 32 present/38 registered. Name solutions

Electrodynamics, Physics 321
Winter 2006

First midterm
Instructor: David Cobden

11.30 am Monday 6 February

Do not turn this page until I say 'go' at 11.30. You have 50 minutes. Hand your exam to me before I leave the room at 12.25.

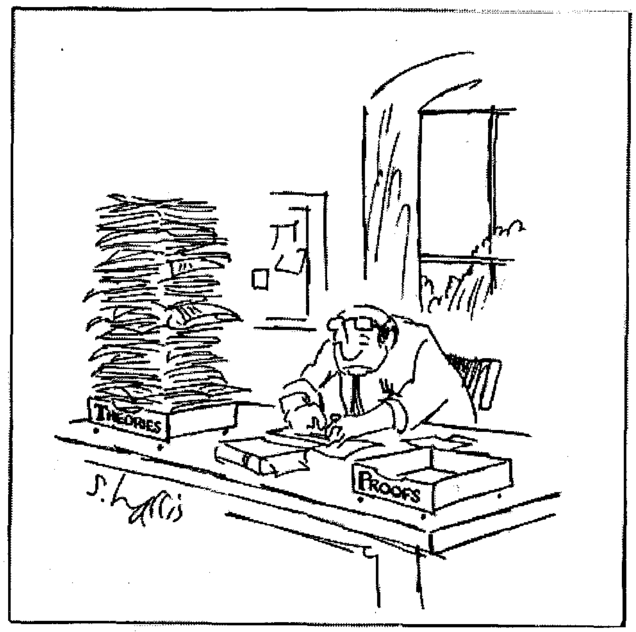
This exam contains 100 points. Be sure to attempt all ten questions.

Please write your name on every page and your SID on the first page.

Write all your working on these question sheets. Use this front page for extra working. It is important to show your calculation or derivation. You probably won't get full marks just for stating the correct answer if you don't show how you get it.

Watch the blackboard/overhead for corrections or clarifications during the exam.

This is a closed book exam. No books, notes or calculators allowed.



Name solutions

1. [8] Carefully state both integral and differential forms of the law relating electric field to charge density.

$$\vec{E}(\vec{r}) = \int \frac{\rho(\vec{r}\,')(\vec{r}-\vec{r}\,')}{4\pi\varepsilon_0 |\vec{r}-\vec{r}\,'|^3} d^3r' \qquad \vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0$$

2. [8] Carefully state both integral and differential forms of the law relating electric potential and charge density.

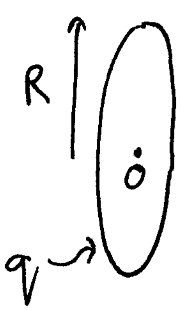
$$V(\vec{r}) = \int \frac{\rho(\vec{r}\,')}{4\pi\varepsilon_0 |\vec{r}-\vec{r}\,'|} d^3r' \qquad \nabla^2 V = -\frac{\rho}{\varepsilon_0}$$

3. [8] Show from the *definition* of the potential $V(\mathbf{r})$ in terms of $\mathbf{E}(\mathbf{r})$ that the electrostatic work done on a point charge q moving from $\mathbf{r}_1$ to $\mathbf{r}_2$ is $q[V(\mathbf{r}_1) - V(\mathbf{r}_2)]$.

$$W = \int_{r_1}^{r_2} \underbrace{\vec{F}}_{\text{e.s. force on particle}} \cdot d\vec{l} = \int_{r_1}^{r_2} q\vec{E}\cdot d\vec{l} = -q\int_{\vec{r}_1}^{\vec{r}_2} \vec{\nabla}V\cdot d\vec{l}$$

$$= q\left[V(\vec{r}_1) - V(\vec{r}_2)\right]$$

4. [14] A thin ring of radius R has a charge q spread uniformly around its circumference. A point particle of charge q and mass m is first held at rest very close to the center of the ring and then released. What is the speed of the particle when it gets far away from the ring?



Refer to q.3 : let $\vec{r}_1 = 0$, $\vec{r}_2 = $ "∞"

work done on particle $= q\left[V(\vec{r}_2) - V(\vec{r}_1)\right] = q[V(\infty) - V(0)]$

If $V(\infty) = 0$ then $V(0) = \frac{q}{4\pi\varepsilon_0 R}$

all charge on ring is distant R from origin

cons of energy: $\frac{1}{2}m\underbrace{v_f^2}_{\text{final speed}} = W = \frac{q^2}{4\pi\varepsilon_0 R}$

$$\therefore v_f = \left(\frac{2}{m}\frac{q^2}{4\pi\varepsilon_0 R}\right)^{1/2} = \frac{q}{\sqrt{2\pi\varepsilon_0 m R}}$$

 Name solutions

5. [10] A voltage V_0 is applied across a vacuum parallel plate capacitor of area A and spacing D. Show that the total energy U stored in the electric field inside capacitor is equal to the energy delivered by a battery in charging it up to V_0.

$$E = \frac{V_0}{D} \qquad U = \int \tfrac{1}{2}\varepsilon_0 E^2 d^3r = \tfrac{1}{2}\varepsilon_0 \left(\frac{V_0}{D}\right)^2 . A . D = \tfrac{1}{2}\varepsilon_0 V_0^2 \frac{A}{D}$$

work done by battery $= \tfrac{1}{2}CV^2 = \tfrac{1}{2}\left(\frac{\varepsilon_0 A}{D}\right)V_0^2 =$ same.

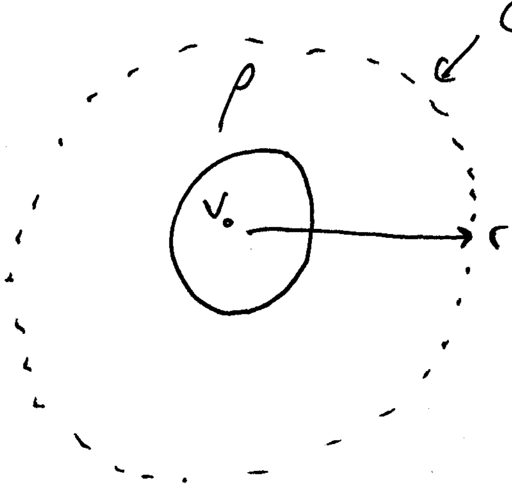
$$\left(W = \int V dQ = \int V C dV = \tfrac{1}{2}CV^2 \right)$$

6. [6] What is the electrostatic force between the plates, including its direction? Write it in terms of U.

e.s. pressure (outwards from conductor) on inner surface $p = \tfrac{1}{2}\sigma E = \tfrac{1}{2}\varepsilon_0 E^2 = \tfrac{1}{2}\varepsilon_0\left(\frac{V_0}{D}\right)^2$

$$\text{Force} = p.A = \tfrac{1}{2}\varepsilon_0 \frac{V_0^2}{D^2} A = \frac{U}{D}$$

7. [18] An infinite cylindrical conducting wire of radius R held at potential V_0 is surrounded by a uniform fixed positive charge density ρ. Find $V(r)$, where r is the distance from the axis, in the vicinity of the wire.



cylindrical Gaussian surface, length L radius r

Apply Gauss's law over this surface:

$$2\pi r L . E = \frac{Q_{enc}}{\varepsilon_0} = \frac{\pi(r^2 - R^2)L\rho}{\varepsilon_0}$$

$$\therefore E(\vec{r}) = \left(r - \frac{R^2}{r}\right)\frac{\rho}{2\varepsilon_0}$$

$$\therefore V(r) = V_0 + \int_R^r (-E) dr'$$

$$= V_0 - \frac{\rho}{2\varepsilon_0}\int_R^r \left(r' - \frac{R^2}{r'}\right) dr'$$

$$= V_0 - \frac{\rho}{2\varepsilon_0}\left[\frac{r'^2}{2} - R^2 \ln r'\right]_R^r$$

$$= V_0 - \frac{\rho}{2\varepsilon_0}\left[\frac{r^2 - R^2}{2} - R^2 \ln\frac{r}{R}\right]$$

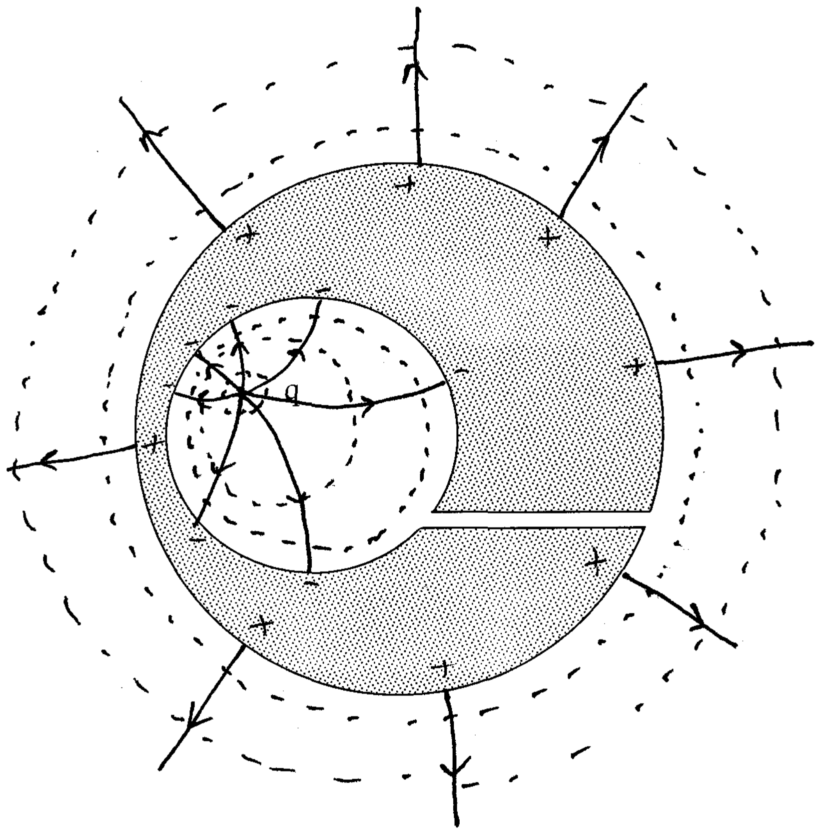
Do you see something wrong with the question?!

b.c. at $r \to \infty$ is not given.

Solution diverges!

 Name solutions

8. [12] A neutral spherical conductor of radius R contains a spherical off-center cavity, as indicated in the cross section drawn below. A thin tunnel ('wormhole') connects the cavity to the outside, as indicated. A positive point charge q is fixed inside the cavity, as indicated, also off-center. Sketch electric field lines and equipotentials everywhere, inside and out. Also, indicate with +'s and –'s where charge is induced in the conductor.



9. [6] In this two-dimensional drawing, is it possible to arrange for the inverse spacing of the field lines to be proportional to the electric field strength? Why (not)?

No. Field lines spread out in 3D, spacing $\sim \frac{1}{r^2}$ but we only draw them in 2D, $\sim \frac{1}{r}$

10. [10] The point charge is now moved out of the cavity through the tunnel and placed far away. What is the change in potential of the conductor during this process?

Potential of conductor is same as for total charge placed at center of sphere.

Before, potential $= \frac{q}{4\pi\varepsilon_0 R}$ after, potential $= 0$ (assuming $V=0$ at ∞)

$\therefore$ change $\overset{\Delta V}{=} \frac{-q}{4\pi\varepsilon_0 R}$