Name Solutions

Electrodynamics, Physics 322
Spring 2006

Second midterm
Instructor: David Cobden

11.30 am, 22 May 2006

26 present.

Do not turn this page until the buzzer goes at 11.30. You have up to 55 minutes: hand your exam to me before I leave the room at 12.25.

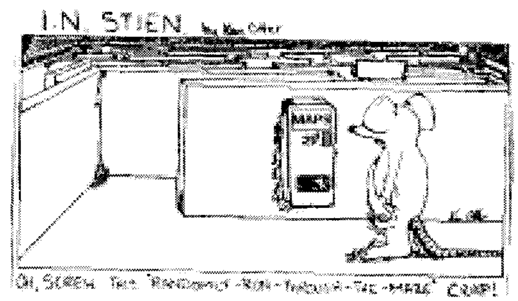
This exam contains 100 points. Be sure to attempt all the questions.

Please write your name on every page and your SID on the first page.

Write all your working on these question sheets. Use this front page for extra working. It is important to show your calculation or derivation. Some of the marks are given for showing clear and accurate working and reasoning.

Watch the blackboard for corrections or clarifications during the exam.

This is a closed book exam. No books, notes or calculators allowed.



Oh, screw this randomly-run-through-the-maze crap

 Name solutions

1. [10] State the four equations specifying the divergence and curl of **E** and **B** in *statics*, (ie, where ρ and **J** are independent of time).

(i) $\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0$ (iii) $\vec{\nabla}\times\vec{E} = 0$

(ii) $\vec{\nabla}\cdot\vec{B} = 0$ (iv) $\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$

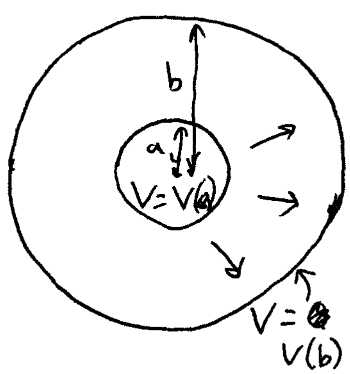
2. [10] State the modifications that must be made to these equations to arrive at Maxwell's four equations in vacuum. Indicate the origin of one of these modifications.

(iii) $\rightarrow \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$ ← comes from Faraday's law of induction which derives from Lorentz force law.

(iv) $\rightarrow \vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$ ← needed to make the equations consistent with conservation of charge.

3. [15] A thin disk of material with moderate electrical resistivity ρ has radius b and thickness $t \ll b$. A circular metal pad of radius a is connected at its center to make one electrical contact, and a metal strip is attached all around the rim to make another. Find the resistance between the inner and outer contacts. (Hint: use charge conservation and symmetry to get the current density $J(r)$ for a given total current I.)

$\vec{J}$ is radial



$$I = J(r)\cdot 2\pi r\cdot t$$

$$\therefore J(r) = \frac{I}{2\pi r t}$$

$$\therefore E(r) = \rho J(r) = \frac{\rho I}{2\pi r t}$$

$$\therefore V(a) - V(b) = \int_b^a \left(\frac{-\rho I}{2\pi r t}\right) dr = \frac{\rho I}{2\pi t}\ln\frac{b}{a}$$

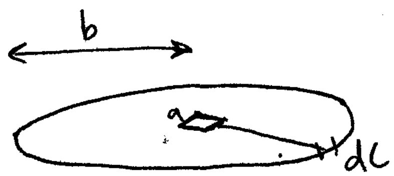
$$= \int(-E)\,dr$$

$$\therefore R = \frac{V(a)-V(b)}{I} = \frac{\rho}{2\pi t}\ln\frac{b}{a}$$

 Name solutions

4. [20] A small square wire loop of side *a* and a large circular wire loop of radius $b >> a$ are coplanar and concentric. Using the properties of mutual inductance, find (approximately) the emf induced in the circular loop when a current $I(t)$ flows around the square loop, assuming the resistance of the circular loop is vey large.

$$M = M_{ba} = M_{ab} = \frac{\Phi_{ab}}{I_b}$$ ← flux through a due to current I_b around b.



$$\Phi_{ab} \simeq (B \text{ at center of loop } b) \times a^2 = B_a a^2$$

Find B_a from Biot-Savart: $$B_a = \oint \frac{\mu_0 I_b dl}{b^2} = \frac{2\pi \mu_0 I_b}{b}$$

$$\therefore M \simeq \frac{B_a a^2}{I_b} = \frac{2\pi\mu_0 a^2}{b}.$$

so $$\mathcal{E}_b = M\frac{dI_a}{dt} \simeq \frac{2\pi\mu_0 a^2}{b}\frac{dI}{dt}$$

5. [5] If you don't assume the resistance of the circular loop is large, is the emf in it smaller or larger, and why?

Smaller. Current flow in this loop induces back-emf due to its self inductance which reduces the total emf (and so reduces the current).

6. [15] Find approximately the electromagnetic energy stored in a solenoid with length *l*, cross-section *A*, and *N* turns, by any means.

Easiest: $$U = \int u_{mag}\, d^3r \simeq \int_{\text{all space}} \frac{1}{2\mu_0} B^2 d^3r$$

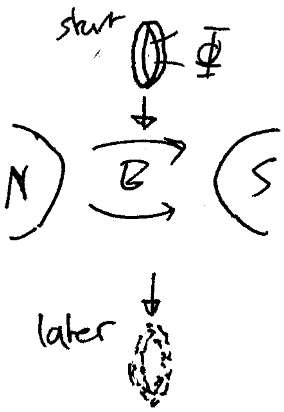
$$\simeq \frac{1}{2\mu_0}\cdot\left(\mu_0 \frac{N}{l} I\right)^2 \cdot (Al)$$

↑ B inside ↑ volume inside (assume $B \simeq 0$ outside)

$$= \frac{\mu_0 N^2 A I^2}{2l}$$

 Name solutions

7. [15] A small gold ring of radius a and resistance R is released from rest with its axis horizontal. As it drops it passes between the poles of a magnet. In the center of the magnet the field reaches a maximum value of B_0. What is the total charge that flows around the ring as it falls past the magnet?



Emf in ring is $\mathcal{E} = -\frac{d\Phi}{dt}$

→ current around ring is $I = \frac{\mathcal{E}}{R} = -\frac{1}{R}\frac{d\Phi}{dt}$

Charge flowed around ring is $Q = \int I\,dt$

$= \int -\frac{1}{R}\frac{d\Phi}{dt}\,dt = -\frac{1}{R}\Delta\Phi$

where $\Delta\Phi$ = ~~difference~~ change in flux during entire process $= 0$

$\therefore Q = 0$.

8. [5] The ring is slowed by its interaction with the magnet. Where does the lost kinetic energy go? (at least three named concepts are involved.)

Lenz's law → (eddy) currents in gold generate force which opposes motion (ie, change of B seen by ring)
→ Joule heating in the gold.

9. [5] What could you do, in principle, to the ring to reduce the amount by which it is slowed down, and why?

Cut a slit in it, so current can't flow around.
(emf will then produce voltage across the slit which does no work)
→ no Joule heating.

10. [extra points] Would a solid disk of gold be slowed more than, less than, or by the same amount as the ring, and why? The answer is not obvious. Specify the relevant factors.

Assume B is uniform over ring → current flow is circular, $\vec{J} = J\hat{\phi}$

→ behavior is as for a set of rings of different diameters superimposed.

Heating in ring $= IV = \frac{V^2}{R_{ring}} \sim \frac{\left(\frac{d\Phi}{dt}\right)^2}{2\pi r \cdot \rho} \propto \frac{(r^2)^2}{r} \propto r^3$

mass of ring $\propto r$ $\quad\therefore$ reduction in $(\text{velocity})^2 \propto \frac{\text{heating}}{\text{mass}} \propto r^2$

conclusion: inner rings are "slowed down less", ie disk slows less.