 Name ________________________________

**Electromagnetism, Physics 322** **First midterm** **8.20 am, January 30, 2004**
**Winter 2004** Instructor: David Cobden

You have 60 minutes. End on the buzzer. Answer all 13 questions.

Write your name on every page and your ID on the first page.

Write all your working on these question sheets. Use this cover page for extra working (you might get credit for it.)

It is important to show your calculation or derivation. You won't get full marks just for stating the correct answer if you don't show how you get it.

Watch the blackboard for corrections or clarifications during the exam.

This is a **closed book** exam. **No notes allowed. No calculators.**

Standard notation for spherical coordinates is used throughout. Thus, for example, $r$ is always the distance from the origin. We also use $r$ for the distance from the exis in cylindrical coordinates.

**Do not turn this page until I say 'go'!**

We write Cartesian unit vectors according to the form $\mathbf{x} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$ .

In cylindrical coordinates, $\mathbf{x} = (r, \phi, z)$, $\hat{\mathbf{r}} \times \hat{\boldsymbol{\varphi}} = \hat{\mathbf{k}}$ ,

and
$$\nabla \times \mathbf{A} = \left[\frac{1}{r}\frac{\partial A_z}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z}\right]\hat{\mathbf{r}} + \left[\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right]\hat{\boldsymbol{\varphi}} + \frac{1}{r}\left[\frac{\partial (rA_\phi)}{\partial r} - \frac{\partial A_r}{\partial \phi}\right]\hat{\mathbf{z}} \ .$$

The field of a dipole $\mathbf{m}$ is given by $\mathbf{B} = \dfrac{\mu_0}{4\pi r^3}[3(\mathbf{m}.\hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{m}]$ .

 Name ________________________________

**Part I.** A solid, nonmagnetic, cylindrical wire of radius $R$ centered on the $z$-axis carries a current density $\mathbf{J}(\mathbf{x}) = \alpha r^2 \hat{\mathbf{k}}$, where $r$ is distance from the axis.

1. [6] State Ampere's law in differential form and derive from it the integral form relating **B** and $I_{enc.}$

2. [8] Find **B** outside the wire.

3. [8] Find **B** inside the wire.

4. [4] Find **H** inside the wire.

 Name ________________________________

5. [10] Find **A** outside the wire, in the Coulomb gauge. Take **A** to vanish at the surface of the wire.

6. [8] At time $t = 0$ a particle with charge $+q$ has a velocity coplanar with the wire, $\mathbf{v}(0) = v_r(0)\,\hat{\mathbf{r}} + v_z(0)\,\hat{\mathbf{k}}$. Subsequently the particle *comes close to the wire but never collides with it.* What can you say about its velocity $\mathbf{v}(t)$ at all later times?

7. [6] Sketch below the motion of the particle if $v_z(0) > 0$ :

**Part II.** A rectangular bar magnet has a square cross-section of side $W$ in the $x$-$y$ plane, a length $L >> W$ in the $z$-direction, and uniform magnetization $\mathbf{M} = M\hat{\mathbf{k}}$ within it.

8. [10] Show that the magnetic field $\mathbf{B}(\mathbf{x})$ produced by the magnet is identical to that produced by a rectangular solenoid.

9. [6] Hence find $\mathbf{B}(\mathbf{x})$ inside the magnet far from the ends.

10. [6] Give an expression, involving a surface integral, for the magnetic dipole moment of a current loop which carries current $I$ along a closed curve C in three-dimensional space.

11. [10] By using this expression applied the bound current density, find the total magnetic dipole moment $\mathbf{m}$ of the bar magnet. Show also that the same value of $\mathbf{m}$ can be obtained in a more direct way.

12. [8] The bar magnet is free to move, but is initially stationary. A uniform magnetic field perpendicular to its axis, $\mathbf{B} = B\hat{\mathbf{i}}$ is then switched on. What happens?

13. [10] The bar magnet is sawn into two equal parts each of length $L/2$, which stick end-to-end. *Estimate roughly* the force of attraction between the two parts. (Assume no externally applied field.)