

Thermal Physics 224
Autumn 2008

First midterm **9.30 am, Monday October 27, 2008**
Instructor: David Cobden

Do not turn this page until the buzzer goes at 9.30. You must hand your exam to me by the time I leave the room at 10.25.

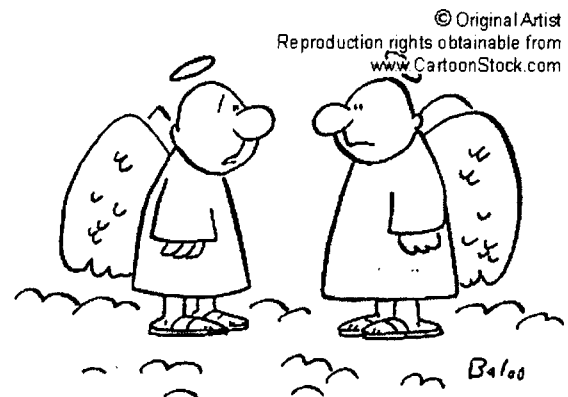
Attempt all the questions.

Please write your name on every page and your SID on the first page.

Write all your working on these question sheets. Use this front page for extra working. It is important to show your calculation or derivation. Some of the marks are given for showing clear and accurate working and reasoning.

Watch the blackboard for corrections or clarifications during the exam.

This is a closed book exam. *No books, notes or calculators allowed.*



"If we have everlasting life,
what about *entropy*?"

1. [5] Define "thermal equilibrium".

A system (eg two bodies in contact) has reached a stationary state in which there is no macroscopic flow of energy (heat).

2. [4] State the equipartition theorem.

In thermal eqm, the average energy in each ^{accessible} degree of freedom is $\frac{kT}{2}$.

3. [4] State the Second Law of Thermodynamics.

Multiplicity, or entropy, tends to increase.

4. [10] Estimate the rms velocity of the shaking of my hand due to bombardment by air molecules. ($kT \approx 0.4 \times 10^{-20}$ J at $T = 300$ K). Do not attempt to quantify effects of caffeine and exhaustion.

$$\left\langle \frac{1}{2} m v^2 \right\rangle = \frac{1}{2} kT \quad \rightarrow \quad v_{\text{rms}} = \sqrt{\frac{kT}{m}}$$

$m = \text{mass of hand} \sim 1 \text{ kg}$

$T = \text{room temp} \sim 300 \text{ K}$

$$v_{\text{rms}} \approx \sqrt{\frac{0.4 \times 10^{-20} \text{ J}}{1 \text{ kg}}} \sim \sqrt{0.4} \times 10^{-10} \text{ m s}^{-1}$$

$$\approx 10^{-10} \text{ m s}^{-1} = 1 \text{ \AA s}^{-1}$$

5. [5] What is the property of real gases that allows an absolute temperature scale to be obtained?

As $p \rightarrow 0$, $pV = NkT$ becomes exact for all gases

so we can define $T = \lim_{p \rightarrow 0} \left(\frac{pV}{Nk} \right)$

6. [10] Find the isothermal compressibility β of a gas obeying the ideal gas law, in terms of p , V , N and T .

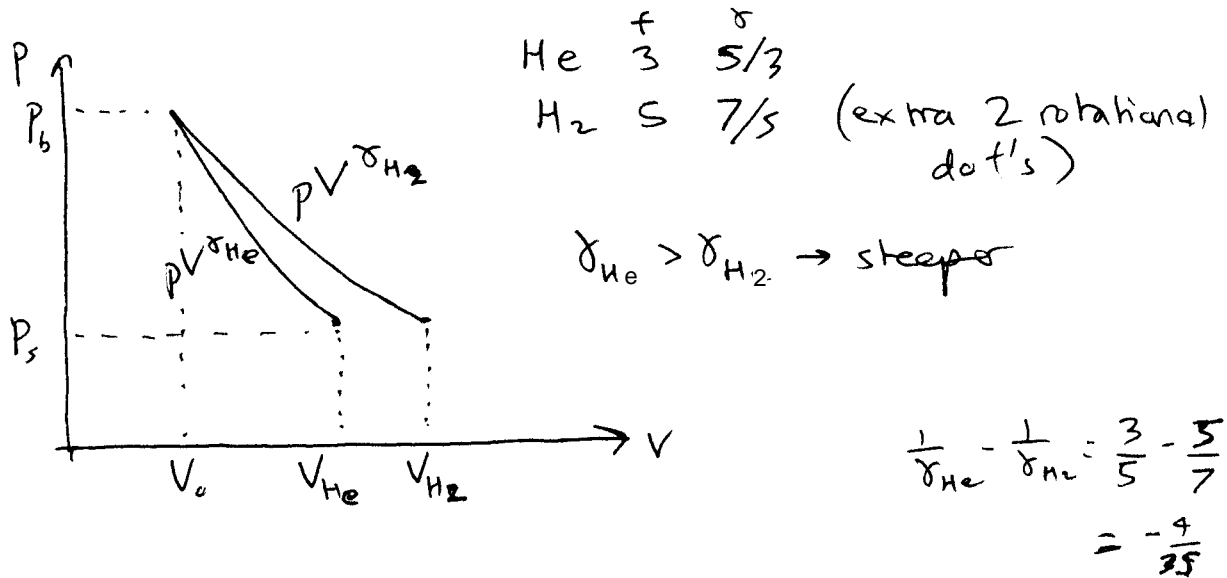
$$\beta = -\frac{1}{V} \left(\frac{\partial V}{\partial p} \right)_T = - \left(\frac{\partial \ln V}{\partial p} \right)_T \quad pV = NkT$$

$$\therefore \ln V = \ln NkT - \ln p$$

$$= -\frac{\partial}{\partial p} \Big|_T (\ln NkT - \ln p)$$

$$= \frac{\partial \ln p}{\partial p} = \frac{1}{p}$$

7. [10] A bubble of helium (monatomic) and a bubble of hydrogen (diatomic), each with volume V_0 , are both initially held in thermal equilibrium at pressure p_b at the bottom of a pool of water with uniform temperature T_0 . Both bubbles are then released, and they rise quickly and hence adiabatically to the surface where $p = p_s$. Sketch the variation of p with V for both bubbles on the same p - V diagram. (Note: $\gamma = 1 + 2/f$, where f is the number of accessible degrees of freedom per molecule).



8. [6] What is the ratio of the final volumes of the two bubbles when they reach the surface?

$$p_s V_{He}^{\gamma_{He}} = p_b V_0^{\gamma_{He}} \quad \therefore V_{He} = \left(\frac{p_b}{p_s}\right)^{1/\gamma_{He}} V_0$$

$$p_s V_{H_2}^{\gamma_{H_2}} = p_b V_0^{\gamma_{H_2}} \quad \therefore V_{H_2} = \left(\frac{p_b}{p_s}\right)^{1/\gamma_{H_2}} V_0$$

$$\frac{V_{He}}{V_{H_2}} = \frac{\left(\frac{p_b}{p_s}\right)^{1/\gamma_{He}} V_0}{\left(\frac{p_b}{p_s}\right)^{1/\gamma_{H_2}} V_0} = \left(\frac{p_b}{p_s}\right)^{1/\gamma_{He} - 1/\gamma_{H_2}} = \left(\frac{p_b}{p_s}\right)^{-4/35}$$

9. [4] Which bubble reaches a lower temperature?

He - $pV = NkT_0$ is smaller.

10. [4] Which bubble does more work against the water?

H₂ - integral $\int p dV$ is larger

11. [4] If instead the bubbles had risen much more slowly and thus isothermally, what would the ratio of their final volumes have been?

The same - $pV = NkT_0$ for both

12. [6] State Fourier's law for heat flow in one dimension, defining all quantities and giving their units.

$$J = -k_t \frac{\partial T}{\partial x}$$

T = temperature (K)

x = position (m)

J = heat flow/unit area/unit time ($J m^{-2} s^{-1}$)

k_t = thermal conductivity ($J m^{-1} s^{-1} K^{-1}$)

13. [8] Define enthalpy H , and show that for a chemical reaction, where the reactants and products are at the same pressure, the heat released is equal to the change in enthalpy.

$$H = U + PV$$

$$\therefore \Delta H = \Delta U + \Delta(PV)$$

$$= \Delta U + P\Delta V \quad \text{if } P_f = P_i$$

$$= W + Q + P\Delta V \quad \text{by First Law.}$$

For chemical reaction, $W = P\Delta V$ (no other work done)
 $\therefore Q = \Delta H$ (heat absorbed!)

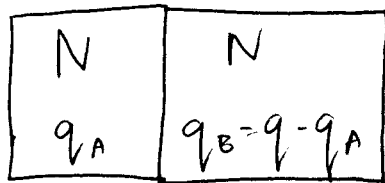
14. [8] The probability of a measurement of a variable x being between x and $x+dx$ is $p(x)dx$. If $p(x) = ae^{-ax}$ and x is positive-definite, show that $p(x)$ is properly normalized, and say why it must be so.

$$\int_{\text{all } x} p(x) dx = \int_0^{\infty} ae^{-ax} dx = a \left[\frac{e^{-ax}}{-a} \right]_0^{\infty} = -(0 - 1) = 1$$

✓ normalized

Σ probabilities of all possible outcomes = 1

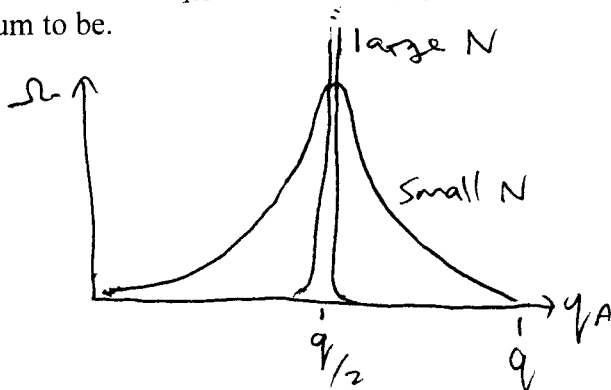
15. [8] Two identical paramagnets, A and B, each having N spins, are in thermal contact and isolated from their environment so that their total energy $E = q\Delta = (q_A + q_B)\Delta$ is fixed. What is the multiplicity of the combined system, Ω , as a function q_A , q and N ?



$$\Omega = \Omega_A \Omega_B = \frac{N!}{q_A!(N-q_A)!} \frac{N!}{q_B!(N-q_B)!}$$

$$= \frac{(N!)^2}{q_A!(N-q_A)!(q-q_A)!(N-q+q_A)!}$$

16. [6] Sketch Ω vs q_A for small N (say, 10) and large N (say, 100), showing where you expect its maximum to be.



Symmetric for $q_A \leftrightarrow q_B = q - q_A$
 max. when energy is uniformly distributed,
 ie $q_A = q_B = \frac{q}{2}$.