Name solutions

Thermal Physics 224 **Second midterm** **9.30 am, Monday 17 November, 2008**
Autumn 2008 Instructor: David Cobden

Do not turn this page until the buzzer goes at 9.30. You must hand your exam to me by the time I leave the room at 10.25.

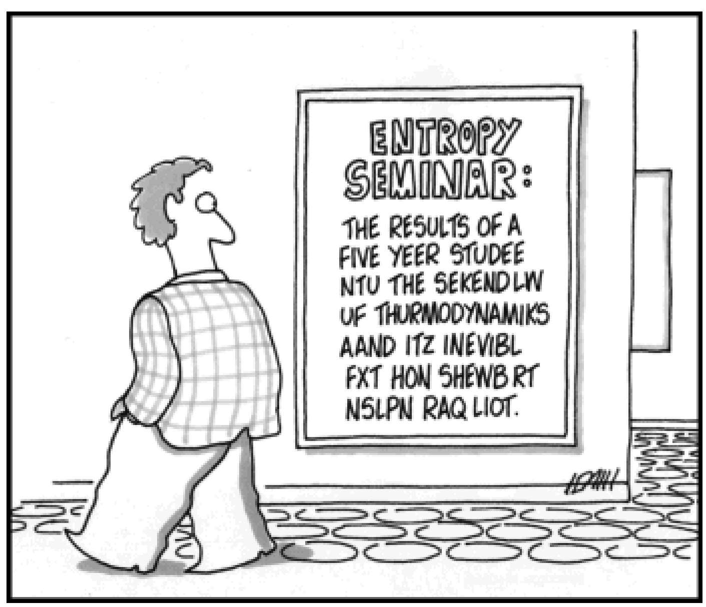
Attempt all the questions.

Please write your name on every page and your SID on the first page.

Write all your working on these question sheets. Use this front page for extra working. It is important to show your calculation or derivation. Some of the marks are given for showing clear and accurate working and reasoning.

Watch the blackboard for corrections or clarifications during the exam.

This is a closed book exam. ***No books, notes or calculators allowed.***



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1. [5] State the "generalized thermodynamic identity" relating U, T, S, P, V, μ and N.

$$dU = TdS - PdV + \mu dN$$

2. [2] How many independent variables are needed to specify the state of a system for which this identity applies?

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3. [5] Deduce an equation relating the chemical potential μ to a partial derivative of S.

Hold U,V constant: $0 = TdS + \mu dN\big|_{U,V}$ $\quad \therefore \mu = -T\left(\frac{\partial S}{\partial N}\right)_{U,V}$

4. [8] Find an expression for μ for an ideal gas using Sackur-Tetrode, $S = Nk\left[\ln\left\{\frac{V}{N}\left(\frac{4\pi mU}{3Nh^2}\right)^{\frac{3}{2}}\right\} + \frac{5}{2}\right]$.

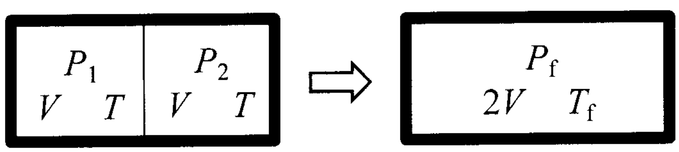
$$\left(\frac{\partial S}{\partial N}\right)_{U,V} = k\ln\left\{\frac{V}{N}\left(\frac{4\pi mU}{3Nh^2}\right)^{3/2}\right\} + \frac{5k}{2} + Nk\left[\frac{\partial}{\partial N}\bigg|_{U,V}\left(\ln N^{-5/2}\right) + 0\right]$$

$$= \quad '' \quad + \quad '' \quad + Nk \cdot \frac{1}{N} \cdot \left(-\frac{5}{2}\right)$$

cancels

$$\therefore \mu = -kT\ln\left\{\frac{V}{N}\left(\frac{2\pi mkT}{3h^2}\right)^{3/2}\right\}$$

Two portions of the same ideal gas, each with volume V, are initially in thermal equilibrium separated by a thin wall within a thermally isolated chamber in a room at temperature T. The pressure P_1 on the left is higher than the pressure P_2 on the right. The thin wall between them is then suddenly removed, and after a short time the gas reaches a steady pressure P_f.



5. [6] Is this process adiabatic? Yes Is it quasistatic? No Is it reversible? No

6. [6] Using the expression for μ from Question 4, and the proper interpretation of μ, show that there is initially a net flow of molecules from left to right.

$$P\frac{V}{N} = kT$$

$$\mu = -kT\ln\left\{\frac{kT}{P}\left(\frac{2\pi mkT}{3h^2}\right)^{3/2}\right\}$$

$$= \text{function of } T + kT\ln P$$

$\therefore P_1 > P_2 \Rightarrow \mu_1 > \mu_2$

$\mu_1 \rightarrow \mu_2$

higher μ wants to give up particles to lower μ

$\therefore$ particles flow from left to right

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7. [10] Using the First Law of thermodynamics, and the fact that the gas is ideal, show that the process is isothermal (ie, that the final temperature T_f is equal to T).

$\Delta U = Q + W$ $\quad Q = 0$ $\quad W = 0$

fast, insulated container — wall is just removed eg by sliding sideways

$\therefore \Delta U = 0 \quad \therefore U_f = U_1 + U_2$

$$\therefore \frac{N_f f k T_f}{2} = \frac{N_1 f k T}{2} + \frac{N_2 f k T}{2} = \frac{(N_1+N_2) f k T}{2}$$

but $N_f = N_1 + N_2$ $\quad \therefore T_f = T$.

8. [5] Explain the absence of a temperature change from a kinetic theory point of view.

The molecules don't exchange any momentum and energy with a moving wall ($W=0$) so their kinetic energy is unchanged - they just suddenly find they can move through a bigger volume.

9. [5] Find P_f.

$$P_f V_f = N_f k T_f \quad \therefore P_f = \frac{(N_1+N_2)kT}{2V} = \frac{P_1+P_2}{2}$$

10. [8] Find the total change in entropy, ΔS_{gas}.

from Sackur-Tetrode, $\frac{U}{N} = \frac{f}{2}kT$ doesn't change, nor does the $Nk \times$constant part

$$\therefore \Delta S = N_f k \ln \frac{V_f}{N_f} - N_1 k \ln \frac{V_1}{N_1} - N_2 k \ln \frac{V_2}{N_2}$$

$$\frac{\Delta S}{k} = (N_1+N_2)\ln \frac{2V}{N_1+N_2} - N_1 \ln \frac{V}{N_1} - N_2 \ln \frac{V}{N_2}$$

$$= (N_1+N_2)\left[\ln 2 - \ln(N_1+N_2)\right] + N_1 \ln N_1 + N_2 \ln N_2 \quad \text{where} \quad N_1 = \frac{P_1 V}{kT}$$

$$\text{or } \frac{T\Delta S}{V} = (P_1+P_2)\left[\ln 2 - \ln(P_1+P_2)\right] + P_1 \ln P_1 + P_2 \ln P_2 \qquad N_2 = \frac{P_2 V}{kT}$$

11. [8] Describe a modified process that would take this system from its initial to its final state *reversibly*. How much heat would flow into the chamber during this process?

One possibility is to move the wall quasistatically and isothermally to the right until the pressures are equal and then remove it. Since the process is now reversible, $\Delta S_{total} = 0 = \Delta S_{outside} + \Delta S_{gas}$

$\therefore \Delta S_{out} = -\Delta S_{gas} = -\frac{Q}{T}$ $\quad \therefore Q = \frac{\Delta S_{gas}}{T}$ must flow in.

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12. [12] A glass containing a mixture of 100 g of water and 100 g of ice at 0 °C = 273 K is placed in a room at 300.3 K. The ice takes half an hour to melt. Calculate the total change in entropy of the *universe* due to this melting. (The latent heat of melting of ice is $L_m = 334$ J/g).

$$\Delta S_{universe} = \Delta S_{ice} + \Delta S_{\substack{water \\ liquid}} + \Delta S_{room}$$

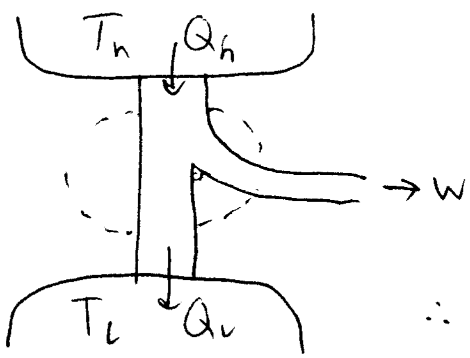
$$= \frac{Q_{latent}}{T_{ice}} + 0 - \frac{Q_{latent}}{T_{room}} = L_{melting} \times mass \times \left(\frac{1}{T_{ice}} - \frac{1}{T_{room}}\right)$$

$$\frac{1}{273K} - \frac{1}{300.3K}$$

$$= 334\, Jg^{-1} \times 100 g \frac{\times 27.3}{273 \times 300.3 K}$$

$$= \frac{334 \times 10}{300.3} JK^{-1} \doteq 11\, JK^{-1}$$

13. [10] Show that if the efficiency of a heat engine is greater that than the ideal limit (of a Carnot engine) then when it operates it decreases the total entropy of the universe. What does this say about such an engine?



$$e = \frac{W}{Q_h} = 1 - \frac{Q_L}{Q_h} > 1 - \frac{T_L}{T_h}$$

$$\therefore \frac{T_L}{T_h} > \frac{Q_L}{Q_h}$$

$$\therefore \Delta S = \frac{Q_L}{T_L} - \frac{Q_h}{T_h} = \left(\frac{Q_L}{Q_h} - \frac{T_L}{T_h}\right)\frac{Q_h}{T_L}$$

$$< 0$$

14. [10] What is the minimum rate, set by thermodynamics, at which an electrical power station generating 100 MW of electricity from steam at 327 °C could be dumping heat into a river at 27 °C?

$\downarrow \dot{Q}_h$

$\longrightarrow \dot{W} = 100\, MW$

$\downarrow \dot{Q}_L$

$$e = 1 - \frac{T_L}{T_h} = 1 - \frac{300}{600} \doteq \frac{1}{2}$$

$$\frac{Q_L}{T_L} > \frac{Q_h}{T_h} \quad \therefore W = Q_h - Q_L < \left(\frac{T_h}{T_L} - 1\right)Q_L \quad \therefore \dot{Q}_L > \frac{\dot{W}}{\frac{T_h}{T_L} - 1}$$

$$\dot{Q}_L > \frac{100 MW}{\frac{600K}{300K} - 1} = 100\, MW$$