 Name ___________solutions____________

Thermal Physics 224 **Second midterm** **9.30 am, Monday November 19, 2007**
Autumn 2007 Instructor: David Cobden

Do not turn this page until the buzzer goes at 9.30. You must hand your exam to me by the time I leave the room at 10.25.

Attempt all the questions.

Please write your name on every page and your SID on the first page.

Write all your working on these question sheets. Use this front page for extra working. It is important to show your calculation or derivation. Some of the marks are given for showing clear and accurate working and reasoning.

Watch the blackboard for corrections or clarifications during the exam.

This is a closed book exam. ***No books, notes or calculators allowed.***

The generalized thermodynamic identity is

$$dU = TdS - pdV + \mu dN. \qquad \text{(i)}$$

The Sackur-Tetrode equation for the entropy of an ideal monatomic gas is

$$S = Nk\left[\ln\left\{\frac{V}{N}\left(\frac{4\pi mU}{3Nh^2}\right)^{\frac{3}{2}}\right\} + \frac{5}{2}\right]. \qquad \text{(ii)}$$

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1. [5] What is meant by a *quasistatic* process?

The system remains close to internal equilibrium
The macroscopic variables remain well defined. Slow.

2. [5] An ideal gas of N atoms initially at temperature T_1 and volume V_1 is expanded to volume V_2. What is the change ΔS in its entropy S if the expansion is adiabatic and quasistatic, and why?

If quasistatic, $dS = \frac{Q}{T}$ $\therefore$ Adiabatic $\rightarrow Q = 0 \rightarrow dS = 0$
$\rightarrow \Delta S = 0$

3. [5] What if the expansion is adiabatic but not quasistatic, and why?

$\Delta S > 0$ because it's irreversible.

4. [10] What if the expansion is isothermal?

$dS = \frac{Q}{T} \rightarrow \Delta S = \frac{Q}{T}$ if T constant

$\Delta U = W + Q$

$= 0$ for ideal gas if $\Delta T = 0$

$$\therefore Q = -W = \int_{V_1}^{V_2} p\,dV = \int_{V_1}^{V_2} \frac{NkT}{V} dV = NkT \ln \frac{V_2}{V_1}$$

(or use the Sackur-Tetrode eqn, (i))

$$\therefore \Delta S = Nk \ln \frac{V_2}{V_1}$$

5. [10] What if it is a free expansion, ie, into vacuum?

Again $\Delta U = 0$ because $W = 0$ and $Q = 0$ $\rightarrow \Delta T = 0$
$T_2 = T_1$

However from Sackur-Tetrode

$S = Nk \ln V$ + stuff indep of V

$\therefore \Delta S = Nk \ln \frac{V_2}{V_1}$ again.

6. [3] Is equation (i) valid for quasistatic processes only, or for any process that leads to a change in U, S, V and N?

Any process. $U(S, V, N)$ is a function of state.

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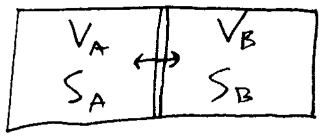
7. [10] Derive from equation (i) the expression for the pressure p which is most natural from the point of view of statistical mechanics. Show, using entropy, that this expression is a quantity that must be equal for two systems in thermal equilibrium which are able to exchange volume (eg. portions of gas on either side of a moveable partition.)

Holding U and N fixed we have $0 = T dS|_{U,N} - p dV|_{U,N}$

$\therefore p = T\left(\frac{\partial S}{\partial V}\right)_{U,N}$ $\quad S_{total} = S_A + S_B$ must be maximum.



Require $\frac{\partial S_{total}}{\partial V_A} = 0$ $\quad dV_A = -dV_B$

$\therefore \frac{\partial S_A}{\partial V_A} - \frac{\partial S_B}{\partial V_B} = 0$ $\quad \therefore \frac{\partial S}{\partial V}$ is same for both.

So is T, in eqm.

8. [8] Use this expression for p combined with equation (ii) to deduce the ideal gas law.

$$\frac{p}{T} = \left(\frac{\partial S}{\partial V}\right)_{U,N} = \frac{\partial}{\partial V}\left(Nk \ln V + \text{stuff indep of } V\right)$$

$$= \frac{Nk}{V}$$

$$\therefore pV = NkT$$

9. [10] Derive from equation (i) an expression for p in terms of U and V which is more natural from the point of view of classical thermodynamics. Explain why it is so, using the relationships between heat and entropy, and between volume and work, for a quasistatic process.

Holding S and N fixed, $dU|_{S,N} = -p dV|_{S,N}$

$\therefore p = -\left(\frac{\partial U}{\partial V}\right)_{S,N}$

For quasistatic process $dS = \frac{Q}{T}$ $\quad \therefore$ Const $S \rightarrow Q = 0$ and $W = -pdV$

Then $dU = Q + W = 0 - pdV$ $\quad$ so $p = \frac{\partial U}{\partial V}$

10. [10] State the definition of temperature T in statistical mechanics, which also can be deduced from equation (i), and use it to deduce the internal energy of the (an ideal monatomic) gas at temperature T.

$T = \left(\frac{\partial S}{\partial U}\right)_{N,V}^{-1}$ $\quad S = Nk \ln U^{3/2} + \text{stuff indep of } U$

$\therefore \frac{\partial S}{\partial U} = \frac{3}{2} Nk \frac{\partial}{\partial U}(\ln U) = \frac{3Nk}{2U}$

$\therefore T = \frac{2U}{3Nk}$ $\quad \therefore U = \frac{3NkT}{2}$

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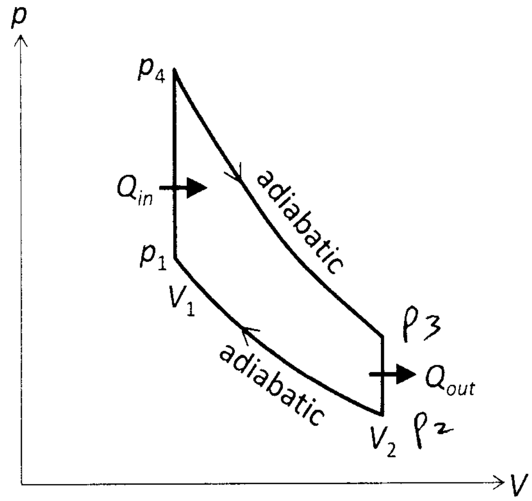
11. [8] A heat engine consists of some ideal gas which is taken quasistatically around the cycle shown. Heat Q_{in} comes from the hot reservoir which is at T_h, and heat Q_{out} is dumped into the cold reservoir at T_c. The efficiency is $e = \frac{W}{Q_{in}}$, where W is the work done by the gas during the cycle.
First, write e in terms of Q_{in} and Q_{out}.

Cons. of energy $\rightarrow W = Q_{in} - Q_{out}$

$\therefore e = \frac{Q_{in} - Q_{out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$



12. [6] Give or find an expression for U in terms of p and V for an ideal gas.

$U = \frac{fNkT}{2}$ $\quad pV = NkT$ $\quad \therefore U = \frac{fpV}{2}$ $\quad f$ = no. degrees of freedom

13. [10] Find Q_{in} and Q_{out}, making use of the First Law, and hence show that $e = 1 - \left(\frac{V_2}{V_1}\right)^a$, where you should determine the parameter a in terms of γ.

$\Delta U_{41} = W_{on} + Q_{in}$ $\quad W_{on} = \int p\,dV = 0$ $\quad \therefore Q_{in} = \Delta U_{41}$

$= \frac{fNk(T_4 - T_1)}{2}$

$= \frac{f}{2}(p_4 V_1 - p_1 V_1)$

Similarly $Q_{out} = \frac{f}{2}(p_3 - p_2)V_2$

$\therefore e = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{(p_3 - p_2)V_2}{(p_4 - p_1)V_1}$

For the adiabats, $pV^\gamma = \text{const}$ $\quad \therefore p_2 V_2^\gamma = p_1 V_1^\gamma$ and $p_3 V_2^\gamma = p_4 V_1^\gamma$

$\therefore (p_3 - p_2)V_2^\gamma = (p_1 - p_4)V_1^\gamma$ $\quad \rightarrow e = 1 - \left(\frac{V_1}{V_2}\right)^{\gamma - 1}$

14. [5] Why must this engine be less efficient than a Carnot Engine? Why is the cycle not reversible?

Q_{in} flows in at temperatures between T_1 and T_4 which must be less than T_h

$\rightarrow$ heat flows from T_h in the reservoir to a lower temperature in the gas $\Rightarrow$ irreversible (entropy is generated)