Electromagnetism, Physics 321 **Second midterm** **8.20 am, November 21, 2003**
Autumn 2003 Instructor: David Cobden

You have 60 minutes. End on the buzzer. Answer all questions. Please do not leave within the last ten minutes of the exam.

Write your name on every page and your ID on the first page.

Write all your working on these question sheets. Use this cover page for extra working (you might get credit for it.)

Watch the blackboard for corrections or clarifications during the exam.

This is a closed book exam. **No notes allowed. NO CALCULATORS!**

Standard notation for spherical coordinates is used throughout. Thus, for example, r is always the distance from the origin. As usual, take $V = 0$ at $r = \infty$.

Make sure you attempt all 14 questions!

Do not turn this page until I say 'go'.

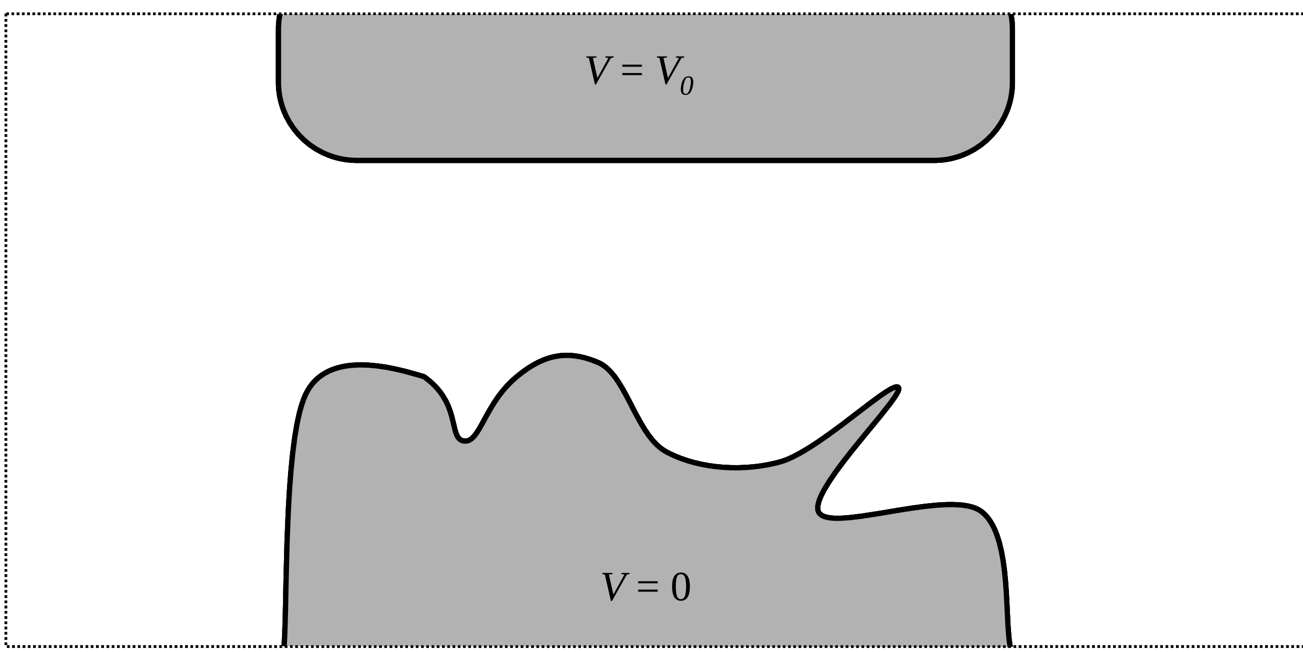
Useful metric conversions

1 trillion microphones = 1 megaphone
1 millionth of a fish = 1 microfiche
1 trillion pins = 1 terrapin
10 rations = 1 decoration
10 millipedes = 1 centipede
3 1/3 tridents = 1 decadent
2 monograms = 1 diagram

1. [10] Explain why the electric field at the surface of a conductor must be normal to the surface, starting with the assumption that a conductor contains charges which are free to move, and ending with the relationship between the field and the surface charge density.

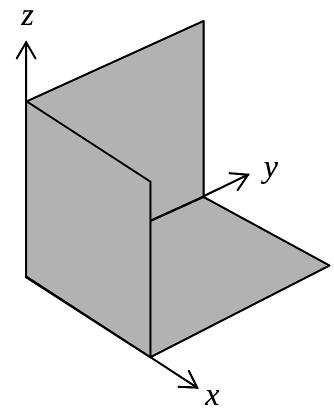
2. [5] State the uniqueness theorem for Laplace's equation in a region $\mathcal{V}$ bounded by surfaces on which either the potential V or its normal derivative $\partial V/\partial n$ is specified.

3. [9] The adjacent sides of two conductors are shown here in cross-section. They are uniform in the direction perpendicular to the paper. A potential V_0 is applied to the upper while the lower is grounded. Sketch equipotentials and a few field lines within the region indicated by the dotted box.



4. [5] As V_0 is increased, electrical breakdown occurs at the point where the electric field is maximum at the surface. Mark this point with an X and explain how you know the field is strongest there.

Three perpendicular, infinite, conducting planes at potential V_0 form a 'cubic corner'. Let them be the planes $x = 0$, $y = 0$ and $z = 0$, and let $\mathcal{V}$ be the region $x > 0$, $y > 0$, $z > 0$.



5. [8] Show that $V(\boldsymbol{x}) = (ax+b)(cy+d)(ez+f)$ obeys Laplace's equation in $\mathcal{V}$.

6. [8] Hence show that the potential in $\mathcal{V}$ is given by $V(\boldsymbol{x}) = V_0 + Cxyz$, where C is a constant.

7. [5] Find the surface charge density at position $(x,y,0)$ on the z=0 surface.

8. [5] Find the electrostatic pressure exerted on the surface at the same position. What is its direction?

9. [5] Explain why V diverges at infinity and why this unphysical behavior is not found in any real system.

The planes are now *grounded*, and a point charge $+q$ is placed at position $\boldsymbol{x} = (a,a,a)$.

10. [10] Describe a set of 7 image charges which may be used to find the potential $V(\boldsymbol{x})$.

11. [5] What is the total induced surface charge on the $x = 0$ plane? (No integration is necessary!)

12. [6] If V is expanded as a power series in r for $r > \sqrt{3}a$, which term, ie, which power of r, will dominate at large r? Why? (Do not try to calculate the term.)

13. [14] An electron of charge e is located at a distance z_0 from the center of a *neutral, isolated,* conducting sphere of radius R. Find the force of attraction of the electron towards the sphere.

14. [5] The electron finally hits the sphere and merges into it. What is the final potential of the sphere?