Electrodynamics, Physics 322
Spring 2006

First midterm
Instructor: David Cobden

Do not turn this page until I say 'go' at 11.30. You have 50 minutes. Hand your exam to me before I leave the room at 12.25 .

This exam contains 100 points. Be sure to attempt all the questions.
Please write your name on every page and your SID on the first page.
Write all your working on these question sheets. Use this front page for extra working. It is important to show your calculation or derivation. You probably won't get full marks just for stating the correct answer if you don't show how you get it.

Watch the blackboard/overhead for corrections or clarifications during the exam.
This is a closed book exam. No books, notes or calculators allowed.

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\mathbf{A}_{\text {dipole }}(\mathbf{x})=\frac{\mu_{0}}{4 \pi} \frac{\mathbf{m} \times \hat{\mathbf{r}}}{r^{2}}
$$


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1. [8] Write down the equation of motion for a particle of mass $m$ and charge $q$ moving in a magnetic field $\mathbf{B}$ and electric field $\mathbf{E}$.
2. [6] A current $I$ is passed along a strip of conducting material with rectangular cross-section having width $W$ and thickness $T$ as indicated in the diagram. If the density of mobile electrons (number per unit volume) in it is $n$, and the charge of an electron is $q$, what is the average electron velocity $\mathbf{v}$ ?

3. [10] A magnetic field $\mathbf{B}=B \hat{\mathbf{z}}$ is applied perpendicular to the strip. An electric field $\mathbf{E}_{\mathbf{H}}$ must then be present in the material (the Hall effect) when the current is steady. Find $\mathbf{E}_{\mathbf{H}}$ and sketch its direction on the diagram. (Neglect the electric field associated with the resistivity.)
4. [8] Hence show that the associated voltage $V_{\mathrm{H}}$ between the edges of the strip is independent of the strip width $W$, and that it reveals the sign of the electron charge.
5. [4] What is the actual source of the Hall electric field?
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6. [6] What is the magnetic dipole moment $\mathbf{m}$ of a circular current loop of radius $r$ centered on the $z$ axis and carrying current $I$ ?
7. [16] A circular disk of radius $R$ has a charge $Q$ distributed uniformly over its upper surface. It rotates at angular velocity $\omega$ about its axis (the z-axis). Find the vector potential A far from the disk, in all directions, to lowest order in $1 / r$.
8. [4] State the condition for the Coulomb gauge.
9. [6] State the integral equation giving the vector potential $\mathbf{A}$ in terms of the current density $\mathbf{J}$ (hint: analogous to the relationship between $V$ and $\rho$ ).
10. [8] Using the relationship in question 9, find the vector potential $\mathbf{A}$ for the disk of question 7 along the axis only. Show that it agrees with the answer to question 7. (more space on next page.)
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A long cylindrical bar of radius $R$, whose axis is the $z$-axis, is made of a material with magnetic susceptibility $\chi$. An insulated wire carrying a current $I$ is coiled around it tightly and uniformly with a turn density $n=N / L \gg 1 / R$. Neglect end effects and neglect contributions from the component of the current parallel to the axis.
11. [6] What are $\mathbf{H}, \mathbf{B}$ and $\mathbf{M}$ inside the bar?
12. [6] What are $\mathbf{H}, \mathbf{B}$ and $\mathbf{M}$ outside the bar?
13. [6] What and where are the bound surface and volume currents present?
14. [6] What is the vector potential A outside the bar, in the Coulomb gauge?
15. [6 extra] Which, if any, of the answers to questions 11-14 above change(s) if the cross-section of the bar is not circular?

