Physics 228 - Winter 2008

HW V

Due Monday 2/11/08

Midterm Exam I is Friday 2/8/08

All problems are in the class text by Boas, except as noted.
Example problems: *Not to be turned in – solutions can be found in the back of the text, or in Appendix B.*

§8.9: 12, 27
§8.10: 5, 15, 17
§8.11: 7, 13, 15, 21
§8.12: 2, 7, 11
§8.13: 3, 15, 30, 47

Assigned problems: *To be turned in. Five problems chosen at random from those assigned in Boas below and the Special Exercise will be graded and are worth 5 points each. Problem A is extra credit: doing extra credit problems can make the difference in borderline grade assignments. (Regular HW points are not the same as Extra Credit points.)*

§14.7: 58
Special Exercise - Use complex contour integral techniques to evaluate the inverse Fourier transform (integral) in the Special Exercise of HW IV.
§8.9: 14, 26
§8.10: 14, 18
§8.11: 8, 16
(Delay §8.12:3 until HW VI)
§8.13: 16 [Assume that the driving term, the right hand side, vanishes for $x < 0$ and also assume boundary conditions $y(0) = y'(0) = 0$.]

A. (Extra Credit – 1 pt) Let's use the built-in functions in *Mathematica* to analyze Fourier and Laplace transforms.

   a) First consider problem 7.12.3 from HW III and the Special exercises. Write a *Mathematica* expression for the appropriate function – the double step function on the range $-\pi \le x \le \pi$ and verify that the plot of the function looks like the figure. Note that the "steps" in this function can be expressed in various ways but *Mathematica* seems to be happiest with the UnitStep[x] function (= 0 for x < 0, and = +1 for x >0, *i.e*. the usual theta function).

b) Find the Fourier transform of this function using the FourierTransform[f[x],x,α] *Mathematica* function and compare to your results in HW III.

c) Find the Inverse Fourier transform (*i.e*., the original function) using the InverseFourierTransform[f[x],x,α] *Mathematica* function and compare to the original function. I find that *Mathematica* seems to prefer to express the result in terms of the Sign[x] function (instead of the UnitStep[x] function), but the original function is reproduced (*e.g*., try plotting it).

d) Now let's play with the LaplaceTransform[f[t],t,p] function to solve the differential equation in problem 8.9.14 (see above), $y'' - 4y' = -4te^{2t}, y(0) = 0, y'(0) = 1$. Note that, by using the derivative function D[y[x],x] $= dy/dx$ to express the left-hand-side of the equation, we can explicitly take the Laplace transform of the left-hand-side, while introducing the boundary conditions. Perform these transforms using the LaplaceTransform function and obtain an expression for the Laplace transform of the full solution of the equation (including the boundary conditions). Note that this can be done all in one step using the Solve[ ] function.

e) Use the InverseLaplaceTransform to find the final solution.

Note: Problem sets must be turned in by the end of class or be in Steve Ellis's Physics Department mailbox by 12:20 PM on the date indicated. Late problem sets are accepted for 1 week with a 50% discount. Solutions will be posted on the web.