Name ________________________________

Electromagnetism, Physics 322 **Winter 2004**

Second midterm
Instructor: David Cobden

8.20 am, February 25, 2004

You have 60 minutes. End on the buzzer. Answer all **11** questions.

Write your name on every page and your student ID on the first page.

Write all your working on these question sheets. Use this cover page for extra working (you might get credit for it.)

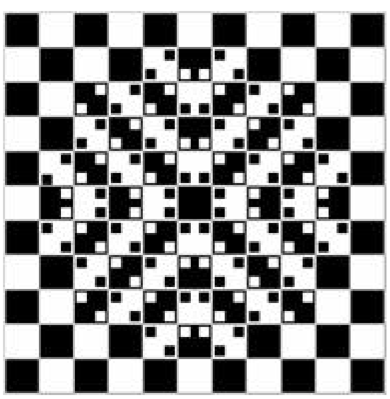
It is important to show your calculation or derivation. You won't get full marks just for stating the correct answer if you don't show how you get it.

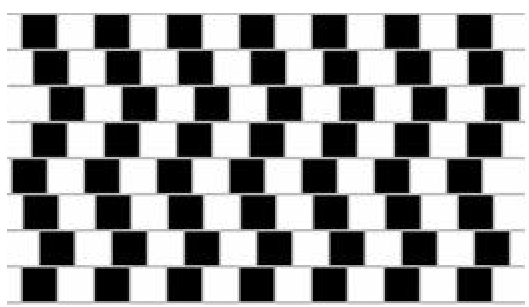
Watch the blackboard for corrections or clarifications during the exam.

This is a **closed book** exam. **No notes. No calculators.**

Do not turn this page until Ruth says 'go'!

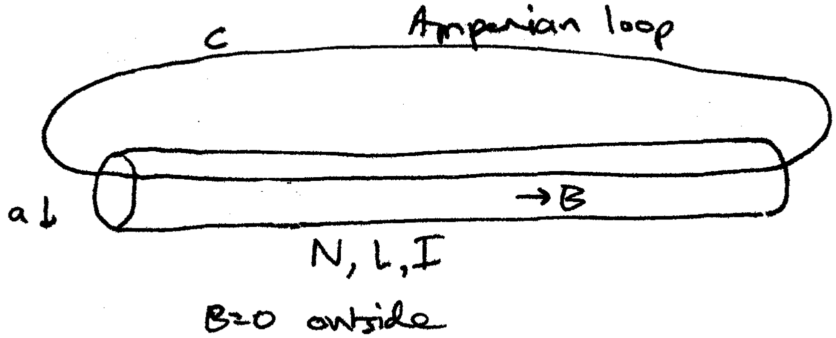
All the lines below are perfectly straight!





Name solutions

1. [8] Consider a solenoid of N turns, length l, and radius $a << l$, carrying a current I. Assuming the magnetic field is zero outside, outline an argument involving a single Amperian loop to show that the field inside is $B = \mu_0 NI/l$.



$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 N I$$

$$= B.L + 0$$

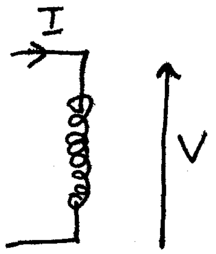
$$\therefore B = \frac{\mu_0 N I}{L}$$

2. [8] Find the energy stored in the solenoid using the magnetic energy density.

$$U = \frac{1}{2\mu_0} \int B^2 \, d^3x = \frac{1}{2\mu_0} \left(\frac{\mu_0 N I}{L}\right)^2 . \pi a^2 L$$

$$= \frac{\mu_0 N^2 I^2 \pi a^2}{2L}$$

3. [8] Show that the energy stored in an object with self-inductance L is $^1/_2 LI^2$.



$$V = L\frac{dI}{dt}$$

$U = W$ (work done)

$$= \int_0^t V I \, dt = \int_0^I L I' dI'$$

$$= \frac{1}{2} L I^2 \quad \text{if } L \text{ is constant}$$

4. [6] Calculate L for the solenoid, by using the results of questions 2 and 3, or otherwise.

$$\frac{1}{2} L I^2 = \frac{\mu_0 N^2 \pi a^2 I^2}{2L}$$

$$\therefore L = \frac{\mu_0 N^2 \pi a^2}{L}$$

 Name solutions

5. [8] If a cube of diamagnetic ($\chi_m < 0$) material, such as wood, of side slightly smaller than a is placed near the end of the solenoid, is it attracted, repelled, or completely uninfluenced? Explain why.

Repelled.

$$U \sim -\vec{M}\cdot\vec{B}$$

$$= -\chi \vec{H}\cdot\vec{B} = -\chi \vec{H}\cdot\left[(1+\chi)\mu_0 \vec{H}\right]$$

$$\simeq -\chi\mu_0 H^2 \simeq -\frac{\chi}{\mu_0} B^2 \quad (|\chi| \ll 1 \text{ for diamagnet})$$

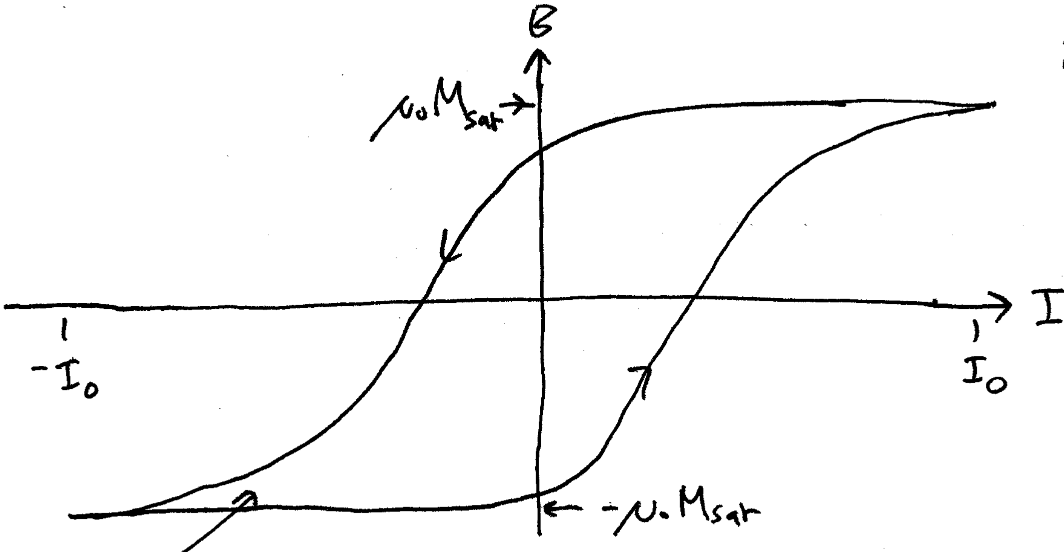
$\therefore$ since $\chi < 0$, U decreases as B decreases

$\rightarrow$ Force $(-\vec{\nabla} U)$ is towards region where B is smaller.

6. [4] If the same piece of material is placed inside the solenoid, does its inductance (a) increase by a large factor, (b) increase slightly, (c) remain the same, $\boxed{\text{(d) decrease slightly}}$ or (e) decrease by a large fraction?

$|\chi| \ll 1$ $\mu_r = 1+\chi$ so μ_r is just < 1; $L \propto \mu_r$

7. [10] A similar solenoid is filled instead with a hard ferromagnetic material (ie, one which has remanant magnetization). The current I is made to oscillate slowly with an amplitude I_0 which is sufficient to saturate the magnetisation at a value M_{sat}. Sketch a graph of the variation of B at the center of the solenoid with I over a cycle of the oscillation. Annotate your graph showing sweep direction and indicating in a few words what is going on. Show the scale (ie, magnitude) of B.



hysteresis loop.

note: area $\propto$ work dissipated as heat per cycle

H follows I.

M lags I due to domains being reluctant to reorient.

$B = \frac{1}{\mu_0} H + M$ $\quad M_{sat} \gg H$

$\therefore B_{sat} \simeq \mu_0 M_{sat}$

8. [10] State Faraday's law of induction in both integral form (relating emf and flux) and differential form, and show how the former leads to the latter.

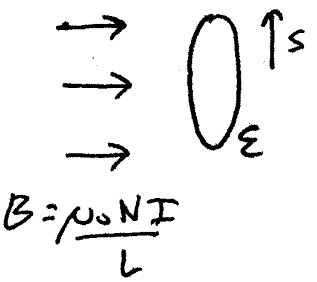
$$\mathcal{E} = -\frac{d\Phi}{dt} \quad \therefore \quad \mathcal{E} = \oint_C \vec{E}\cdot d\vec{l} = \int_S \vec{\nabla}\times\vec{E}\cdot d\vec{S} \text{ by Stokes}$$

$$\updownarrow$$

$$= -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{S} = \int_S -\frac{\partial\vec{B}}{\partial t}\cdot d\vec{S} \quad \text{for any } S \text{ bounded by } C$$

$$\therefore \quad \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

9. [12] Now consider the empty solenoid carrying current $I = I_0 \cos \omega t$. A thin circular wire loop of radius $s < a$ and resistance R is located in the center of the solenoid with its plane perpendicular to the axis. What current I_{loop} flows in the loop?

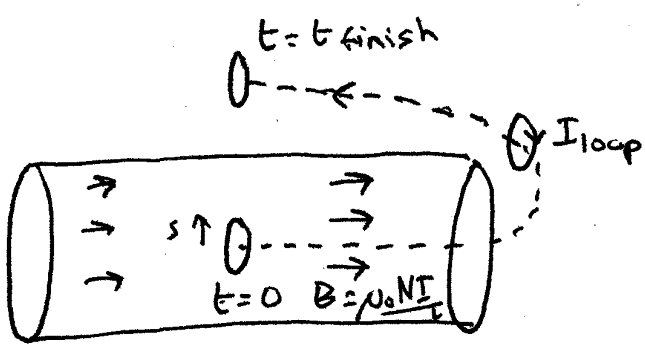


$$I_{loop} = \frac{\mathcal{E}}{R} = -\frac{1}{R}\frac{d\Phi}{dt} = -\frac{1}{R}\frac{d}{dt}\left(\pi s^2 \cdot \frac{\mu_0 N I}{L}\right)$$

$$= -\frac{\pi s^2 \mu_0 N}{RL}\frac{dI}{dt}$$

$$= \frac{\pi s^2 \mu_0 N \omega I_0 \sin\omega t}{RL}$$

10. [12] Now the solenoid current is set to a constant value of I_0. The loop is then slowly and carefully removed from the solenoid and placed next to it. Calculate the total charge that flows around the loop during this procedure.



$$I_{loop} = \frac{\mathcal{E}}{R} = \frac{1}{R}\frac{d\Phi}{dt}$$

$$Q_{loop} = \int_0^{t_{finish}} I_{loop}\, dt$$

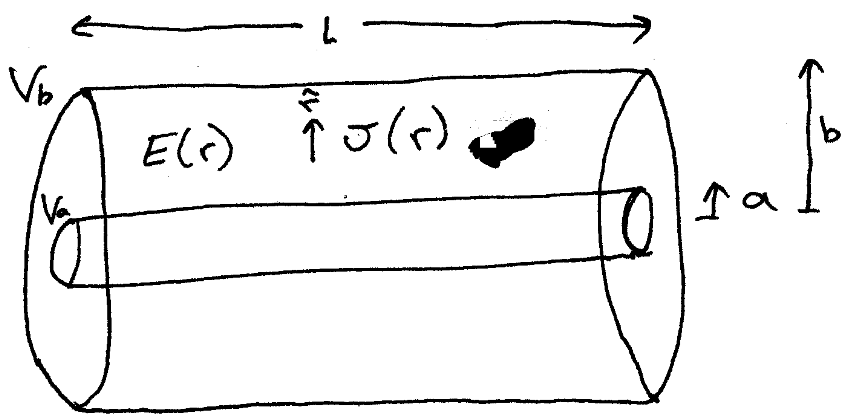
$$= -\frac{1}{R}\int \frac{d\Phi}{dt}\, dt$$

$$= -\frac{1}{R}\int_{\Phi_{start}}^{\Phi_{finish}} d\Phi = -\frac{1}{R}\left(\overset{0}{\Phi_{finish}} - \Phi_{start}\right)$$

$$= \frac{\Phi_{start}}{R} = \pi s^2 B = \frac{\pi s^2 \mu_0 N I}{RL}$$

 Name solutions

11. [14] A metal tube of radius a and length l is coaxial with another tube of radius $b > a$ having the same length. The space between the tubes is filled with a material of conductivity σ. Find the resistance R between the two cylinders.



By symmetry: $\vec{J} = J(r)\,\hat{r}$

Ohm's law: $\vec{J} = \sigma\vec{E} = \sigma E(r)\,\hat{r}$

Continuity (of charge) $\rightarrow$ current through and cylinder of radius r is the same. $= I$

$$I = 2\pi r L\, J(r) \qquad \therefore\ J(r) = \frac{I}{2\pi r L} = \sigma E(r)$$

$$V = V_a - V_b = -\int_b^a E(r)\,dr = \int_a^b \frac{J(r)}{\sigma}\,dr = \int_a^b \frac{I}{2\pi r L\sigma}\,dr$$

$$= \frac{I}{2\pi L\sigma}\ln\frac{b}{a}$$

$$\therefore\ R = \frac{V}{I} = \frac{\ln b/a}{2\pi L\sigma}$$