Name ______*solutions*____________________

Thermal Physics 224 **First midterm** **9.30 am, Wednesday October 31, 2007**
Autumn 2007 Instructor: David Cobden

Do not turn this page until the buzzer goes at 9.30. You must hand your exam to me by the time I leave the room at 10.25.

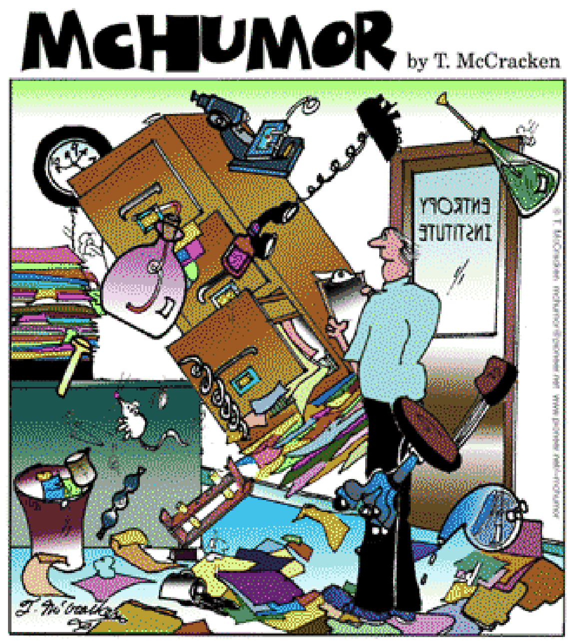
Attempt all the questions.

Please write your name on every page and your SID on the first page.

Write all your working on these question sheets. Use this front page for extra working. It is important to show your calculation or derivation. Some of the marks are given for showing clear and accurate working and reasoning.

Watch the blackboard for corrections or clarifications during the exam.

This is a closed book exam. ***No books, notes or calculators allowed.***



The Entropy Institute.

 Name solutions

1. [4] State the equipartition theorem.

$$\langle \text{energy in one quadratic degree of freedom} \rangle = \frac{kT}{2}$$

2. [10] Find an expression for the root mean square speed v_{rms} of a dust particle of mass m suspended in air at temperature T.

$$\left\langle \tfrac{1}{2} m v_x^2 \right\rangle = \left\langle \tfrac{1}{2} m v_y^2 \right\rangle = \left\langle \tfrac{1}{2} m v_z^2 \right\rangle = \frac{kT}{2} \quad \text{by above.}$$

$$\therefore \left\langle \tfrac{1}{2} m v^2 \right\rangle = \left\langle \tfrac{1}{2} m (v_x^2 + v_y^2 + v_z^2) \right\rangle = \frac{3kT}{2}$$

$$\therefore \langle v^2 \rangle = \frac{3kT}{m} \qquad \therefore v_{rms} = \sqrt{\frac{3kT}{m}}$$

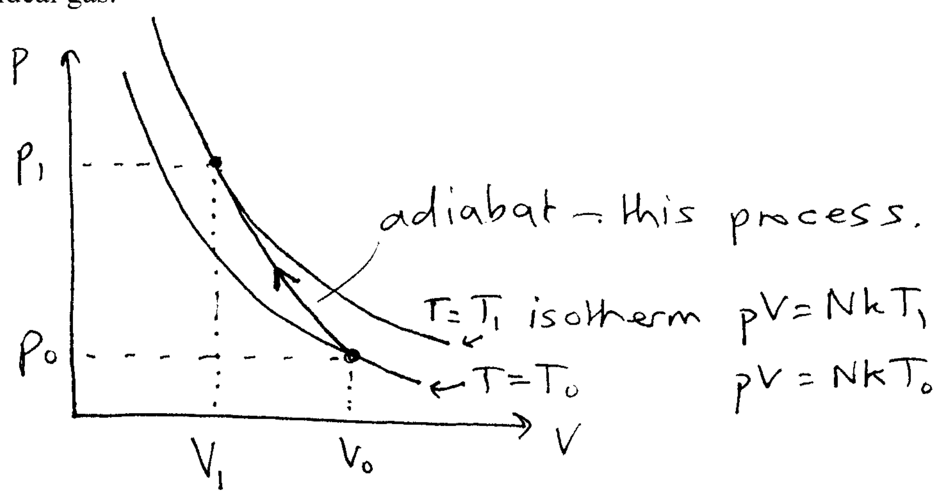
3. [6] Find an *approximate* expression for the root mean square angular velocity ω_{rms} of the dust particle if its radius is approximately r.

Roughly, $\left\langle \tfrac{1}{2} I \omega^2 \right\rangle = \dfrac{3kT}{2}$ by analogy.

Also roughly, $I = mr^2$ (to within a numerical factor)

$$\therefore \omega_{rms} \simeq \sqrt{\frac{3kT}{mr^2}}$$

4. [8] In the course of pumping up a bicycle tire, a volume V_0 of air at atmospheric pressure p_0 is compressed adiabatically to a pressure p_1. Sketch the process on a p-V diagram, indicating also some isotherms. Assume air is an ideal gas.



5. [8] What is the final volume V_1 of this air after compression? Use $\gamma = 1 + 2/f$, where f has its usual meaning, and the fact that air is mostly diatomic nitrogen and oxygen.

$$pV^\gamma = \text{const} \quad \therefore p_1 V_1^\gamma = p_0 V_0^\gamma \quad \therefore V_1 = V_0 \left(\frac{p_0}{p_1}\right)^{1/\gamma}$$

For diatomic molecule $f = \underset{\substack{\text{kinetic} \\ v}}{3} + \underset{\substack{\text{rotational} \\ \omega}}{2} = 5 \quad \therefore \gamma = 1 + \frac{2}{5} = \frac{7}{5}$

$$\therefore V_1 = V_0 \left(\frac{p_0}{p_1}\right)^{5/7}$$

 Name solutions

6. [8] How much work is done in compressing the air?

$$pV^\gamma = p_0 V_0^\gamma$$

$$W = -\int_{V_0}^{V_1} p\,dV = \int_{V_1}^{V_0} p_0 \frac{V_0^\gamma}{V^\gamma}\,dV$$

$$= p_0 V_0^\gamma \left[\frac{-1}{\gamma-1}\frac{1}{V^{\gamma-1}}\right]_{V_1}^{V_0} = \frac{p_0 V_0^\gamma}{\gamma-1}\left(\frac{1}{V_1^{\gamma-1}} - \frac{1}{V_0^{\gamma-1}}\right)$$

$$\gamma - 1 = \frac{2}{5} \qquad = \frac{5}{2} p_0 V_0^{7/5}\left(V_1^{-2/5} - V_0^{-2/5}\right)$$

7. [8] If the temperature of the air is initially T_0, what is the temperature T_1 after compression?

$$pV = NkT \quad \therefore V = \frac{NkT}{p} \qquad \therefore p_1\left(\frac{NkT_1}{p_1}\right)^\gamma = p_0\left(\frac{NkT_0}{p_0}\right)^\gamma$$

$$\therefore \left(\frac{T_1}{T_0}\right)^\gamma = \left(\frac{p_0}{p_1}\right)^{1-\gamma} \qquad \therefore T_1 = T_0\left(\frac{p_1}{p_0}\right)^{1-1/\gamma} = \left(\frac{p_1}{p_0}\right)^{2/5}$$

8. [5] The heat equation in one dimension is $\frac{\partial^2 T}{\partial x^2} = \frac{C}{k_t}\frac{\partial T}{\partial t}$. What is k_t and what are its units?

k_t is the thermal conductivity. $[k_t] = J m^{-2} s^{-1} / K m^{-1}$
$= J m^{-1} s^{-1} K^{-1}$

9. [6] What is C? Give an expression for C in terms of V, U and T (with their usual definitions). Mention any approximation involved.

C is the heat capacity per unit volume.

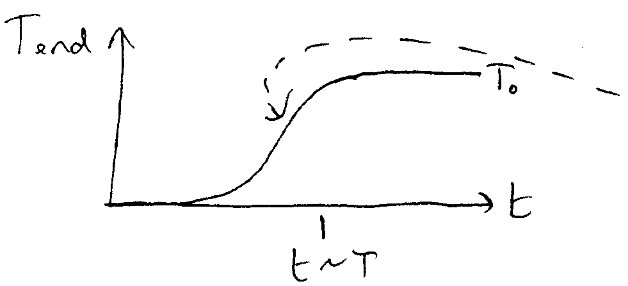
$$C = \frac{1}{V}\frac{\Delta Q}{\Delta T} \doteq \frac{1}{V}\left(\frac{\partial U}{\partial T}\right)_V$$

We use $C_p \doteq C_v$ because thermal expansion is small for solids.

10. [10] A solid rod of length L is heated to temperature T_0 at its center. Use the form of the heat equation to *estimate roughly* the maximum rate at which the temperature subsequently rises at the ends of the rod in terms of the above quantities.

Dimensions of heat equation: $\frac{T}{L_0^2} = \frac{C}{k_t}\frac{T}{\tau}$

L_0 = characteristic size
τ = " time. $\therefore \tau = \frac{C L_0^2}{k_t}$



slope $\simeq \frac{T_0}{\tau}$ $\qquad L_0 \sim \frac{L}{2}$ (heated in middle)

$$\simeq \frac{4 k_t T_0}{C L^2}$$

Name solutions

11. [4] State the Second Law of Thermodynamics.

Entropy never decreases (or equivalent)

12. [6] Show from the definition of entropy S in terms of multiplicity Ω that it is an extensive quantity.

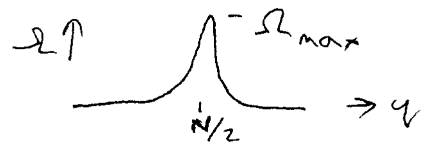
$S = k \ln \Omega$ For one system. If system is split into parts A and B, $\Omega = \Omega_A \Omega_B$

$\therefore S = k \ln(\Omega_A \Omega_B) = k \ln \Omega_A + k \ln \Omega_B = S_A + S_B$ ← definition of extensive.

13. [5] Consider a two-state paramagnet with N dipoles of which q point up. Assume that the paramagnet is completely isolated and the magnetic field is zero, so there is no energy difference between up and down spins. The problem is equivalent to that of finding q pennies with their heads up in a box of N. What is Ω for the macrostate specified by q?

No ways to choose q from N is $\Omega = \dfrac{N!}{q!\,(N-q)!}$

14. [4] What is the most probable value of q, assuming that all microstates are equally likely?



$q_{peak} = \dfrac{N}{2}$

15. [10] Use this, and Stirling's approximation ($\ln m! \approx m \ln m - m$) to find a simple expression for the entropy of the paramagnet when N is large. Demonstrate that it is indeed extensive for this system.

$S = k \ln \Omega \rightarrow k \ln \Omega_{max}$ for large N

$$\Omega_{max} = \Omega\left(q = \tfrac{N}{2}\right) = \frac{N!}{\left[\left(\frac{N}{2}\right)!\right]^2}$$

$$\therefore S = k\left[\ln N! - 2 \ln\left(\tfrac{N}{2}\right)!\right]$$

$$\simeq k(N \ln N - N) - 2k\left(\tfrac{N}{2} \ln \tfrac{N}{2} - \tfrac{N}{2}\right) \quad \text{by Stirling}$$

$$S/k = N \ln N - N \ln \tfrac{N}{2} = N \ln 2$$

$$S = Nk \ln 2$$

If we split the system into parts, $N = N_A + N_B$, clearly $S_{tot} = S_A + S_B$ because $S \propto N$.