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Part I. [18 points] A spherical star with the mass and radius of our sun rotates about its axis with an angular speed of $\omega_i = 5.0 \times 10^{-6}$ rad/s.

1. [4 points] The star completes 1 revolution every

   A. 0.15 days
   B. 2.3 days
   C. 7.3 days
   D. 14.5 days
   E. 873 days

1 revolution = 2π rad

$\frac{2\pi}{5\times10^{-6}} = 1256637$ seconds $= 14.5$ days

The star collapses, becoming a white dwarf. After 12 hours, its angular velocity has increased to 0.1 rad/sec. Assume that no mass is lost in the process.

2. [5 points] What is the final radius of the white dwarf?

   A. 1.9 km
   B. 7.8 km
   C. 4.9×10^3 km
   D. 7.8×10^3 km
   E. 2.8×10^6 km

$L = I\omega$ $\quad I = \frac{2}{5}MR^2$ $\quad M = 1.99\times10^{30}$ kg $\quad R_i = 6.96\times10^8$ m

$L_i = L_f$

$\frac{2}{5}MR_i^2\omega_i = \frac{2}{5}MR_f^2\omega_f$

$R_f^2 = R_i^2 \frac{\omega_i}{\omega_f} = (6.96\times10^8 \text{ m})^2 \frac{5\times10^{-6}}{0.1} = 2.4\times10^{13}$ m

$\Rightarrow \boxed{R_f = 4.9\times10^3}$

3. [4 points] What is the magnitude of the average angular acceleration of the star over the 12 hours?

   A. 5.4×10^{-11} rad/s^2
   B. 1.2×10^{-6} rad/s^2
   C. 2.3×10^{-6} rad/s^2
   D. 1.4×10^{-4} rad/s^2
   E. 8.3×10^{-3} rad/s^2

$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{0.1 - 5\times10^{-6}}{12\cdot60\cdot60} = 2.3\times10^{-6}$ r/s^2

4. [5 points] The star completes its collapse when it reaches a radius of $R_f = 2.84 \times 10^3$ km and an angular velocity of $\omega_f = 0.3$ rad/sec. What is the change (from start to finish) in the kinetic energy of the star?

   A. 0 J
   B. $\frac{1}{5}MR_f^2\left[\omega_f^2 - \omega_i^2\right]$ J
   C. $\frac{2}{5}M\left[R_f^2\omega_f - R_i^2\omega_i\right]$ J
   D. $\frac{2}{5}M^2\left[R_f^2\omega_f^2 - R_i^2\omega_i^2\right]$ J
   E. $\frac{1}{5}M\left[R_f^2\omega_f^2 - R_i^2\omega_i^2\right]$ J

$KE = \frac{1}{2}I\omega^2$

$KE_i = \frac{1}{2}\cdot\frac{2}{5}MR_i^2\omega_i^2$

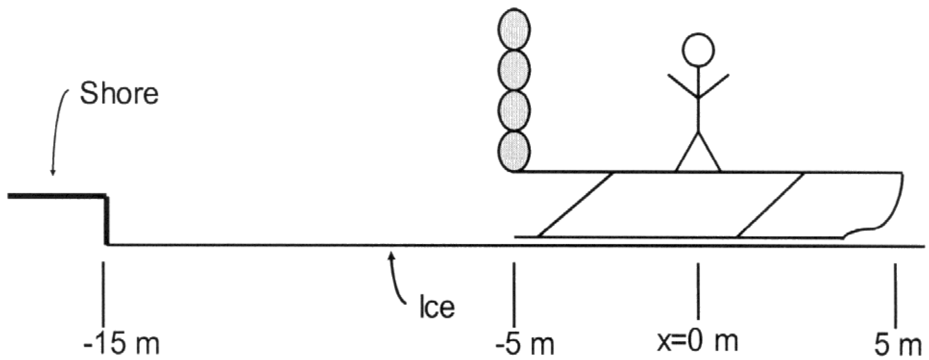
$\Delta KE = \frac{2}{10}MR_f^2\omega_f^2 - \frac{2}{10}MR_i^2\omega_i^2$

$= \frac{2}{10}M\left[R_f^2\omega_f^2 - R_i^2\omega_i^2\right]$

Name Solutions **Student ID** ________ **Score** ________

last *first*

Part II. [24 points] M&M on ice – A Massless Man is standing on a 10m long sled. The center of the sled is positioned at x=0 m, and the edge of the shore is at -15 m. The ice is *very* thick and *very* slippery (*i.e.* no friction, and it won't break). There is a stack of 4 large snowballs on the very left edge of the sled. Each has a mass of 200 kg. Assume that the sled is massless for questions 5 to 8. The massless man is not allowed to throw the snowballs off the edge of the sled.



5. [4 points] What is the x coordinate of the center of mass of this system (the massless man, the sled, and the snowballs)?

   A. -15 m
   B. -5.0 m
   C. -2.5 m
   D. 0.0 m
   E. 5.0 m

   ∃ mass only @ -5m → everything else is massless

6. [4 points] The center point of the sled is initially at 0 m. If the massless man moves two of the snowballs to the far right end of the sled, at what x coordinate will the center of the sled be?

   A. -15 m
   B. -5.0 m
   C. -2.5 m
   D. 0.0 m
   E. 5.0 m

   The COM of the sled, man, snowballs will now be the center of the sled. Since ∄ external force, it won't move! So ~~COM~~ center of sled must now be -5m.

7. [4 points] Which statement best describes the situation after the massless man has moved the two snowballs from the left end of the sled to the right?

   A. The sled will be at rest, to the left of its initial position
   B. The sled will be at rest, to the right of its initial position
   C. The sled will be moving in the +x direction
   D. The sled will be moving in the –x direction
   E. None of the above

8. [3 points] By moving the snowballs from one end of the sled to the other will the massless man ever be able to make the sled touch the shore?

   A. Yes
   B. No
   C. Not enough information

   MM can move the COM 10m → so he is 10m from the shore.

Name Solutions Student ID ____________________ Score__________

last *first*

For the next two questions, assume instead that the sled has a mass of 100 kg, uniformly distributed about its center (but the man is still massless). Assume the snowballs are again stacked on the left end of the sled, and that the sled is again in the position shown above (x=0 m).

9. [4 points] What is the x-position of the system's center of mass (sled, massless man, snowballs)?

   A. -4.9 m
   B. -4.4 m
   C. -2.1 m
   D. 2.1 m
   E. 5.0 m

$$x_{cm} = \frac{4 \cdot 200\,kg \cdot -5\,m + 0 \cdot 100\,kg}{4 \cdot 200\,kg + 100\,kg} = -4.4\,m$$

10. [4 points] By moving the snowballs from one end of the sled to the other will the massless man ever be able to make the sled touch the shore?

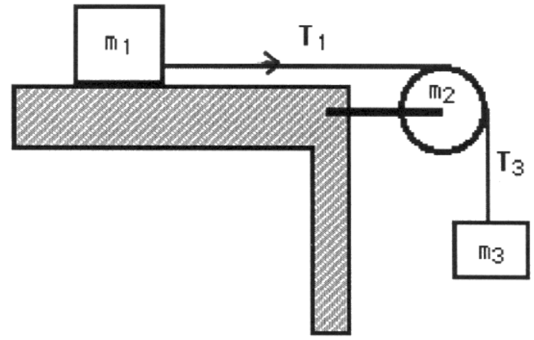
   A. Yes
   B. No
   C. Not enough information

— furthest he can move is 8.8m → not quite enough.

Name Solutions Student ID ____________________ Score__________

last *first*

Part III. Mixed Topics: A block of mass $\mathbf{m_1}$ rests on a table with which it has a coefficient of friction $\boldsymbol{\mu}$. A string attached to the block passes over a pulley to a block of mass $\mathbf{m_3}$. The pulley is a uniform disk of mass $\mathbf{m_2}$ and radius $\mathbf{r}$. As the mass $\mathbf{m_3}$ falls, the string does not slip on the pulley.



11. [6 pts] With what acceleration does the mass $\mathbf{m_3}$ fall?

A. $(m_3-\mu\bullet m_1)g/(m_1+m_3+m_2)$ m/s^2
B. $[(m_3-\mu\bullet m_1)g - (T_3-T_1)]/(m_1+m_3+m_2/2)$ m/s^2
C. $(m_3-\mu\bullet m_1)g/(m_1+m_3-m_2/2)$ m/s^2
D. $[(m_3-\mu\bullet m_1)\,g + (T_3-T_1)]/(m_1+m_3+ m_2)$ m/s^2
E. $(m_3-\mu\bullet m_1)g/\,(m_1+m_3+m_2/2)$ m/s^2

$T_3 - m_3 g = -m_3 a$

$T_1 - \mu N = m_1 a$

$\vec{\alpha}$ into page

$\Sigma\tau = I\vec{\alpha} = -\frac{Ia}{r}$

$I = \frac{1}{2} m_2 r^2$

$\tau_1 = \vec{r} \times T_1$ out of page

$= +r(\mu N + m_1 a)$

$\tau_2 = \vec{r} \times T_3$ into page

$= -r m_3 (g-a)$

$r(\mu N + m_1 a) - r m_3 (g-a) = -\frac{Ia}{r}$

$a(r m_1 + r m_3 + \frac{I}{r}) = (g r m_3 - r m_1 \mu g) \Rightarrow$

$a = \frac{g(m_3 - \mu m_1)}{m_1 + m_3 + m_2/2}$

12. [3 pts] A particle is at $\mathbf{r} = (2.0\mathbf{i} + 7.0\mathbf{j} + 5.0\mathbf{k})$ m, acted on by a force of $\mathbf{F} = (14.0\mathbf{j} - 3.0\mathbf{k})$ N what is the resulting torque, $\boldsymbol{\tau}$, calculated about the origin?

A. $(49\mathbf{i} + 6.0\mathbf{j} - 28\mathbf{k})$ m•N
B. $(-21\mathbf{i} + 28\mathbf{j} + 6\mathbf{k})$ m•N
C. $(-91\mathbf{i} + 6.0\mathbf{j} + 28\mathbf{k})$ m•N
D. $(49\mathbf{i} + 10\mathbf{j} + 28\mathbf{k})$ m•N
E. $(91\mathbf{i} + 28\mathbf{j} + 6\mathbf{k})$ m•N

$$\begin{vmatrix} i & j & k \\ 2.0 & 7.0 & 5.0 \\ 0 & 14.0 & -3.0 \end{vmatrix}$$

$= i(-7\cdot 3 - 5\cdot 14) - j(-2\cdot 3 - 0\cdot 5) + k(2\cdot 14 - 7\cdot 0)$

$= -91 i + 6j + 28k$

Name Solutions **Student ID** ____________________ **Score** __________

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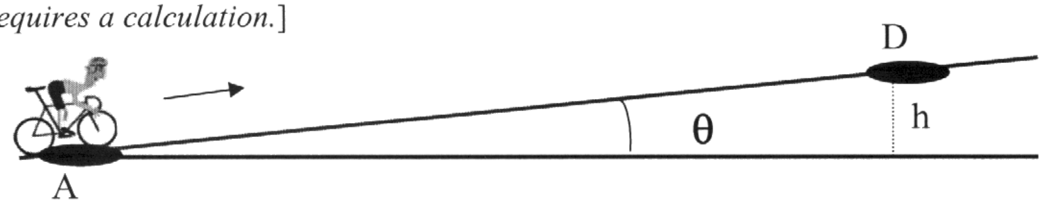
Part IV. [30 pts] A nice day to bike. – A cyclist is riding up a hill. At point A the bike - cyclist has a center of mass velocity of $v_{CM} = 6.10$ m/s. The rider and bike frame have a combined mass M = 82.0 kg while each tire has a mass $m_w = 2.20$ kg (so the total mass is $M_T = M + 2m_w$). Assume each tire can be approximated as a thin hoop of radius R=.340 m and that the tires are rolling without slipping. [*For each question in this section highlight answers by drawing a box around the algebraic equation and the numerical answer, if the question requires a calculation.*]

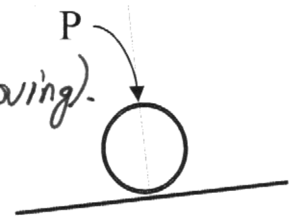


13. [6 pts] At point A, what is the magnitude of the angular velocity, ω, in rad/s ~~and the period~~, T, in seconds of one of the bike wheels?

$V_{cm} = 6.1\,m/s \qquad R\omega = V \Rightarrow \boxed{\omega = \frac{V}{R}} \qquad \boxed{T = \frac{2\pi}{\omega} = 0.35\,s}$

$\omega = \frac{6.1\,m/s}{0.34\,m} = 17.9$ rad/s

14. [6 pts] At point A, relative to the ground what is the magnitude and direction of the linear velocity, v_P, in m/s for the point P on the tire furthest from the ground?



It is twice the speed of rot: you get $v = R\omega$, and also the V_{cm} (cause axis of rotation is moving). since $R\omega = V_{cm}$.

$V_{cm} + R\omega = 2V_{cm} = 12.2$ m/s

15. [10 pts] What is the total kinetic energy at point A?

$KE = KE_I + KE_T \qquad I = M_W R^2$

$KE_I = \frac{1}{2}I\omega_1^2 + \frac{1}{2}I\omega_2^2 = I\omega^2 \qquad \omega = \frac{V_{cm}}{R}$

$KE_T = \frac{1}{2}M_T V_{cm}^2$

$KE = M_W R^2 \frac{V_{cm}^2}{R^2} + \frac{1}{2}M_T V_{cm}^2$

$= M_W V_{cm}^2 + \frac{1}{2}M_T V_{cm}^2 = \boxed{V_{cm}^2\left(M_W + \frac{1}{2}M_T\right) = KE}$

$KE = (2.2 + \frac{1}{2}\cdot 86.4)(6.1)^2 = \boxed{1689.3\,J}$

16. [8 pts] Assume that the cyclist is not pedaling and comes to a stop at point D. The height gained between Point A and Point D is *h*. What is the distance, *d*, between points A and D? Express *d* only as an analytical expression. For the total kinetic energy at point A use the symbol K_A (**Not the expression derived in Question 15 above**). Neglect air resistance, internal friction between the wheels and axles, and assume no use of brakes.

All KE goes into mgh, and h is related to d: $d\sin\theta = h$

$mgh = \cancel{KE}\; K_A$

$h = \frac{K_A}{mg} \qquad d = \frac{h}{\sin\theta} = \boxed{\frac{K_A}{mg\sin\theta}} = d$

Name (please print): ____ Total Points:_____
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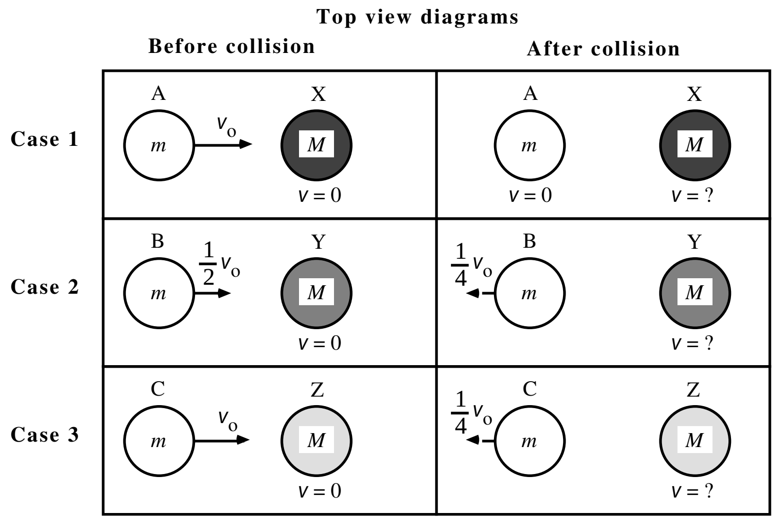
V. [20 points] Three experiments are performed with 6 pucks, A, B, C, X, Y, and Z, on horizontal, frictionless surfaces. Pucks A, B, and C each have mass *m.* Pucks X, Y, and Z each have mass *M* ($M > m$) and are all initially at rest. Pucks X, Y, and Z are made of different materials.

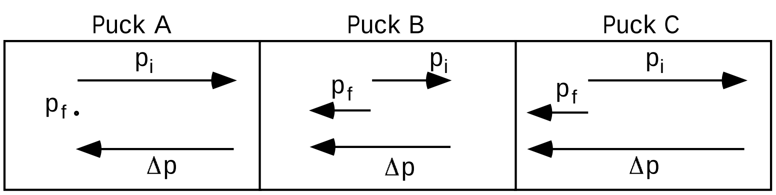


In Case 1, puck A moves with initial speed v_0 toward puck X. After the collision, puck A is at rest.

In Case 2, puck B moves with initial speed $(1/2)v_0$ toward puck Y. After the collision, puck B moves to the left with speed $(1/4)v_0$.

In Case 3, puck C moves with initial speed v_0 toward puck Z. After the collision, puck C moves to the left with speed $(1/4)v_0$.

17. [6 pts] Rank the *change in momentum* vectors ($\Delta\vec{p}$) of pucks **A**, **B**, and **C** in order of decreasing magnitude from greatest to smallest. Explain.



C > A > B.

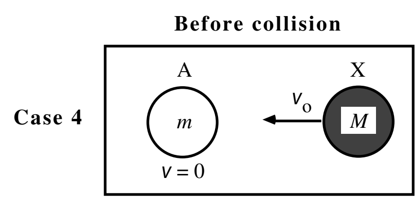
$|\Delta\vec{p}_C| = (5/4)|\Delta\vec{p}_A|$

$|\Delta\vec{p}_B| = (3/4)|\Delta\vec{p}_A|$

18. [8 pts] Rank the final speeds of pucks **X**, **Y**, and **Z** from greatest to smallest. Explain.

Z > X >Y. Because there is no net force on either system, the momentum of each system is constant. When the momentum is constant in a system of two objects, a change in momentum of one object is equal and opposite to the change in momentum of the second object. Thus, the change in momentum of puck A is equal in magnitude to the change in momentum of puck X, etc. Therefore the change in momentum ranking for pucks X, Y, and Z is Z > X > Y. Because X, Y, and Z have equal mass and all start from rest, this is also the final speed ranking.

Suppose that a different experiment, Case 4, had been performed with pucks A and X.



In Case 4, puck X is launched toward puck A with initial speed v_0 (the same speed with which puck A is launched toward puck X in Case 1).

19. [6 pts] Would the magnitude of the *change in momentum* of puck A in Case 4 be *greater than, less than,* or *equal to* the magnitude of the *change in momentum* of puck A in Case 1? Explain.

Equal to. Case 4 is identical to case 1 except that we are looking at the collision from a different reference frame. The free-body diagram for either puck will look the same from either frame, so the impulse delivered to puck A by puck X is the same for both cases. Thus, the change in momentum of puck A will be the same as viewed from either reference frame.