Name ______________solutions___________________

**Electrodynamics, Physics 322** **First midterm** **8.20 am, January 31, 2005**
**Winter 2005** Instructor: David Cobden

You have 60 minutes. End on the buzzer at 9.20. Answer all **12** questions.

Write your name on every page and your ID on the first page.

Write all your working on these question sheets. Use this cover page for extra working (you might get credit for it.)

It is important to show your calculation or derivation. You won't get full marks just for stating the correct answer if you don't show how you get it.

Watch the blackboard for corrections or clarifications during the exam.

This is a **closed book** exam. **No notes allowed. No calculators!**

Do not turn this page until the buzzer goes!

$$\mathbf{A}_{dipole}(\mathbf{x}) = \frac{\mu_0}{4\pi}\frac{\mathbf{m}\times\hat{\mathbf{r}}}{r^2}$$


from Nearingzero.com

 Name solutions

I. You're given a beam of monoenergetic charged particles of velocity $v_0$, whose path you can visualize by passing it through a low density gas which fluoresces.

1. [10] Show how you could find the charge/mass ratio $q/m$ of the particles by observing the shape of the beam when it passes through a uniform magnetic field $\mathbf{B} = B\hat{\mathbf{z}}$.

Lorentz: $m\frac{d\vec{v}}{dt} = q\vec{v}\times\vec{B}$

Launch beam perpendicular to $\vec{B}$, eg $\vec{v} = v_0\hat{x}$

→ trajectory is circle in x-y plane, radius $r_c$

$$\frac{mv_0^2}{r_c} = qv_0B \quad \rightarrow r_c = \frac{mv_0}{qB}$$

$$\therefore \frac{q}{m} = \frac{v_0}{r_cB}$$

← get from below (whoops! Q2 should have been first)

↑ measure with ruler

2. [10] Show how you could find $v_0$ by applying a uniform electric field $\mathbf{E}$ in addition to this magnetic field such that the beam becomes straight.

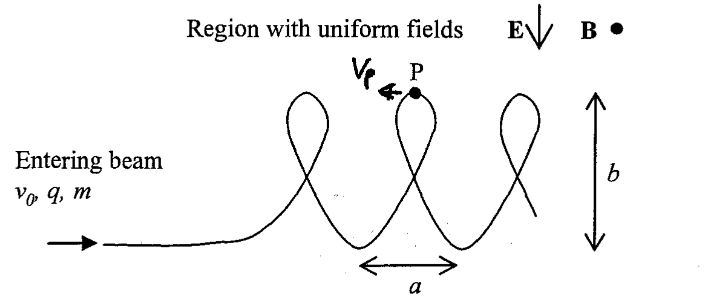
$$m\frac{d\vec{v}}{dt} = q(\vec{E} + \vec{v}\times\vec{B}) = 0 \quad \text{when} \quad \vec{E} = -\vec{v}\times\vec{B}$$

∴ if we apply $\vec{E}$ normal to $\vec{B}$, ie $\vec{E} = E\hat{y}$ for example,

then $E = v_0 B$ so $v_0 = \frac{E}{B}$

Name Solutions

3. [10] When the beam enters a particular combination of crossed electric and magnetic fields **E** and **B** the trajectory of the beam lies in a plane and is as sketched below. What is the velocity of the particles at point P? (Consider kinetic energy.)




$\vec{B}$ does not do work

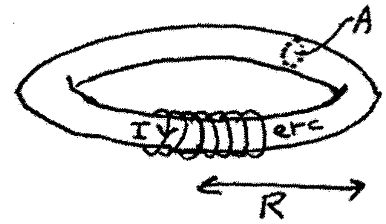
Cons. of energy $\rightarrow$ $\frac{1}{2} m v_0^2 = \frac{1}{2} m v_P^2 + qEb$

$v_P < v_0$ because curvature is smaller

$\therefore v_P^2 = v_0^2 - 2m^{-1} q E b$

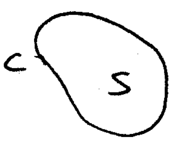
$\therefore v_P = \sqrt{v_0^2 - 2m^{-1} q E b}$

II. A toroidal solenoid is a wire coil which is curved into a circle of radius $R$ and a cross-sectional area $A$, as indicated. It has $N$ turns uniformly spaced around its perimeter and carries current $I$.




4. [8] Derive Ampere's law in integral form starting from Ampere's law in differential form.

$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$

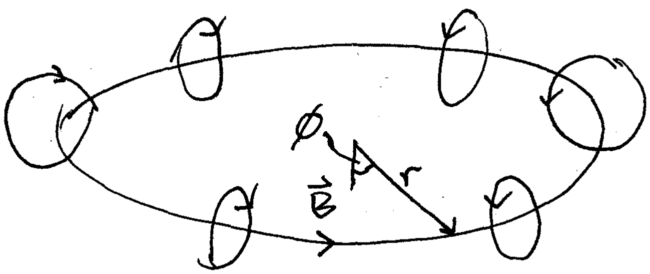



$\int_S \vec{\nabla} \times \vec{B} \cdot d\vec{S} = \oint \vec{B} \cdot d\vec{l}$

S bounded by C

$= \int_S \mu_0 \vec{J} \cdot d\vec{S} = \mu_0 I_{enc}$

5. [12] Use it in a single step to find the approximate magnetic field inside the coil. Do the same for the field outside. Make clear your reasoning and approximation(s) made.




Consider a circular loop threading along the coil inside with radius r, coaxial with torus.

$\vec{B} \simeq B_\phi(r)\hat{\phi}$ by symmetry + looks like a straight solenoid close-up.

$\oint \vec{B} \cdot d\vec{l} = 2\pi r B_\phi$

$= \mu_0 I_{enc}$

$= \mu_0 N I$ inside.

$\therefore B_\phi \simeq \frac{\mu_0 N I}{2\pi r} \simeq \frac{\mu_0 N I}{2\pi R}$

Outside, $I_{enc} = 0$ so $\vec{B} = 0$.

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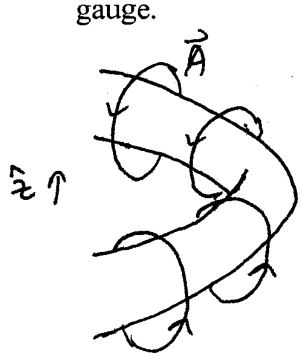
6. [3] Does the answer to the previous question depend on the cross-sectional shape of the coil?

No.

7. [4] How much magnetic flux is enclosed by one loop of the coil?

$$\Phi = \int \vec{B}\cdot d\vec{S} \doteq B_\phi A \doteq \frac{\mu_0 N I A}{2\pi R}$$

8. [6] Sketch below the form of the lines of the vector potential **A** outside the solenoid, in the Coulomb gauge.




$\vec{A}$ follows the current (roughly) and curls round the B-field, so

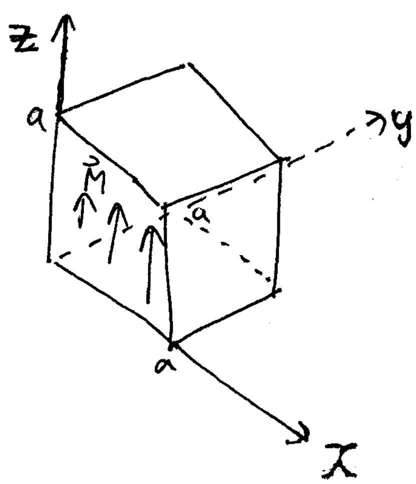
← Close to the surface it must do this.

[ $\oint \vec{A}\cdot d\vec{l} = \int \vec{B}\cdot d\vec{S} = \Phi \rightarrow A \sim \frac{1}{s}$ in this regime, where $s$ = distance from local axis of coil ]

Far from it, the toroid looks like a current loop, and eventually a dipole, $\vec{m} = I\pi R^2 \hat{z}$

III. A cube of side $a$ with one corner at the origin has a magnetization $\mathbf{M}(\mathbf{r}) = K_0(x/a)\,\hat{\mathbf{z}}$ which is perpendicular to the upper and lower faces ($z = 0$ and $z = a$) and increases linearly from zero at one side ($x = 0$) to a magnitude $K_0$ at the other ($x = a$).

9. [11] Find all the bound currents in the system.




$\vec{K}_b = \vec{M} \times \hat{n}$

$= 0$ at $x = 0$ where $M = 0$

and at $z = 0$ and $z = a$ where $\vec{M} \parallel \hat{n}$

At $x = a$, $\vec{K}_b = K_0 \hat{z} \times \hat{x} = K_0 \hat{y}$

At $y = 0$, $\vec{K}_b = K_0 \frac{x}{a} \hat{z} \times (-\hat{y}) = K_0 \frac{x}{a} \hat{x}$

At $y = a$, $\vec{K}_b = K_0 \frac{x}{a} \hat{z} \times \hat{y} = -K_0 \frac{x}{a} \hat{x}$

$$\vec{J}_b = \vec{\nabla} \times \vec{M} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & 0 & K_0 \frac{x}{a} \end{vmatrix} = -\hat{y}\frac{\partial}{\partial x}\left(\frac{K_0 x}{a}\right) = -\hat{y}\frac{K_0}{a}$$

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10. [10] Find the vector potential **A**(**r**) *far away* from the cube ($r >> a$) in the Coulomb gauge.

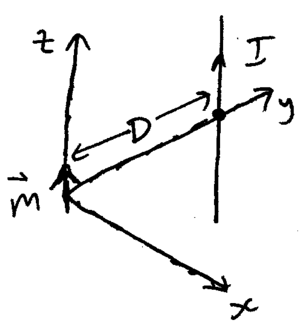
far away ($r >> a$), dipole term dominates so

$\vec{A} \doteq \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$ and we just need $\vec{m}$:

$$\vec{m} = \int \vec{M} \, d^3r = a^2 \int K_0 \frac{x}{a} \hat{z} \, dx = \frac{K_0 a^3}{2} \hat{z} = m\hat{z} \quad \text{(for below)}$$

$$\therefore \vec{A} = \frac{\mu_0 K_0 a^3}{8\pi} \frac{\hat{z} \times \hat{r}}{r^2} \quad \left( = \frac{\mu_0 K_0 a^3}{8\pi} \frac{\cos\theta}{r^2} \hat{\phi} \text{ in sph polars} \right)$$

11. [8] A current $I$ is passed along a long straight horizontal wire a distance $D >> a$ above the cube (parameterized by $y = D$, $x = 0$). Find the approximate torque on the cube.




$$\vec{N} = \vec{m} \times \vec{B} = m\hat{z} \times \frac{\mu_0 I}{2\pi D} \hat{x} = \frac{m \mu_0 I}{2\pi D} \hat{y}$$

$$= \frac{K_0 a^3 \mu_0 I}{4\pi D} \hat{y}$$

(give or take a sign depending on direction of $I$)

12. [8] Find the approximate net force exerted on the cube by this current.

$$\vec{F} = \vec{\nabla}(\vec{m} \cdot \vec{B}) = \vec{\nabla} \left[ (m\hat{z}) \cdot \frac{\mu_0 I}{2\pi y} \hat{x} \right] \Bigg|_{y=-D} = \vec{\nabla}(0) = 0 \, !$$