Name Solutions 26 present

Electrodynamics, Physics 322 **First midterm** **11.30 am, 3 May 2006**
Spring 2006 Instructor: David Cobden

Do not turn this page until I say 'go' at 11.30. You have 50 minutes. Hand your exam to me before I leave the room at 12.25.

This exam contains 100 points. Be sure to attempt all the questions.

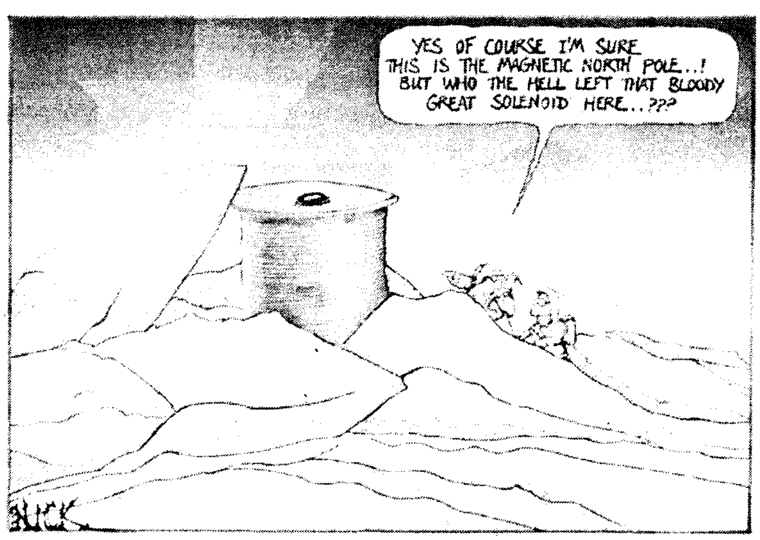
Please write your name on every page and your SID on the first page.

Write all your working on these question sheets. Use this front page for extra working. It is important to show your calculation or derivation. You probably won't get full marks just for stating the correct answer if you don't show how you get it.

Watch the blackboard/overhead for corrections or clarifications during the exam.

This is a closed book exam. No books, notes or calculators allowed.

$$\mathbf{A}_{dipole}(\mathbf{x}) = \frac{\mu_0}{4\pi}\frac{\mathbf{m}\times\hat{\mathbf{r}}}{r^2}$$

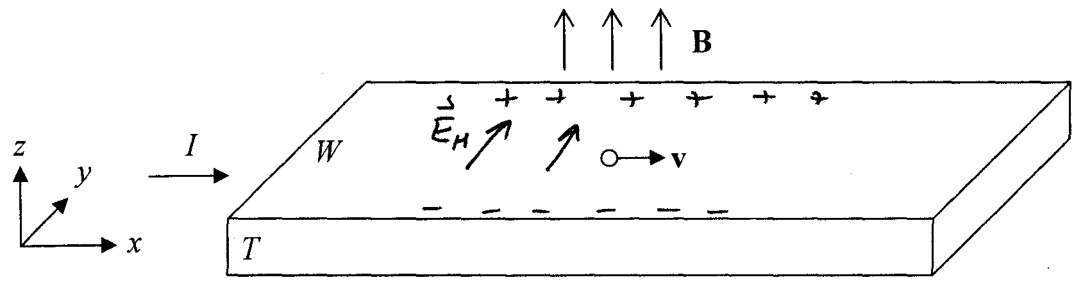


 Name solutions

1. [8] Write down the equation of motion for a particle of mass m and charge q moving in a magnetic field **B** and electric field **E**.

$$m\frac{d\vec{v}}{dt} = q\left(\vec{E} + \vec{v}\times\vec{B}\right)$$

2. [6] A current I is passed along a strip of conducting material with rectangular cross-section having width W and thickness T as indicated in the diagram. If the density of mobile electrons (number per unit volume) in it is n, and the charge of an electron is q, what is the average electron velocity **v**?



$$I = JA = \rho v A = qnvWT \quad \therefore \vec{v} = \frac{I}{nqWT}\hat{x}$$

3. [10] A magnetic field $\mathbf{B} = B\hat{\mathbf{z}}$ is applied perpendicular to the strip. An electric field $\mathbf{E_H}$ must then be present in the material (the Hall effect) when the current is steady. Find $\mathbf{E_H}$ and sketch its direction on the diagram. (Neglect the electric field associated with the resistivity.)

Steady current → average net force on each electron must be zero.

$$\therefore \vec{E}_H + \vec{v}\times\vec{B} = 0 \quad \therefore \vec{E}_H = -\vec{v}\times\vec{B} = -\frac{I}{nqWT}\hat{x}\times B\hat{z}$$

$$= +\frac{IB}{nqWT}\hat{y}$$

4. [8] Hence show that the associated voltage V_H between the edges of the strip is independent of the strip width W, and that it reveals the sign of the electron charge.

$$V_H = E_H W = \frac{IB}{nqT} \quad \therefore \text{indep. of } W$$

it q changes sign, so does V_H.

5. [4] What is the actual source of the Hall electric field?

Charge builds up on either side of the strip, as indicated (here we assume q is negative).

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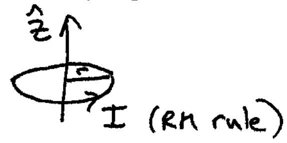
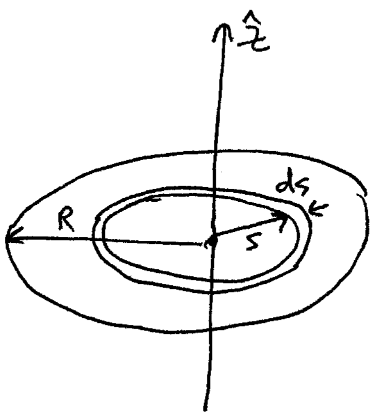
6. [6] What is the magnetic dipole moment **m** of a circular current loop of radius r centered on the z-axis and carrying current I ?



$$\vec{m} = I\int d\vec{S} = \pi r^2 I\,\hat{z}$$

7. [16] A circular disk of radius R has a charge Q distributed uniformly over its upper surface. It rotates at angular velocity ω about its axis (the z-axis). Find the vector potential **A** far from the disk, in all directions, to lowest order in $1/r$.



$$\vec{m} = \int d\vec{m} = \int dI.\pi s^2\,\hat{z}$$

dI = current in ring of thickness ds

$$dI = K\,ds = \sigma v\,ds = \frac{Q}{\pi R^2}.\omega s.ds$$

$$\therefore \vec{m} = \int_0^R \frac{Q}{\pi R^2}.\omega s.\pi s^2\,ds\;\hat{z}$$

$$= \frac{\omega Q}{R^2}\int_0^R s^3\,ds\;\hat{z} = \frac{\omega Q R^2}{4}\,\hat{z}$$

from cover, for large r, $\vec{A} = \vec{A}_{dipole} = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\hat{r}}{r^2}$

$$= \frac{\mu_0}{4\pi}\frac{\omega Q R^2}{4}\frac{\hat{z}\times\hat{r}}{r^2} = \frac{\mu_0\omega Q R^2}{16\pi r^2}\,\hat{\phi}$$

8. [4] State the condition for the Coulomb gauge. $\vec{\nabla}.\vec{A} = 0$

9. [6] State the integral equation giving the vector potential **A** in terms of the current density **J** (hint: analogous to the relationship between V and ρ). $\vec{A}(\vec{r}) = \int \frac{\mu_0}{4\pi}\frac{\vec{J}(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}\,d^3r'$

10. [8] Using the relationship in question 9, find the vector potential **A** for the disk of question 7 *along the axis only*. Show that it agrees with the answer to question 7. (more space on next page.)

For every current element $\vec{J}(\vec{r}\,')$ on one side there's an equal and opposite current element the same distance away on the other side, $\vec{J}(-\vec{r}\,') = -\vec{J}(\vec{r}\,')$, and their contributions cancel. $\therefore$ Total $\vec{A} = 0$, along z-axis.

This agrees with $\hat{z}\times\hat{r} = 0$ in Q7 when $\hat{z} = \hat{r}$.

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A long cylindrical bar of radius R, whose axis is the z-axis, is made of a material with magnetic susceptibility χ. An insulated wire carrying a current I is coiled around it tightly and uniformly with a turn density $n = N/L >> 1/R$. Neglect end effects and neglect contributions from the component of the current parallel to the axis.

11. [6] What are **H**, **B** and **M** inside the bar?

$\vec{H} = nI\hat{z}$ everywhere (from $\vec{\nabla}\times\vec{H} = \vec{J}_{free}$)

$\vec{B} = \mu_r\mu_0\vec{H} = (1+\chi)\mu_0 nI\hat{z}$

$\vec{M} = \chi\vec{H} = \chi nI\hat{z}$

12. [6] What are **H**, **B** and **M** outside the bar?

$\vec{H} = 0$

$\vec{B} = \mu_0\vec{H} = 0$

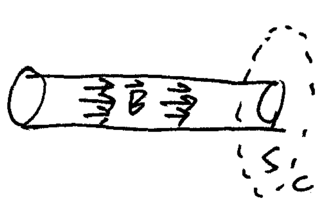
$\vec{M} = 0$ (nothing there!)

13. [6] What and where are the bound surface and volume currents present?

$\vec{J}_b = \vec{\nabla}\times\vec{M} = 0$ inside and outside

$\vec{K}_b = \vec{M}\times\hat{n} = (\chi nI\hat{z})\times\hat{r} = \chi nI\hat{\phi}$

14. [6] What is the vector potential **A** outside the bar, in the Coulomb gauge?



$\vec{\nabla}\times\vec{A} = \vec{B} \rightarrow \oint_C \vec{A}\cdot d\vec{l} = \int_S \vec{B}\cdot d\vec{S}$

Symmetry $\rightarrow$ $\vec{A} = A_\phi(r)\hat{\phi}$

$\therefore 2\pi r A_\phi(r) = \pi R^2 \underbrace{(1+\chi)\mu_0 nI}_{B \text{ inside}}$ $\therefore \vec{A} = \hat{\phi}\dfrac{R^2(1+\chi)\mu_0 nI}{2r}$

15. [6 extra] Which, if any, of the answers to questions 11-14 above change(s) if the cross-section of the bar is not circular?

Only $\vec{A}$ outside changes (can't use symmetry to say $\vec{A} = A_\phi(r)\hat{\phi}$ now).