Name ___Solutions___ **Student ID** ____________ **Score**________

last *first*

Constants you may find useful

$G = 6.67 \times 10^{-11}\ Nm^2/kg^2$

$M_{Earth} = 5.97 \times 10^{24}$ kg

$M_{Sun} = 1.99 \times 10^{30}$ kg

$r_{Earth} = 6.38 \times 10^{3}$ km

$r_{Sun} = 6.96 \times 10^{5}$ km

$d_{Sun\text{-}Earth} = 149.6 \times 10^{6}$ km

$g = 9.8 m/s^2$

$\varepsilon_0 = 8.85 \times 10^{-12}\ C^2/(Nm^2)$

$\mu_0 = 4\pi \times 10^{-7}$ Tm/A

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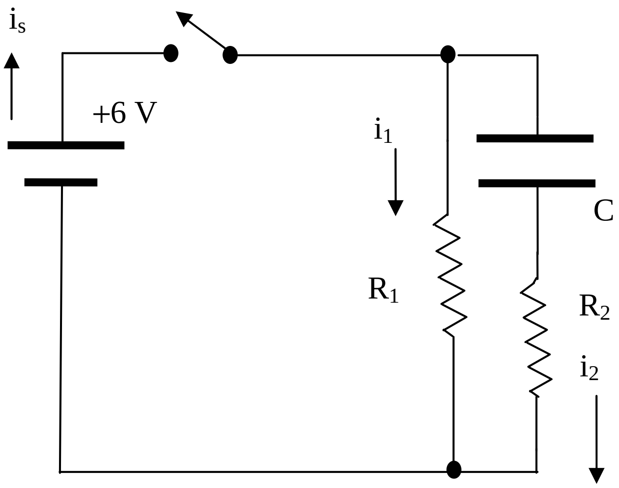
I (25 pts)



In the circuit to the left
$R_1 = 12\ \Omega$
$R_2 = 24\ \Omega$.
$C = 0.1$ F.

The switch is initially open. It is closed, left closed for a long time, and then opened again.

The currents are positive when going in the direction of the arrows.

At the instant after the switch is closed:

The 6 V appears across R1, so V=iR gives $6 = i_1 12$ thus $i_1 = 6/12 = ½$ A

1. i_1 = A. –2 A, B. –½ A, C. 0, D. ½ A, or E. 2 A?

The 6 V appears across C and R_2 which are in series. C has no charge initially, so the entire 6 V appears across R_2 initially, thus $i_2 = 6/24 = ¼$ A.

2. i_2 = A. 0, B. 1/6 A, C. ¼ A, D. ½ A, or E ¾ A?

At the instant after the switch is opened:

Now the circuit has the two resistors in series with C which has been charged to 6 V. Since the two resistors are in series $i_1 = -\ i_2$ (note the arrow) and $V = i\ (R_1 + R_2)$ gives $6 = i(12+24)$ so $i = 6/36 = 1/6$ A. This i is i_1 since the top of R_1 is connected to the +6 volts of the capacitor.

3. i_1 = A. –¼ A, B. –1/6 A, C. 0, D. 1/6 A, or E ¼ A?

4. i_2 = A. –¼ A, B. –1/6 A, C. 0, D. 1/6 A, or E ¼ A?

5. At what time after the switch is opened does Q, the charge on the capacitor = (1/e) x (the value just before the switch was opened)?

   A. 0.8 s, B. 1.2 s, C. 2.4 s, D. 3.6 s, or E. none of those.

We learned in lab that the time constant (time for the capacitor charge to change by a factor of e) is RC where R here is the series resistance, $R_1 + R_2$, so $\tau = (12+24)(0.1) = 3.6$ s

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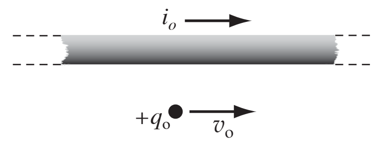
II. [20 Total Pts.]

In the lab frame a positive charge q_0 moves with speed v_0 to the right near a long straight wire carrying current to the right. In the lab frame it is observed that the charge moves in a straight line without changing speed.



6. Which one of the following statements about the charge densities in the wire is most correct in the lab frame?

A. The density of *positive* charges is *greater than* that of *negative* charges.
B. The density of *positive* charges is *less than* that of *negative* charges.
C. The density of *positive* charges is *equal to* that of *negative* charges.
D. More information is needed to compare the *positive* and *negative* charge densities.
E. There are no positive or negative charges in the wire.

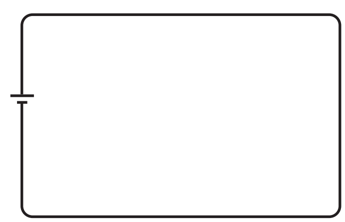
If the charge is to move in a straight line, the electric force on the charge must point down to cancel the upward magnetic force.

7. Which one of the following statements about the charge densities in the wire is most correct in the rest frame of the charge q_0?

A. The density of *positive* charges is *greater than* that of *negative* charges.
B. The density of *positive* charges is *less than* that of *negative* charges.
C. The density of *positive* charges is *equal to* that of *negative* charges.
D. More information is needed to compare the *positive* and *negative* charge densities.
E. There are no positive or negative charges in the wire.

The charge must also have zero net force on it in the rest frame of the charge. There is no magnetic force in that frame. So, there can be no electric force either.

A simple circuit consisting of a battery and a wire is shown in the lab frame at right. The entire circuit has no net charge in the lab frame. Consider a frame *R* in which the circuit moves to the right .



8. Which one of the following statements about the total charge density in the top wire is most correct in frame *R?*

A. The total charge density on the top wire is *positive.*
B. The total charge density on the top wire is *negative.*
C. The total charge density on the top wire is *zero.*

The positive charges are moving in this frame, so the positive charge density increases. The negative charges move slower. So, their density goes down.

9. Which one of the following statements about the total charge density in the bottom wire is most correct *R?*

A. The total charge density on the bottom wire is *positive.*
B. The total charge density on the bottom wire is *negative.*
C. The total charge density on the bottom wire is *zero.*

The total charge on the circuit must remain zero.

10. Which one of the following statements about the total charge on the entire circuit in frame *R* is most correct?

A. The total charge density on the circuit is *positive.*
B. The total charge density on the circuit is *negative.*
C. The total charge density on the circuit is *zero.*

The total charge is not affected by changing reference frames.

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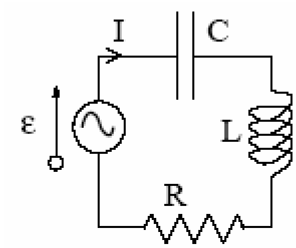
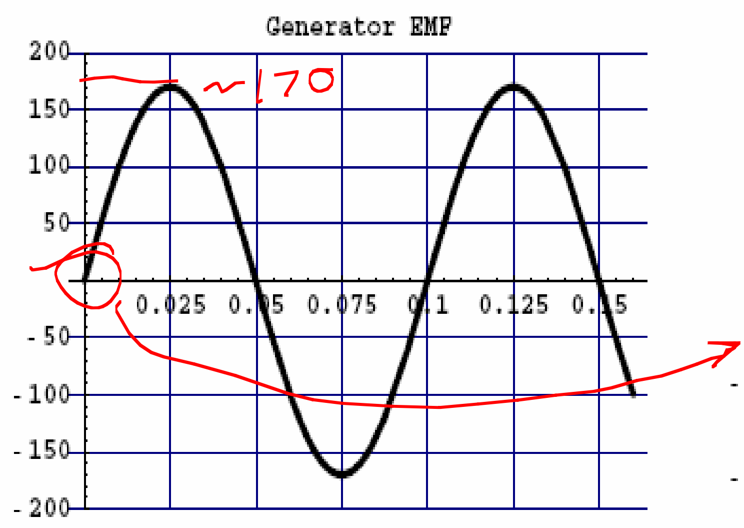
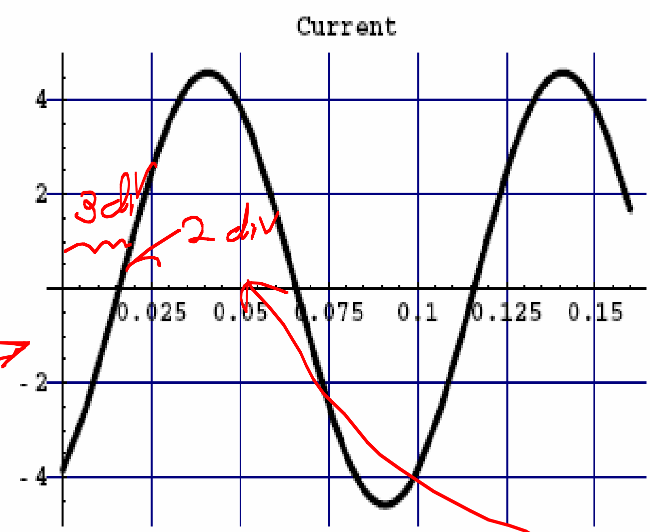


Part III. [28 points] A generator is connected to a LRC circuit as shown to the right. The EMF across the generator is $V=V_0\sin(\omega t)$, and, after a short time, the current in the circuit is given by $I=I_0\sin(\omega t-\phi)$. The period $T=2\pi/\omega = 0.1$ s. The EMF and current are shown on the plots below.

$\omega = \frac{2\pi}{0.1s} = 62.8$ rad/sec





The y-axis of the first plot is Volts, and the second is Ampere's. The horizontal axis of both is seconds.

11. [4 points] What is the RMS (root mean square) voltage of the generator?

- **A.** 240 V
- **B.** 170 V
- **C.** 150 V
- **D.** 120 V
- **E.** 85 V

$$\mathcal{E}_{RMS} = \frac{V_0}{\sqrt{2}} = \frac{170V}{\sqrt{2}} = 120.3V$$

12. [4 points] What is the phase offset, ϕ, between the voltage and the current?

- **A.** $\pi/2$ rad
- **B.** 1.0 rad
- **C.** 0.74 rad
- **D.** 0 rad
- **E.** -1.0 rad

$\mathcal{E}$ leads I $\therefore$ $\phi > 0$. ($\frac{3}{5}$ from # divisions)

$\Delta t = \frac{3}{5}\,0.025s = 0.015$

$\Delta\phi = \omega\Delta t = (62)(0.015) = 0.942$

13. [4 points] Given the information you have so far, which one of the following statements is correct?

- **A.** One can determine R, L, and C.
- **B.** One cannot determine R, L, or C
- **C.** One can determine L and C, but not R.
- **D.** One can determine R and LC, but not L or C separately.
- **E.** One can determine R, but not L or C.

Recall $V_0 = i_0\sqrt{R^2+(X_L-X_C)^2}$

$\tan\phi = \frac{X_L - X_C}{R}$

2 eqns, 3 unknowns.
→ can get R and diff between X_L & X_C

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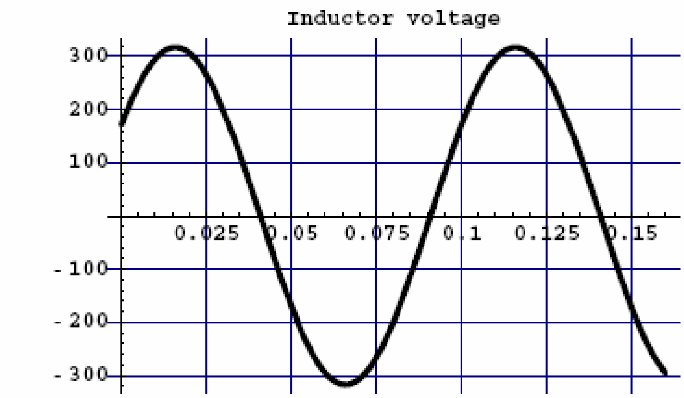
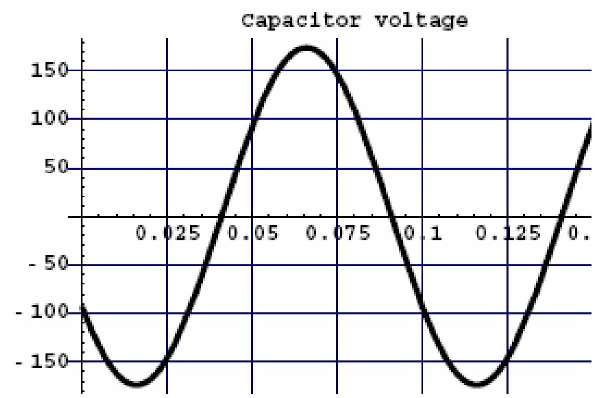
Using the additional information in the below two plots, answer questions 14-17. Both graphs are in units of Volts along the y-axis, and seconds along the x-axis.





14. [4 points] What is the inductance L?

$\Delta V_L = -L\frac{dI}{dt} \Rightarrow L = -\frac{\Delta V_L}{dI/dt} = \frac{-\Delta V_L}{I_0\omega\cos(\omega t-\phi)}$

- **A.** 0.01 H
- **B.** 0.05 H
- **C.** 0.11 H
- **D.** 0.55 H
- **E.** 1.1 H

do it @ $t=0$

$\Delta V_L = 175$, $\frac{dI}{dt} = -I_0\omega\cos(\phi) = -(4.5)(62.8)\cos(0.7) = 166.2$

$L = 1.05$

15. [4 points] What is the capacitance C?

- **A.** 0.42 mF
- **B.** 0.8 mF
- **C.** 1.1 mF
- **D.** 4.2 mF
- **E.** 80 μF

$C = \frac{Q}{\Delta V}$

$Q = -I_0\frac{1}{\omega}\cos(-\phi) = I_0\omega^{-1}\cos\phi$

$C = \frac{I_0\cos\phi}{\Delta V_C\,\omega}$ @ $t=0$, $\Delta V_C = -100$ ∴ $C = 0.42$ mF

16. [4 points] What is the resistance, R?

- **A.** 10 Ω
- **B.** 20 Ω
- **C.** 50 Ω
- **D.** 100 Ω
- **E.** 200 Ω

$\tan\phi = \frac{X_L - X_C}{R}$ ∴ $R = \frac{X_L - X_C}{\tan\phi}$

$X_L = \omega L$. $X_C = \frac{1}{\omega C}$

$\Rightarrow R = 20.6\ \Omega$

17. [4 points] The frequency ω is increased, and V_0 is held fixed. Which of the following best describes what happens to the current and phase?

- **A.** I_0 decreases and the offset ϕ increases.
- **B.** I_0 decreases and the offset ϕ decreases.
- **C.** I_0 increases and the offset ϕ increases.
- **D.** I_0 increases and the offset ϕ decreases.
- **E.** I_0 and ϕ are unchanged.

$X_{LRC} = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$

$\tan\phi = \frac{\omega L - \frac{1}{\omega C}}{R}$

so ⇒ $\phi > 0$, so we are above resonant freq ⇒ this means $\omega\uparrow$ we are further away → $I_0\downarrow$ but ϕ gets larger.

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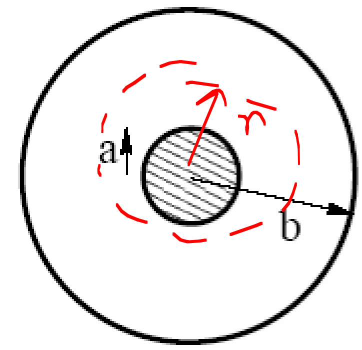
last *first*

Part IV. [30 points] The cylindrical capacitor consists of an outside conducting shell with inner radius b=3cm and an inner conducting rod of radius a=1 cm, and is L=2m long. The cross sectional view is shown to the right. The capacitor is charged so that it holds a charge Q = 100 μC on the inner conductor and –Q on the outer conductor. It is then disconnected from the charging circuit. Ignore fringing effects. Make sure to **show** your work!



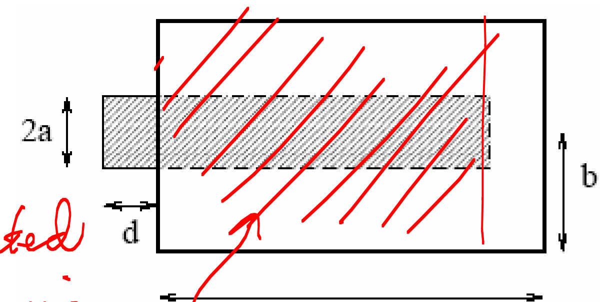
18. [10 points] Use Gauss' law to show the capacitance is $C = 2\pi\varepsilon_0 L/\ln(b/a)$, where L is the length of the capacitor.

$\varepsilon_0 \oint E\cdot dA = \varepsilon_0 2\pi r L E = q_{enc} = Q \Rightarrow E = \frac{Q}{2\pi\varepsilon_0 L r}$

$V = \int_a^b E\cdot dl = \frac{Q}{2\pi\varepsilon_0 L}\int_a^b \frac{1}{r}\,dr = (\ )(\ln b - \ln a) = \frac{Q}{2\pi\varepsilon_0 L}\ln\left(\frac{b}{a}\right)$

$Q = CV \Rightarrow \boxed{C = \frac{2\pi\varepsilon_0 L}{\ln(b/a)}}$

19. [10 points] The central rod is now pulled out of the cylinder along the central axis a distance d = 10cm, as shown. By what factor does the electric field at the surface of the inner conductor, well away from the ends, change? Please express the answer as a number.



Conducting rod → charge can flow. Because charge + is attracted to minus cap will live in L-d region

$E_{old} = \frac{Q}{2\pi\varepsilon_0 L r}$ $E_{new} = \frac{Q}{2\pi\varepsilon_0 (L-d) r}$

$\frac{E_{new}}{E_{old}} = \boxed{\frac{L}{L-d}} = \frac{2m}{1.9m} = \boxed{1.05}$

20. [10 points] How much work was done to pull the central wire out this far? Please express the answer as a number. Is the work done positive or negative?

Positive or negative

Look @ total energy of the capacitors.

$U = \frac{1}{2}\frac{Q^2}{2C}$

$\Delta U = \frac{1}{2}Q^2\left(\frac{1}{C_f} - \frac{1}{C_i}\right) = \frac{Q^2}{2}\frac{\ln(b/a)}{2\pi\varepsilon_0}\left(\frac{1}{L-d} - \frac{1}{L}\right)$

$= \frac{Q^2}{4\pi\varepsilon_0}\ln(b/a)\left(\frac{1}{L-d} - \frac{1}{L}\right)$

$= \boxed{-2.6\,J}$ decrease in energy stored.

$\Rightarrow$ $\boxed{\text{Negative work done}}$

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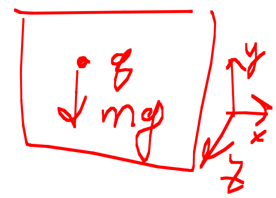
Part V. [27 points] A particle of charge q and mass m is subject to gravity acting downward on the page, and a magnetic field of magnitude B acting in a direction perpendicular to the page, pointing into the page. We assume that the initial velocity of the particle is zero. A) [9 pts] Show that the motion will only be in the plane of the page. B) [9 pts] If there are many such particles, and their mutual Coulomb repulsion is negligible, what is the direction of the average current? C) [9 pts] Check your conclusions by writing out the form of Newton's second law for the three components of the motion. If there is an oscillatory component to the particle's motion, determine the angular frequency. Please **clearly** note where each answer is for parts A, B, and C.

$\perp B$ $\otimes$



A 9pts Total Force is $F = -m\vec{g} + q\vec{v}\times\vec{B}$

starts at rest.

(4pts) $m\vec{g}$ is in plane of page ⇒ so mg won't cause any out of plane motion.

(5pts) $q\vec{v}\times\vec{B}$ will be in plane as long as $\vec{v}$ remains in plane. But $\vec{v}$ comes from $m\vec{g}$ (in plane) and $q\vec{v}\times\vec{B}$ will be in plane.

$q\vec{v}\times\vec{B} \Rightarrow q\vec{v}\times\vec{B}$

B 9pts Since we ignore coulomb repulsion, we consider a single particle and generalize.

The motion depends on the m, q, and B!

Assume $q>0$ → general motion will be down and to the right. (6 pts)

If $mg \gg qB$ then initial motion will be mostly down. Opposite, and it will be horizontal and even oscillate! (3 pts)

C 9pts

$$\vec{v}\times\vec{B} = \hat{i}(v_y B_z - \cancel{v_z} B_y) - \hat{j}(v_x B_z - \cancel{v_z} B_x) + \hat{k}\cancel{(v_x B_y - v_y B_x)}$$

$$\boxed{m\frac{d^2x}{dt^2} = \frac{dy}{dt}Bq \quad ② \; m\frac{d^2y}{dt^2} = -mg - \frac{dx}{dt}Bq}$$ (4 pts)

Hard $\frac{d}{dt}\left[\frac{dy}{dt} = \frac{m}{Bq}\frac{d^2x}{dt^2}\right] \Rightarrow \frac{d^2y}{dt^2} = \frac{m}{Bq}\frac{d^3x}{dt^3}$

② ⇒ $\frac{d^3x}{dt^3} = -\frac{Bqg}{m} - \frac{B^2q^2}{m^2}\frac{dx}{dt}$ → integrate → $\frac{d^2x}{dt^2} = -\frac{Bqg}{m}t - \frac{B^2q^2}{m^2}x + C$

Can see oscil → $\boxed{\omega = \frac{Bq}{m}}$ (5 pts)

Soln: $x(t) = H + It + J\sin(\omega t + \phi)$

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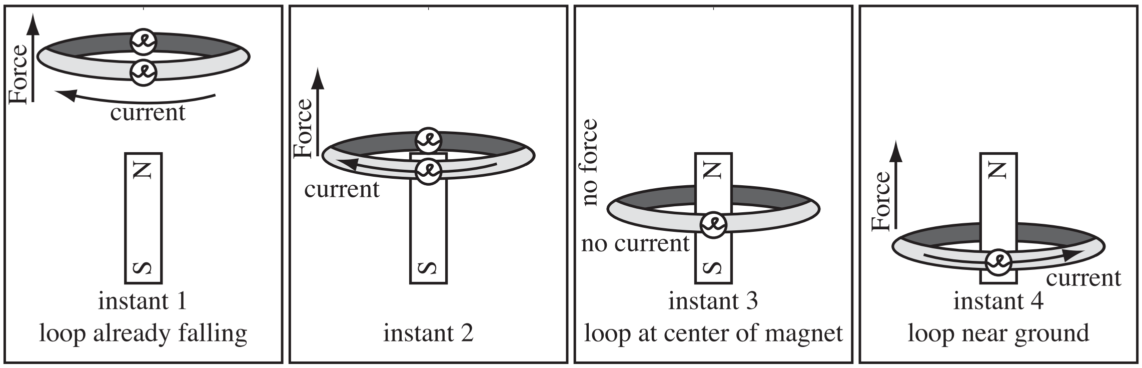
last first

IV. [20 Pts.]

A loop made of two identical bulbs and conducting wire is dropped over a strong bar magnet. Assume the loop remains parallel to the floor as it falls and that the bar magnet is perfectly symmetric. The loop is shown at four different instants during its fall.



i. [4 pts] Draw and label an arrow on each diagram that shows the direction of the current in the front of the loop at that instant. Explain.

Choose an area vector pointing upward. In the first two cases the external magnetic flux is positive and increasing. Thus, by Lenz' law, a current will flow to the left in the front part of the loop to cause an induced flux which is negative. The external flux is neither increasing nor decreasing at instant 3. Hence, there is no current. At instant 4 the external flux is decreasing. So, a current is induced to the right in the front of the loop. These same answers can be obtained by considering the magnetic forces on charges in the loop.

ii. [4 pts] Draw and label an arrow on each diagram that shows the direction of the magnetic force on the loop at that instant. Explain.

The current loop can be thought of like a bar magnet with the north end down at instants 1 and 2. Thus the force is upward in cases 1, 2. At instant 3 the force is zero because the current is zero. At instant 4 consider the force on a small portion of the current carrying wire in the field of the bar magnet. The right hand rule shows that there is an upward component of that force for every part of the wire. Thus, the force is also upward at instant 4.

iii. [3 pts] Is the magnitude of the magnetic force on the loop at instant 1 greater than, less than, or equal to the magnitude of the magnetic force on the loop at instant 2? Explain.

During a small interval of time around instant 2 the flux through the loop increases more than for a small interval of time around instant 1. Thus, the current is larger at instant 2. Since the current and the field are both larger at instant 2, the force is also larger.

iv. [3 pts] Is the brightness of the bulbs at instant 1 greater than, less than, or equal to the brightness of the bulbs at instant 2? Explain.

Since the current is larger at instant 2 the bulbs are also brighter.

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Another loop is constructed with four identical bulbs and conducting wire. The four-bulb loop and the two-bulb loop are released from rest over identical bar magnets.

v. [6 pts] Will the loops hit the ground at the same time? If not, which loop hits first? Explain.

The loop with the larger resistance has a smaller current. So, there is less of an upward force. Hence, the four-bulb circuit hits first.