the lines of radii.

This is a 2d wave so y_m is really $y_m(r)$!

$$y_m(r) = \frac{1}{r} Y$$

a)
At point P:

$$y = y_1 + y_2 = y_m \left[\frac{1}{r_1} \sin(kr_1 - \omega t) + \frac{1}{r_2} \sin(kr_2 - \omega t) \right]$$

We can't combine these guys easily unless $r_1 \sim r_2$.

see $\longrightarrow$ http://www.pitt.edu/~wek3/sineadd/node1.html
for nice deriv of $\sin(a+b) = \sin(a)\cos(b) \ldots$

$$r = \frac{r_1 + r_2}{2} \qquad r_1 = 2r - r_2 \qquad r_2 = 2r - r_1$$

$$\frac{1}{r_1} = \frac{1}{2r - r_2} = \frac{1}{r + (r - r_2)} = \frac{1}{r} \left[\frac{1}{1 + \frac{r - r_2}{r}} \right]$$

Note: $\frac{r - r_2}{r}$ is small $\Rightarrow$ expand w/ Taylor, as we did last quarter!!

$$\frac{1}{1+x} \approx 1 + \frac{1}{(1+x)^2}\, x \Big|_{x = \frac{r-r_1}{r}}$$

$$1+x\Big| = \frac{2r-r_1}{r} = \frac{r_2}{r}$$

$$\approx 1 + \frac{r^2}{r_2^2}\frac{(r-r_1)}{r} = 1 + \underbrace{\frac{r}{r_2}}_{\sim 1}\underbrace{\left[\frac{r-r_1}{r_2}\right]}_{\sim 0!}$$

Thus

$$\frac{1}{r_1} \approx \frac{1}{r}(1) \quad \text{and} \quad \frac{1}{r_2} \approx \frac{1}{r}$$

and we can now use our law of <u>sines/cosines</u>

$$y = \frac{2Y}{r}\left[\sin\left(\tfrac{1}{2}k(r_1-r_2)-\omega t\right)\cos\left(\tfrac{1}{2}k(r_1-r_2)\right)\right]$$

so ↑ and ⟶ are really the amplitude here.

$$\Rightarrow \boxed{y_m \to \frac{2Y}{r}\cos\tfrac{1}{2}k(r_1-r_2)}$$

b) Look at $\Delta r = n\lambda$

$$\cos\tfrac{1}{2}kn\lambda = \cos\tfrac{1}{2}\frac{2\pi}{\lambda}n\lambda$$

$$= \cos n\pi \quad \Rightarrow \text{will be either } \pm 1!$$

That is the best you'll do!

$\cos\frac{1}{2}k(n+\frac{1}{2})\lambda = \cos(n+\frac{1}{2})\pi \Rightarrow \cos\frac{\pi}{2}, \frac{3\pi}{2}, \ldots = 0!$ destructive!

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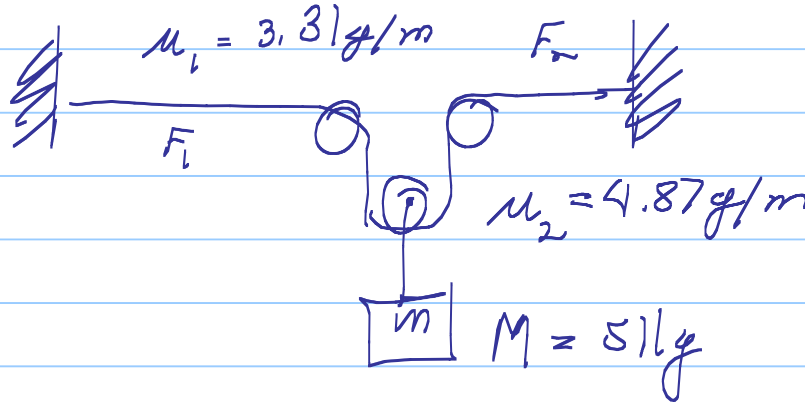
a) v in each string?

$$v = \sqrt{\frac{F}{\mu}}$$



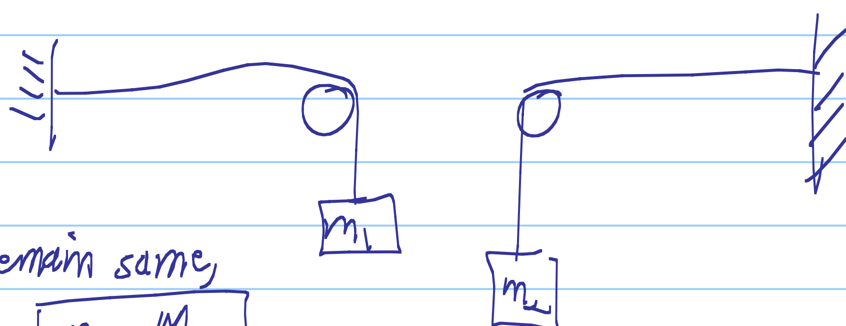
$F_1 + F_2 = Mg$ and, in Equilibrium, $\underbrace{|F_1| = |F_2|}$

if not, then strings would move.

$$v_1 = \sqrt{\frac{Mg}{2\mu_1}} = \boxed{27.50391}\ \text{m/s}$$

$$v_2 = \sqrt{\frac{Mg}{2\mu_2}} = \boxed{22.67483}\ \text{m/s}$$

b)



for v to remain same,

$\frac{Mg}{2} = F_j$ $\boxed{m_1 = \frac{M}{2} \quad m_2 = \frac{M}{2}}$

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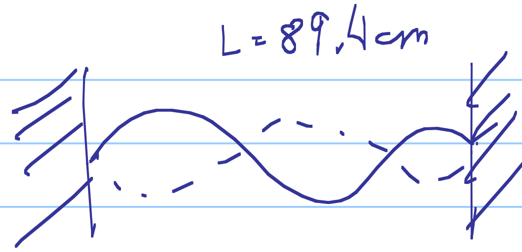
$\mu = 7.16\,g/m$
$F = 152\,N$



a) $v = \sqrt{\frac{F}{\mu}} =$ 145.7019 m/s

b) $L = 1.5\lambda \Rightarrow \lambda = \frac{L}{1.5} =$ 5.96E-01 m

c) $f = ?$ $\quad vT = \lambda, \quad f = \frac{1}{T} = \frac{v}{\lambda} =$ 2.44E+02 Hz

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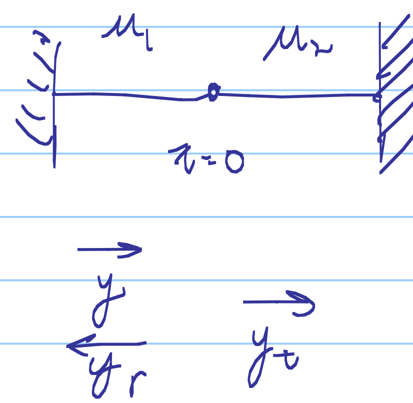


$y = A \sin k_1 (x - v_1 t)$
$y_r = C \sin k_1 (x + v_1 t)$
$y_t = B \sin k_2 (x - v_2 t)$

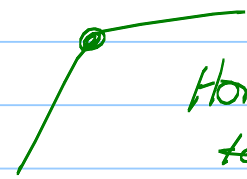
a) $k_1 v_1 = k_2 v_2 = \omega$ (assume)

at $x = 0$, $y + y_r = y_t$

$A \sin k_1(-v_1 t) + C \sin k_1(+v_1 t) = B \sin k_2(-v_2 t)$
$A \sin(-\omega t) + C \sin(\omega t) = B \sin(-\omega t)$
$\sin(-\omega t) = -\sin(\omega t)$
$A - C = B \Rightarrow A = B + C$

b) Assume both strings @ knot have same slope

Why:



Horizontal component of tension would lead to net horizontal force

$\Rightarrow$ knot would move back and forth.

Lets do same trick with derivative

$$\frac{d(y_i+y_r)}{dx} = Ak\cos k_1(x-v_1t) + C k_1 \cos k_1(x+vt)$$

$$\frac{d(y_t)}{dx} = B k_2 \cos k_2(x-v_2t)$$

Put together, take at $x=0$, and $v_i k_i = \omega$:

$$Ak_1 \cos -\omega t + Ck_1 \cos \omega t = Bk_2 \cos -\omega t \qquad \cos\omega t = \cos -\omega t$$

$$k_1(A+C) = Bk_2 \qquad A = B + C \Rightarrow B = A - C$$

$$k_1(A+C) = k_2(A-C)$$

$$A(k_1-k_2) = C(-k_2-k_1)$$

$$\boxed{C = \frac{k_2-k_1}{k_1+k_2}A} \qquad kv=\omega \text{ so } k=\frac{\omega}{v}$$

$$c = \frac{\frac{1}{v_2} - \frac{1}{v_1}}{\frac{1}{v_1} + \frac{1}{v_2}} A = \frac{\frac{v_1 - v_2}{v_1 v_2}}{\frac{v_2 + v_1}{v_1 v_2}} A = \boxed{\frac{v_1 - v_2}{v_1 + v_2} A}$$

When is $c < 0$?

$$v_1 - v_2 < 0 \Rightarrow v_1 > v_2$$

$$\sqrt{\frac{F}{\mu_1}} > \sqrt{\frac{F}{\mu_2}}$$

$$\sqrt{\frac{1}{\mu_1}} > \sqrt{\frac{1}{\mu_2}}$$

$\mu_2 > \mu_1 \Rightarrow c < 0 \Rightarrow$ phase change of π on the reflected wave

$\Rightarrow$ so @ reflection it must have had this added.

When go from $1 \to 2$

$\mu_1 > \mu_2$ – get no phase chg

$\mu_1 < \mu_2$ – get π