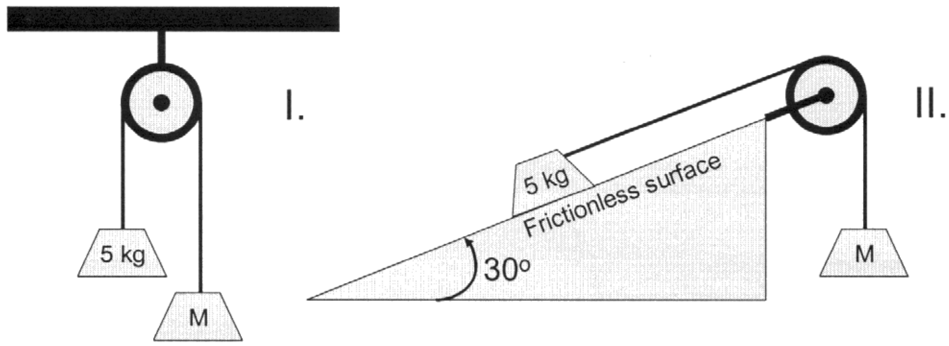
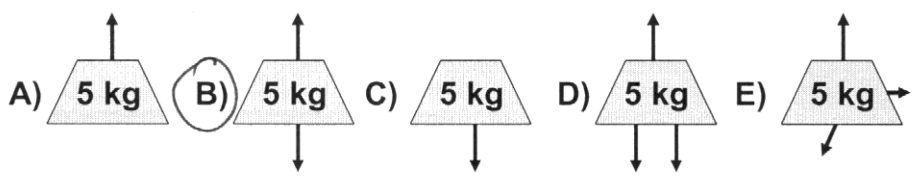
Name Solutions Student ID 121C Score________
last *first*

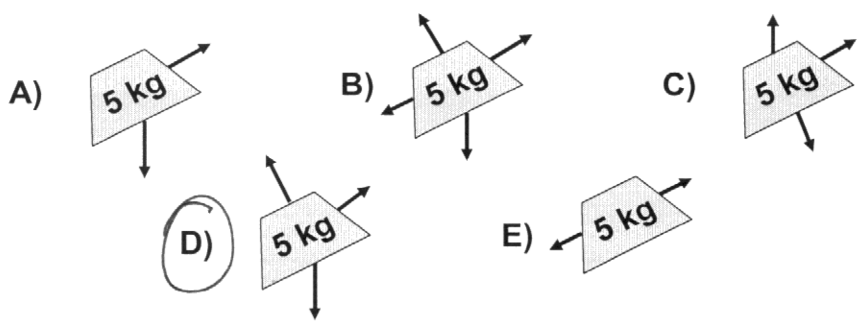
Part I. [17 points] For each of the following questions, consider each pulley machine. In all cases, the system is not moving. Ignore friction. Choose the correct free body diagram, and calculate the mass M for each.



1. [4 points] Choose the correct free body diagram for the **5 kg** mass shown hanging on the end of a string attached to a pulley and another mass M (see figure I).



2. [4 points] Choose the correct free body diagram for the **5 kg** mass resting on the frictionless incline plane as shown in figure II.



3. [4 points] For the system in I to remain at rest, in the configuration shown, what mass must M be?
   - **A.** 1.0 kg
   - **B.** 2.5 kg
   - **(C.)** 5.0 kg
   - **D.** 7.5 kg
   - **E.** 10 kg

4. [5 points] For system II to remain at rest, in the configuration shown, what mass must M be?
   - **(A.)** 2.5 kg
   - **B.** 4.3 kg
   - **C.** 4.9 kg
   - **D.** 5.0 kg
   - **E.** 25 kg

← some net force.

$m g \sin\theta = M g \Rightarrow M = m \sin\theta = 2.5\ kg$

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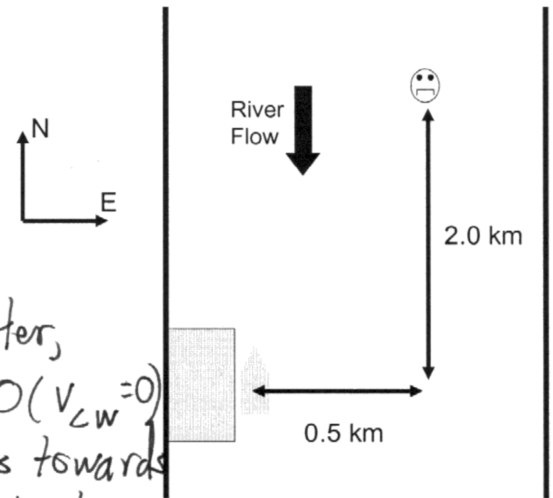
Part II. [18 points] A child in danger of drowning in a river is being carried downstream by a current that flows due South uniformly with a speed of 3 km/hr. The child is 0.5 km from the shore and 2 km North (i.e., upstream) of a boat landing when a rescue boat sets out.



5. [4 points] The boat proceeds at its maximum speed of 24 km/hr with respect to the water. What is the speed of the boat relative to the child?
   - **A.** 18.1 m/s
   - **B.** 21.1 m/s
   - **C.** 21.0 m/s
   - **(D.)** 24.0 m/s
   - **E.** 27.0 m/s

   child is in water, so $v_{rel} = 0$ ($v_{cw} = 0$) ∴ boat moves towards child @ 24 km/s

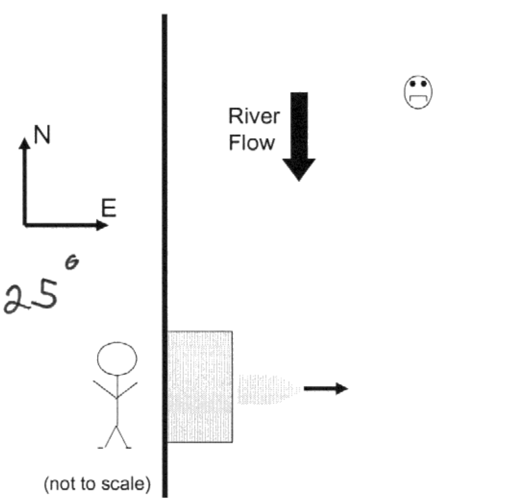
6. [4 points] And at what angle relative to NORTH should the pilot point the bow of the boat in order to reach the child in the minimum amount of time?
   - **A.** 1.33 degrees
   - **(B.)** 14.0 degrees
   - **C.** 15.0 degrees
   - **D.** 23.7 degrees
   - **E.** 76.0 degrees

   Since boat & child are in same ref frame, should point directly.

   $\tan\theta = \frac{2.0}{0.5} = 4$

   $\theta = 75.96°$. But that is rel to East. $90 - 75.96$ is rel to N.

Another pilot, who has not taken 121 yet, sets out at the same time in an identical boat. He instead sets out directly across the river, perpendicular to the bank.



7. [5 points] To a person standing on the rescue boat dock, at what angle relative to ~~the horizontal~~ EAST is the boat moving? (neg is below)
   - **(A.)** -7.1 degrees
   - **B.** -0.12 degrees
   - **C.** 0 degrees
   - **D.** 0.12 degrees
   - **E.** 7.1 degrees

   $\theta = \tan\frac{-3}{24} = -7.125°$

8. [5 points] And what is the boat's ground speed?
   - **A.** 21.0 m/s
   - **B.** 23.8 m/s
   - **C.** 24.0 m/s
   - **(D.)** 24.2 m/s
   - **E.** 27.0 m/s

   $\sqrt{(3)^2 + (24)^2} = 24.2$ m/s

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Part III. [20 points] The space shuttle is in a circular orbit 400 km above the earth. The orbital period is 90 minutes. *The radius of the earth is 6.38×10^3 km.*

9. [4 points] What is the velocity, $\mathbf{v_r}$, of the shuttle?

   A. 9.22 km/s
   (B.) 7.89 km/s
   C. 350 km/s
   D. 5.67 km/s
   E. 473 km/s

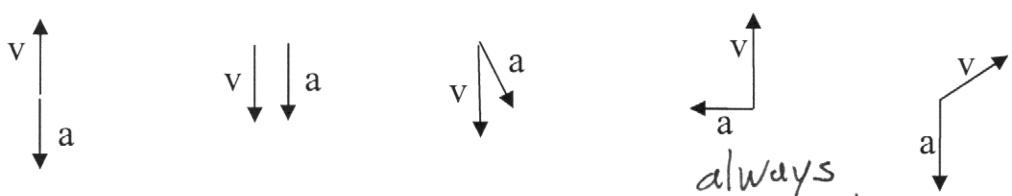
$V_r = \frac{d}{T} = \frac{2\pi r_s}{T} = \frac{2\pi(6.8\times10^3 km + 400 km)}{90\cdot60} = \frac{2\pi(6.8\times10^6 m + 400\times10^3)}{90\cdot60} = 7.89$ km/s

10. [3 points] What is the centripetal acceleration experience by the shuttle (in units of g)?

   A. 0.0 g
   B. 0.011 g
   C. 9.2 g
   D. 0.15 g
   (E.) 0.94 g

$a = \frac{v^2}{r} = \frac{(7.89\times10^3 m/s)^2}{(6.8\times10^6 m + 400\times10^3 m)} =$

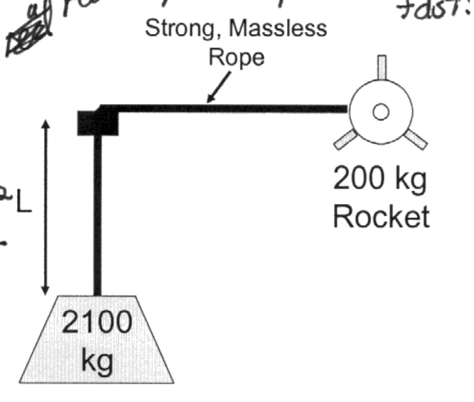
11. [3 points] Qualitatively, which vector diagram correctly represents the relationship between the acceleration and velocity vectors at some point in its orbit?



A. **B.** **C.** **(D.)** ⊥!! **E.**

A rocket is attached to the end of a strong, massless rope. The rocket's engine moves the rocket at a speed of 120 m/s (shown coming out of the page in figure). The rocket has a mass of 230 kg. The rope is 200 m long. It goes through a collar and is attached at the other end to a large mass of 2100 kg. Ignore gravity's effect on the rocket (but not the large mass at the end of the rope!).

read my own problem too fast!



12. [4 points] If the rope were fixed so the rocket rotated with a radius of 100 m, what would the magnitude of the centripetal force be?

   A. 0.6 N
   B. 144 N
   C. 15650 N
   (D.) 33120 N
   E. 42134 N

$ma = \frac{mv^2}{r} = \frac{(230 kg)(120 m/s)^2}{100 m} = 33{,}120 N$

13. [6 points] If the rope is allowed to slip through the collar until the system reaches equilibrium, how far below the collar does the large mass of 2100 kg hang?

   (A.) 39.1 m
   B. 88.2 m
   C. 160 m
   D. 198 m
   E. 0 m (Mass rises to and hits collar)

M ≡ large mass, m is mass of rocket. Then we have:

$Mg = \frac{mv^2}{r}$ - solve for r

$r = \frac{mv^2}{Mg} = \frac{(230 kg)(120 m/s)^2}{(2100 kg)(9.8 m/s^2)} = 160.9$ m

Then 200 − r is length mass hangs down.

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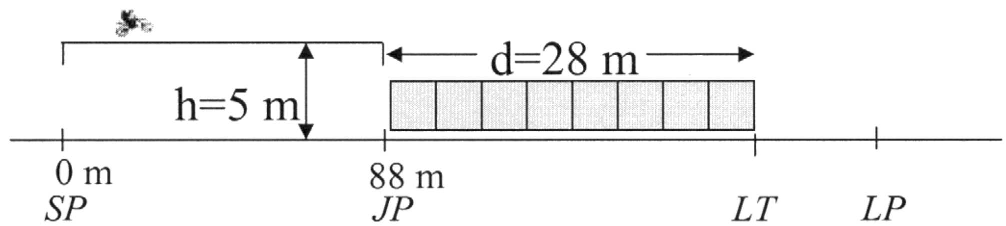
Part IV. [25 points] A motorcycle stunt driver is attempting to jump over 8 trucks. *For questions 1, 2, 3, & 5 show equations for each answer, along with the numerical answer.*



14.[5 points] Starting from rest at point SP, on a level platform 5 m above the ground, the driver accelerates at a constant rate of 11 m/s^2. What is the velocity at the point JP where the motorcycle leaves the platform ($\mathbf{v}_{JP}$)?

$v^2 = v_0^2 + 2a(x - x_0) \Rightarrow v^2 = 2 \cdot (11\,m/s^2)\,88 \Rightarrow$ $\boxed{v = 44\,m/s}$

15. [7 points] What height above the ground as it passes over the top of the last truck, at point LT?

1) Time to get to LT. 2) how far above ground is it?

1) in air, no accel, so $v = 44\,m/s$ is const.

$x = vt \Rightarrow t = \frac{x}{v} = \frac{28\,m}{44\,m/s} = \frac{7}{11} = \boxed{0.6\overline{36}\,s}$

2) so – $y = y_0 - \frac{1}{2}gt^2 = 5m - \frac{1}{2}(9.8\,m/s^2)(0.636)^2 = 5m - 1.98 = \boxed{3.01\,m}$

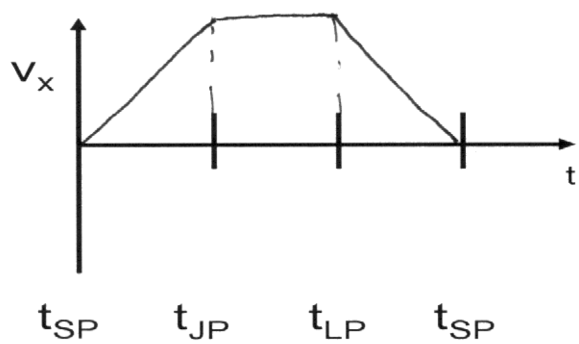
16. [5 points] Upon landing, if the driver brakes with an acceleration of –7.5 m/s^2, how much time will it take to come to a stop?

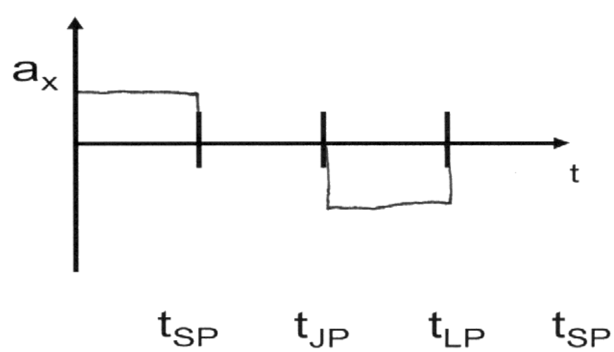
@ landing $V_x = 44\,m/s$ still; $V_y = 0$ (shock absorbers!)

so $V(t_f) = V_0 + at$

$0\,m/s = 44\,m/s - (7.5\,m/s^2)t \Rightarrow t = \frac{44\,m/s}{7.5\,m/s^2} = \boxed{5.87\,s}$

17. [8 points] Make qualitatively correct sketches of the **x** component of the velocity and acceleration of the motorcycle *vs.* time. t_{SP} is the starting time, t_{JP} occurs as the motorcycle leaves the platform, t_{LP} as the motorcycle lands, and t_{stop} as it comes to rest (see points on diagram above).





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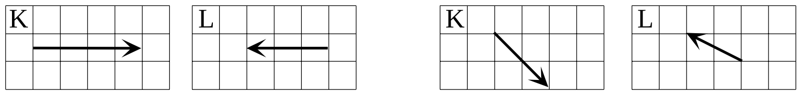
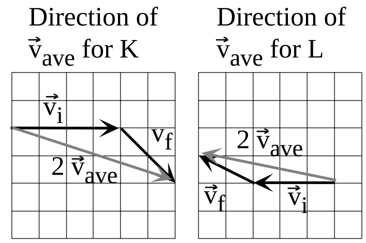
V. [20 points total]

Two skaters, K and L, approach on a path and nearly collide. The velocity vectors of the skaters just before, t = 0 s, and just after, t = 2 s, the near collision are shown below and drawn to scale. Assume that the acceleration of each student is constant between t = 0 s and t = 2 s.



t = 0 s   t = 2 s

18. [7pts] Draw an arrow in each of the boxes at right to indicate the *direction* of the average velocity for each skater between t = 0 s and t = 2 s. Label the arrows clearly on the diagram. Explain your reasoning.

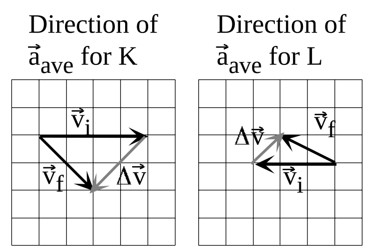


During constant acceleration, average velocity is equal to the average of the initial and final velocity, or $\vec{v}_{ave} = (\vec{v}_i + \vec{v}_f)/2$. *As shown in the diagram at right, the average velocity vector can be found graphically by adding the initial and final velocity vectors head to tail. The resultant vector has twice the magnitude of the average velocity vector but the same direction.*

19. [3pts] Is the *magnitude* of the average velocity of skater K *greater than, less than,* or *equal to* the *magnitude* of the average velocity of skater L? Briefly explain your reasoning.

The above diagram shows that the average velocity vector found for K is longer than the average velocity vector found for L, so the magnitude of the average velocity for skater K is greater.

20. [7pts] Draw an arrow on each of the grids at right to indicate the direction of the average acceleration for each skater between t = 0 s and t = 2 s. Label the arrows clearly on the diagram. Explain your reasoning.



Average acceleration is defined as the change in velocity divided by the corresponding change in time, or $\vec{a}_{ave} = \Delta\vec{v}/\Delta t$. *According to this relationship, the average acceleration vector and change in velocity vector have the same direction. The change in velocity vectors for skaters K and L are found graphically in the diagram shown at right. Change in velocity can be thought of as the vector that must be added to the initial velocity in order to get the final velocity, or* $\Delta\vec{v} + \vec{v}_i = \vec{v}_f$.

21. [3pts] Is the *magnitude* of the average acceleration of skater K *greater than, less than,* or *equal to* the *magnitude* of the average acceleration of skater L? Briefly explain your reasoning.

The above diagram shows that the average acceleration vector found for K is longer than the average acceleration vector found for L, so the magnitude of the average acceleration for skater K is greater.