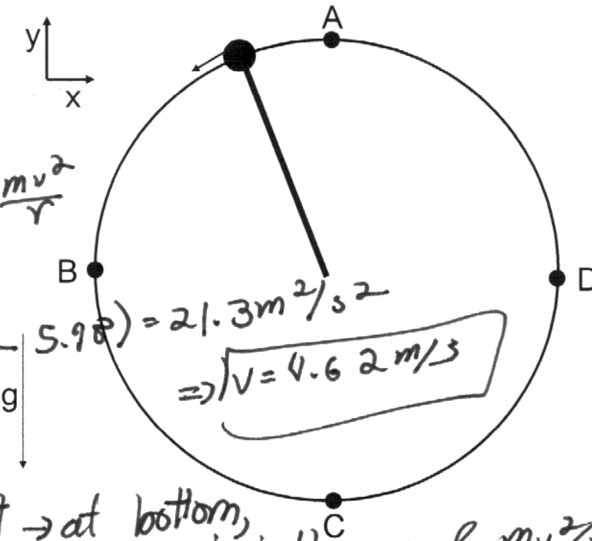
Name Solutions **Student ID** ____________________ **Score**__________

last *first*

Part I. [10 points] A 5.0 kg ball is attached to the end of a massless rod 1.5 m long, which moves in uniform circular motion. Gravity points down, as shown in the picture. The rod will break if the tension in it exceeds 120 N.



1. [4 points] What is the maximum magnitude of the ball's velocity such that the rod *does not break*?

   A. 4.6 m/s
   B. 6.0 m/s
   C. 7.1 m/s
   D. 21 m/s
   E. 36 m/s

   T, mg

   $\Sigma F = T - mg = ma = \frac{mv^2}{r}$

   $v^2 = \frac{r}{m}(T - mg)$

   $= \frac{1.5\,m}{5.0\,kg}(120 - 5 \cdot 9.8) = 21.3\,m^2/s^2$

   $\Rightarrow \boxed{v = 4.62\,m/s}$

2. [3 points] Instead of rotating the rod at the speed calculated in problem 1, it spins just a bit faster. Where is the rod most likely to break?

   A. Near point A
   B. Near point B
   C. Near point C
   D. Near point D
   E. Almost anywhere along the path

   Where T is largest → at bottom, where T must support both mg & mv^2/r.

3. [3 points] During uniform circular motion, the rod is broken just as the ball passes point B. Which statement best describes the path of the ball?

   A. The ball travels in a straight line down.
   B. The ball travels in a straight line up.
   C. The ball travels down and curves towards point C
   D. The ball travels down and curves away from point C
   E. None of the above

   — no forces afterwards ↳ straight line.

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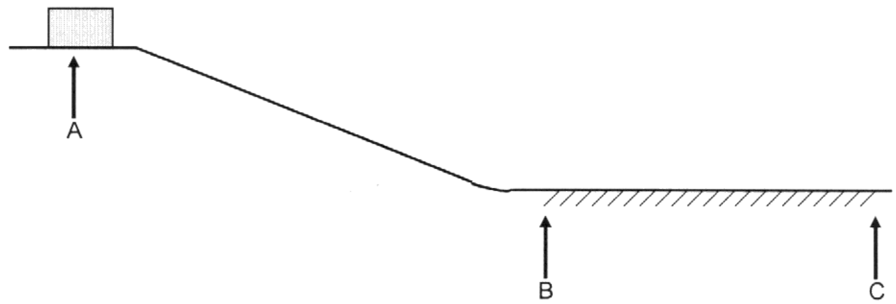
last *first*

Part II. [xx points] A 5 kg block starts out at 2 m/s across a frictionless surface (*A*), slides down a frictionless inclined plane, and then across a flat surface (*B-C*) with a coefficient of kinetic friction μ_k. It comes to rest *at* point C. When the block reaches point B it is moving with a speed of 8 m/s. The distance between points B and C is 4 m.



4. [xx points] What is the change in height between points A and B?

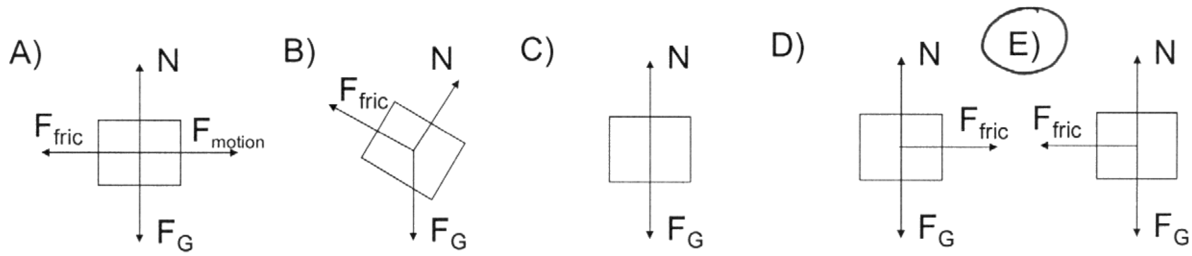
   A. 0.61 m
   B. 3.1 m
   C. 3.3 m
   D. 15 m
   E. 30 m

Energy conserv: $E_i = \frac{1}{2} m v_i^2 + mgh$

$E_f = \frac{1}{2} m v_f^2$

$E_i = E_f$ $\quad v_f^2 = v_i^2 + 2gh \Rightarrow h = \frac{v_f^2 - v_i^2}{2g} = \frac{64-4}{2 \cdot 9.8} = \boxed{3.06\text{m}}$

5. [xx points] Select is correct free body diagram for the block when it is located on the flat surface between points B and C.



deacceleration

6. [xx points] As previously stated, the block is moving with a speed of 8 m/s at point B, and comes to a complete stop *at* point C. What is the coefficient of kinetic friction, μ_k?

   A. 0.051
   B. 0.16
   C. 0.46
   D. 0.82
   E. 4.1

$v_f^2 = v_i^2 + 2a\Delta x$ $\qquad F = \mu_k N = \mu_k mg = ma$

$v_f^2 = v_i^2 + 2\mu_k g \Delta x$ $\qquad$ a is negative $\qquad a = \mu_k g$

$\Rightarrow \mu_k = \frac{v_f^2 - v_i^2}{2g\Delta x} = \frac{0-64}{2 \cdot 9.8 \cdot 4} = \boxed{0.816}$

7. [xx points] Which ranking of the total mechanical energy of the block, E, at points A, B, and C is correct?

   A. $E_A > E_B > E_C$
   B. $E_A > E_B = E_C$
   C. $E_A = E_B > E_C$
   D. $E_A = E_B = E_C$
   E. $E_B > E_A > E_C$

- As long as no fric & all conservative forces, $\Delta E = 0$. $\therefore E_A = E_B$
- As it moves ~~to~~ $B \to C$, looses energy to fric $E_B > E_C$

$\therefore$

$E_A = E_B > E_C$

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Part III. [13 points] Consider a star just like our sun, with the same mass and radius (values for solar and planetary masses and radii can be found on the second page of this exam)

8. [5 points] If an object of mass 7.5 kg is released 10 m above the surface of the star, how long will it take to hit the "ground" of the star (*i.e.* fall the 10m)?

   A. 0.00027 s
   B. 0.073 s
   C. 0.099 s
   D. 0.27 s
   E. 1.4 s

$x = \frac{1}{2}at^2 \quad a = \frac{F}{m} = \frac{1}{m} G \frac{mM}{r^2} = \frac{GM}{r^2}$

$t^2 = \frac{2x}{a} = \frac{2xr^2}{GM} = \frac{20 \cdot (6.96 \times 10^{8})^2}{(6.67 \times 10^{-11})(1.99 \times 10^{30})} = 7.3 \times 10^{-4}\ s^2$

$\sqrt{\ } \Rightarrow \boxed{0.27 s}$

Our sun may eventually become a white dwarf, and collapse to a much smaller radius – this can happen when a main sequence star uses up all of its fuel. Assume that the star in Problem 8 burns off half its mass (so it is ½ the mass of the sun now) and collapses to a much smaller radius.

9. [5 points] The acceleration due to gravity, g', is measured to be 1.0 x 10^{12} m/s^2 at the surface of the white dwarf. What is the radius of white dwarf?

   A. 3.0 x 10^3 m
   B. 8.2 x 10^3 m
   C. 1.2 x 10^4 m
   D. 2.2 x 10^4 m
   E. 6.6 x 10^7 m

$mg' = G\frac{mM}{r^2} \Rightarrow r^2 = \frac{GM}{g'} =$

$\frac{(6.67 \times 10^{-11})(\frac{1}{2})(1.99 \times 10^{30})}{1 \times 10^{12}} = 6.6 \times 10^{7}$

$\Rightarrow \boxed{r = 8.2 \times 10^{3} m}$

10. [3 points] Which statement best describes the differences between dropping the same object as in problem 8 on the original star and the white dwarf?

   A. The object will take just as long to fall the 10 m on the sun as the white dwarf.
   B. The object will take longer to fall the 10 m on the sun than the white dwarf.
   C. The object will take longer to fall the 10 m on the white dwarf than the sun.

$g' \gg$ original g!

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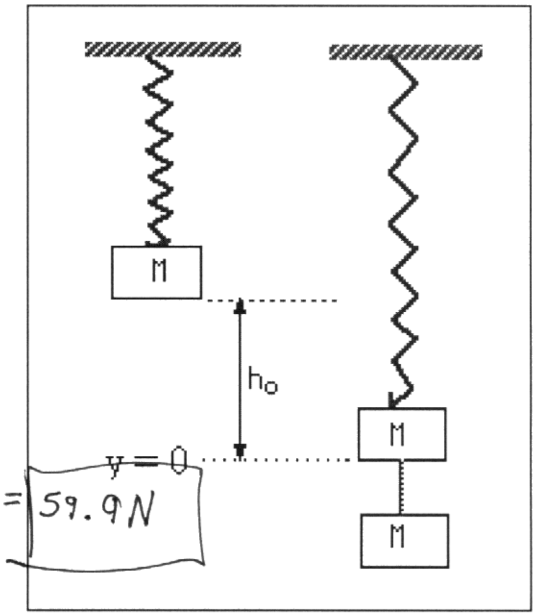
last *first*

A block of mass $M = 11$ kg hangs on a spring. When a second block with an identical mass of 11 kg is tied with a string to the first, the spring stretches an additional $h_o = 1.8$ m. The two blocks are initially at rest.



11. [yy pts] What is the value of the spring constant k?

A. 38 N/m
B. 60 N/m
C. 120 N/m
D. 32 N/m
E. 27 N/m

$k h_o$

Mg

$k h_o = Mg$

$k = \frac{Mg}{h_o} = \frac{(11\,kg)(9.8\,m/s^2)}{1.8\,m} = \boxed{59.9\,N}$

Now the string connecting the two blocks is burned and the second block falls off. The block that remains attached to the spring starts to move.

12. [yy pts] How far above $y = 0$ is the bottom of the block attached to the spring when it attains its maximum speed?

A. 1.8 m
B. 2.7 m
C. 1.3 m
D. 3.6 m
E. 0.9 m

Block M will have max velocity as it passes through equilib position. This equilib position is just the original position $\Rightarrow$ $\boxed{1.8\,m}$ above its stretched position.

13. [yy pts] What is the maximum speed attained by the block attached to the spring?

A. 3.8 m/s
B. 2.1 m/s
C. 6.5 m/s
D. 4.2 m/s
E. 3.6 m/s

We can do conserv. of mech energy.

$\Delta E = 0$ – all conservative forces.

$E_i = \frac{1}{2} k h_o^2$ – at stretched position.

$E_f = \frac{1}{2} m v^2$ – only KE @ equilib position.

$\Rightarrow v^2 = \frac{k}{m} h_o^2 = \frac{60\,N/m}{11\,kg} \cdot (1.8\,m)^2 = 17.7\,m^2/s^2$

$\Rightarrow \boxed{v = 4.20\,m/s}$

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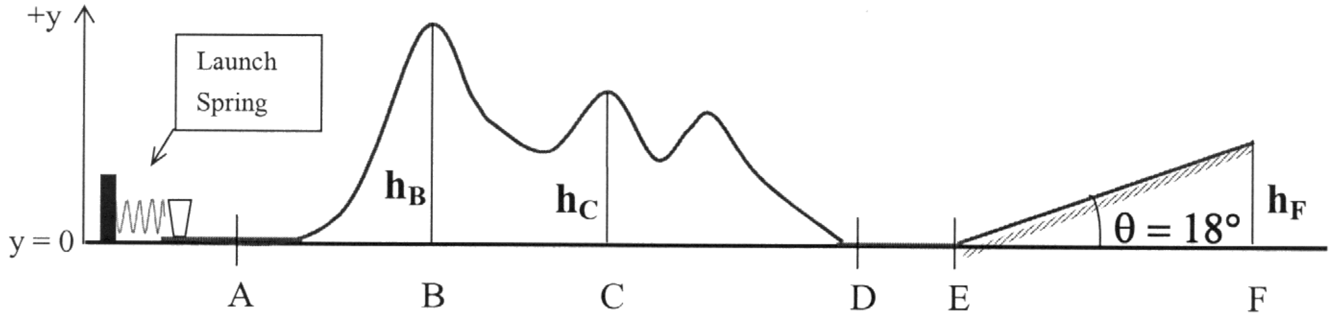
II. [30 pts] Mindbender Roller Coaster. - A car of mass *m* travels along a roller coaster track. Assume there is no friction or air resistance, unless otherwise stated in the question. The spring used for launching the car has a spring constant of *k*. [*For each question in this section highlight answers by drawing a box around the algebraic equation and the numerical answer, if the question requires a calculation.*]



14. [8 pts] By what distance x, should the spring be compressed before launch so that the car reaches Point B with a velocity of v_B? Express the distance *x* in terms of the quantities given or depicted in the figure.

Use $\Delta E = 0$ (conservative forces)

$E_i = PE$ in spring $= \frac{1}{2}kx^2$

$E_f = PE + KE$ @ point B

$= \frac{1}{2}mv_B^2 + mgh_B$

Solve for x!

$\frac{1}{2}kx^2 = \frac{1}{2}mv_B^2 + mgh_B$

$\Rightarrow \boxed{x = \sqrt{\frac{mv_B^2 + 2mgh_B}{k}}}$

15. [10 pts] Given $m = 900$ kg, $h_B = 30$ m, $h_C = 22$ m and $v_B = 4$ m/s, what are the potential and kinetic energies for the car at Point C? Show equations and numerical answers.

$\Delta E = 0$, $E_B = E_C$. Further, we know PE @ point C by mgh_C!

$E_B = \frac{1}{2}mv_B^2 + mgh_B$ ← know it all!

$E_C = \frac{1}{2}mv_C^2 + mgh_C$

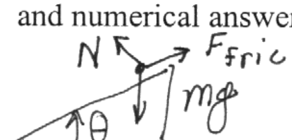
$\boxed{PE_C = mgh_C = 900 \cdot 9.8 \cdot 22 = 194040\ \text{J}}$

$\boxed{KE = E_B - PE_C = 77760\ \text{J}}$

$E_B = (\frac{1}{2})(900)(4)^2 + (900)(9.8)(30\,m)$

$= 271,800$ J

16. [8 pts] After reaching Point F and coming to a stop, what is the minimum value of the static friction, μ_s, required so that the car remains at rest and does not roll back down the inclined track? Show equation and numerical answer.



keep from sliding down incline

$F_{fric} + mg \sin\theta = 0$!

$-\mu_k N + mg \sin\theta = 0$

$-\mu_k mg \cos\theta + mg \sin\theta = 0$

$\mu_k = \frac{\sin\theta}{\cos\theta}$

$= \tan\theta$

$= .32$

$\boxed{\mu_k \geq 0.32}$

17. [4 pts] Explain how the velocity, v_D, of the car at Point D will vary if the height of the track at Point C is changed (raised or lowered)?

Gravity is conservative ⇒ the height @ point C won't affect the car's motion <u>unless</u> mgh_C is larger than the total energy of the car → then the car can't get over.

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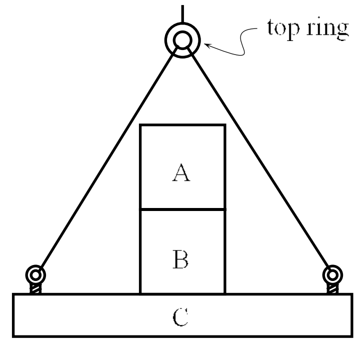
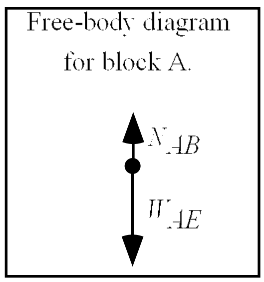
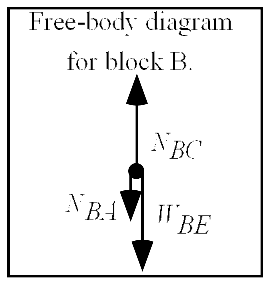
IV. [20 Total Pts.] The system shown at right is composed of two blocks, A and B, of mass, *m*; and a third block, C, of mass, 2*m*. Ignore air resistance.



1. The system is lowered with acceleration g/2 downward, equal to half the acceleration it would have in free fall.

   a. [8 *Pts.*] Draw free body diagrams for boxes A and B in the space provided below. Label the forces. Be sure to indicate the object on which the force is exerted and the object exerting the force.





   b. [4 *Pts.*] Let the weight of object A be W_o. Calculate the magnitudes of the forces on box B in terms of W_o.

Since the acceleration is downward, the net force is downward. The magnitude of the net force on both block A and B is the mass times the acceleration. That is *mg/2*, or *Wo*/2. To get this net force on block A, $N_{AB} = W_o/2$. By Newton's third law $|N_{AB}|=|N_{BA}|$. For the net force on block B to be *Wo/2*, N_{BC} must be *Wo*. In summary: $|N_{BC}|=|W_{BE}|=Wo$, $|N_{BA}|=Wo/2$

2. [4 *Pts.*] The cable is now cut, and the system is in free fall. Rank the magnitudes of the forces acting on block B from greatest to smallest. If any of the forces present in the previous problem are now zero, explicitly say so.

If the system is in free fall, the net force on the block B is $mg=W_o$. The weight of block B accounts for this force alone, so the other forces are zero.
In summary: $|N_{BC}|=|N_{BA}|=0$, $|W_{BE}|=Wo$

3. [4 *Pts.*] The system is now taken far enough out into space that gravitational effects are negligible, and the top ring is attached to a spacecraft. The spacecraft then tows it with an increasing speed. Rank the magnitudes of the forces acting on block B from greatest to smallest. If any of the forces present in problem 1 are now zero, explicitly say so.

Block B now accelerates in the direction of the top ring. So there is a net force in that direction. There is no weight force. So the force on block B from block C must exceed the force on block B from Block A.
In summary: $|N_{BC}|>|N_{BA}|$, $|W_{BE}|=0$