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Electrodynamics, Physics 321 **Final exam** **8.20 am, December 14, 2004**
Autumn 2004 Instructor: David Cobden

You have 120 minutes. End on the buzzer at 10.20. Answer all 16 questions. They are divided into four sections each worth 50 points, 1 section per sheet of paper. So aim to spend half an hour on each sheet.

Write your name on every page and your ID on the first page.

Write all your working on these question sheets. Use this cover page for extra working (you might get credit for it.)

It is important to show your calculation or derivation. You won't get full marks just for stating the correct answer if you don't show how you get it.

Watch the blackboard for corrections or clarifications during the exam.

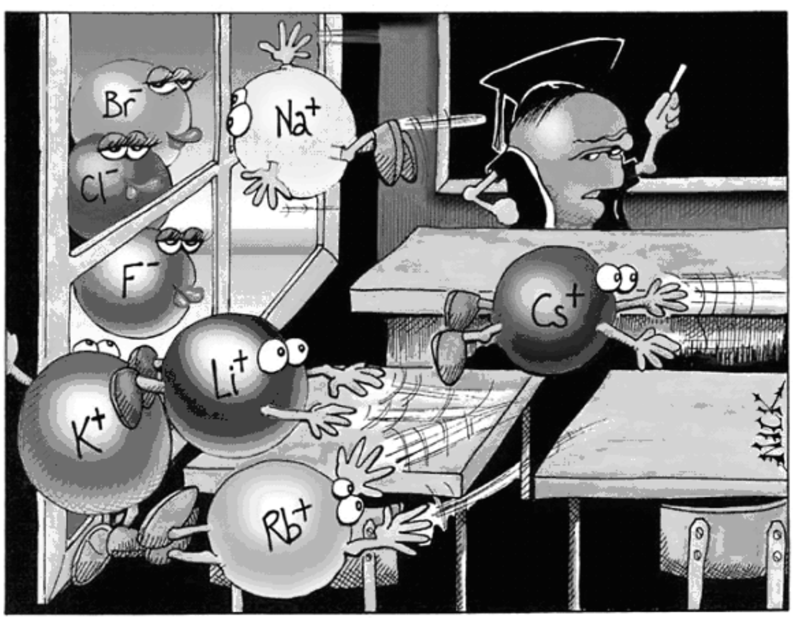
This is a **closed book** exam. **No notes allowed. No calculators!**

Do not turn this page until I say GO!

$P_l(1) = 1$

$P_0(x) = 1$ $P_1(x) = x$ $P_2(x) = (3x^2-1)/2$ $P_3(x) = (5x^3 - 3x)/2$ $P_4(x) = (35x^4 - 30x^2 + 3)/8$

$$-\nabla\left(\frac{\mathbf{p}.\hat{\mathbf{r}}}{4\pi\varepsilon_0 r^2}\right) = \frac{3(\mathbf{p}.\hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{p}}{4\pi\varepsilon_0 r^3} \qquad \varepsilon_0 \approx 10^{-11}\ \text{F/m}$$



"Perhaps one of you gentlemen would mind telling me just what it is outside the window that you find so attractive..?"

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I. *The questions in this section ask for estimates. Show your reasoning, explain clearly assumptions or approximations made, and do all calculations algebraically first.*

A spaceship has a net charge $Q = +10^{-5}$ C on it when it leaves the earth's surface. Assume the earth is a conductor left with a net charge $-Q$, ie, the earth-spaceship system is neutral. [$R_{earth} \approx 6{,}000$ km.]

1. [10] *Estimate* the resulting change in the electrostatic potential of the earth between the point when the spaceship departs and when it leaves the solar system.

2. [7] Estimate the electrostatic force on the spaceship when it is a distance $d = 1$ km from the ground.

3. [7] Estimate the force when $d = 10^6$ km.

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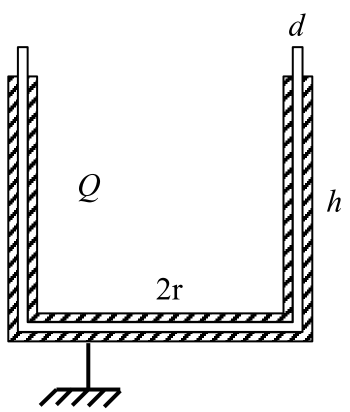
4. [11] Estimate the force when $d = 10^4$ km (ie, similar to R_{earth}).

5. [15] A Leyden jar consists of a cylindrical beaker made of glass ($\varepsilon_r = 10$) of thickness $d = 0.2$ cm, with height $h = 20$ cm and radius $r = 10$ cm, coated on the inside and outside with a metal films, as indicated below. The top edges of the beaker are not coated. A charge $Q = +10^{-5}$ C is deposited on the inner metal using a van de Graaff generator with the outside of the beaker grounded. Estimate the voltage developed between the inner and outer metal films.



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II. A ring of radius R centered on the origin in the x-y plane has uniform line charge density λ around its circumference. It is imbedded in an infinite, uniform dielectric of dielectric constant ε_r.

6. [20] Show that the potential along the z-axis is $V(z) = \dfrac{\lambda R}{2\varepsilon_r \varepsilon_0 z\sqrt{1+R^2/z^2}}$.

7. [20] By matching the result in the previous question to the eigenfunction expansion of the potential in spherical coordinates, find $V(r,\theta)$ away from the axis for $r > R$ to 3rd order in $1/r$.

8. [10] Give the definitions of the monopole moment Q and the dipole moment **p** for some localized charge distribution $\rho(\mathbf{r})$. When is **p** independent of coordinate system?

III. A spherical capacitor consists of two concentric spherical metal shells of radii R_{in} and R_{out} with the space between them filled by a uniform linear dielectric of dielectric constant ε_r. The inner shell holds a charge Q and the outer is grounded.

9. [25] Find **E**, **D** and **P** within the dielectric, all bound charge, and the capacitance C.

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10. [12] Show that the energy U stored in the dielectric is equal to $Q^2/2C$.

11. [5] What is the electrostatic pressure on the outer shell?

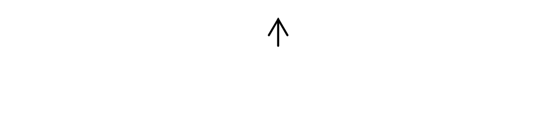
12. [8] Draw cartoons to indicate why $\mathbf{P}.\hat{\mathbf{n}} = \sigma_b$ at the surface of a dielectric and $\nabla.\mathbf{P} = -\rho_b$ in the bulk.

IV. A point dipole **p** is located a distance D above the conducting surface $z = 0$, oriented at an angle θ to the z-axis.

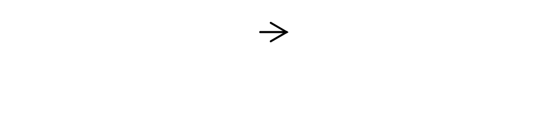
13. [12] Find an expression for the total potential $V(\mathbf{r})$ by using the method of images (think of the point dipole as a physical dipole).

14. [12] Sketch the field lines

(i) for $\theta = 0$



(ii) for $\theta = \pi/2$.



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15. [12] Find an expression for the net force on the dipole valid for any θ.

16. [14] Find an expression for the torque valid for any θ. Show that the torque is zero at both $\theta = 0$ and $\theta = \pi/2$.