 Name ______solutions__

Electrodynamics, Physics 322 **Second midterm** **8.20 am, February 28, 2005**
Winter 2005 Instructor: David Cobden

You have 60 minutes. End on the buzzer at 9.20. Answer all 12 questions.

Write your name on every page and your ID on the first page.

Write all your working on these question sheets. Use this cover page for extra working (you might get credit for it.)

It is important to show your calculation or derivation. You won't get full marks just for stating the correct answer if you don't show how you get it.

Watch the blackboard for corrections or clarifications during the exam.

This is a **closed book** exam. **No notes allowed. No calculators!**

Do not turn this page until the buzzer goes!

"And the award for Best Newcomer goes to..."

Name solutions

I. A straight cylindrical wire of radius a and length $l >> a$ has magnetic susceptibility χ. When connected between the terminals of a battery with emf V_0 the current density within it is found to be $J(r) = Cr^2$, where r is the distance from the axis in cylindrical coordinates.

1. [10] What is the resistivity $\rho(r)$ of the wire?

$$E = \frac{V_0}{l} \quad \text{uniform over wire} \quad \left(\text{you can see this from Laplace eqn}\right)$$

$$\therefore \rho(r) = \frac{E}{J} = \frac{V_0/l}{Cr^2} = \frac{V_0}{lCr^2}$$

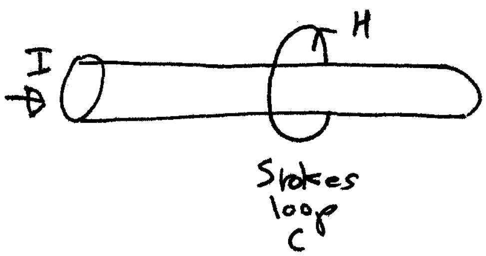
2. [10] What is total rate of heat dissipation in the wire?

$$\text{Power} = VI = V_0 \int_{\text{cross sec}} J\, ds = V_0 \int_0^a Cr^2 . 2\pi r\, dr$$

$$= \frac{\pi a^4 V_0 C}{2} \qquad I = \frac{\pi a^4 C}{2}$$

3. [10] Find $\mathbf{H}(r)$ and $\mathbf{B}(r)$ outside the wire.

By Biot-Savart and symmetry, $\vec{H} = H(r)\hat{\phi}$



$$\vec{\nabla} \times \vec{H} = \vec{J}_{free} \rightarrow \oint_C \vec{H} . d\vec{l} = I_{enc}$$

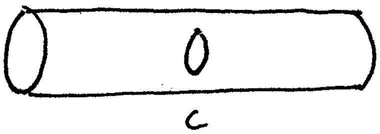
$$\therefore 2\pi r H = I \qquad \therefore \vec{H} = \frac{I\hat{\phi}}{2\pi r}$$

$$\vec{B} = \mu_0 \vec{H} = \frac{\mu_0 I \hat{\phi}}{2\pi r} \qquad (I \text{ as above})$$

(outside $\chi = 0$)

4. [10] Find $\mathbf{H}(r)$ and $\mathbf{B}(r)$ inside the wire.

Now use Stokes loop inside:



$$2\pi r H = I_{enc} = \int_0^r Cr'^2\, 2\pi r'\, dr'$$

$$\therefore H = \frac{1}{r} \int_0^r Cr'^3\, dr' = \frac{Cr^3}{4}$$

$$\therefore \vec{H} = \frac{Cr^3}{4}\hat{\phi}$$

$$\vec{B} = (1+\chi)\mu_0 \vec{H} = \frac{(1+\chi)\mu_0 C r^3 \hat{\phi}}{4}$$

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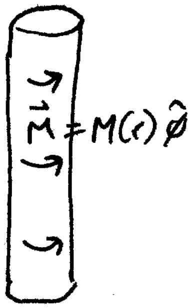
5. [10] Now consider instead a wire of the same size but made of ferromagnetic material having a permanent azimuthal magnetization $\mathbf{M}(r) = M(r)\hat{\varphi}$. Find $\mathbf{B}(r)$ outside the wire when it is not connected to the battery.



Symmetry $\rightarrow$ $\vec{H} = H(r)\hat{\phi}$ $\quad \vec{B} = \mu_r \mu_0 \vec{H}$

Outside, $2\pi r H(r) = I_{free} = 0$

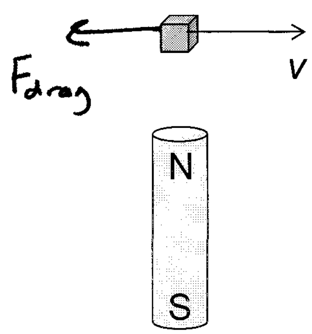
$\therefore \vec{H} = 0$

$\therefore \vec{B} = 0$ outside.

(Bound currents flow up on surface and back down in the bulk; total bound current is zero as it must be)

II. A metal cube with negligible susceptibility is thrown past a fixed bar magnet, as sketched below.

6. [10] What force acts on the cube, what is its direction, and what principles are involved?



Bar magnet produces nonuniform B-field

Motion of metal through this produces electric fields inside it given by $\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}$

This generates eddy currents which, by Lenz's law, generate Lorentz forces that oppose the changing field, ie, slow down the cube.

$\chi = 0 \Rightarrow \vec{M} = 0 \Rightarrow$ no magnetic force of the form $\vec{\nabla}(-\vec{M}\cdot\vec{B})$

Of course there is also gravity and air resistance — no marks for those.

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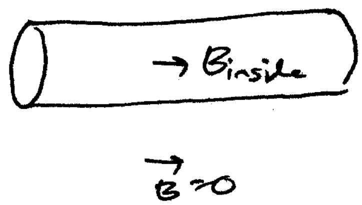
III. A solenoid of length l, radius $a << l$, N turns, and resistance R, initially carries a steady current I_0.

7. [10] Find the self-inductance L of the solenoid, starting from the definition of L.



$B_{inside} = \frac{\mu_0 N I}{l}$ flux linked $\Phi = \pi a^2 B_{inside} N$.

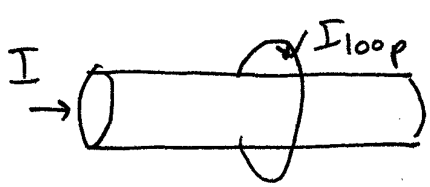
$$L = \frac{\Phi}{I} = \frac{\pi a^2 N \left(\frac{\mu_0 N I}{l}\right)}{I} = \frac{\mu_0 \pi a^2 N^2}{l}$$

8. [10] The solenoid is suddenly shorted end-to-end. If the current at the moment where it is shorted is I_0, how much heat is subsequently generated in the solenoid windings?

All stored energy must be dissipated as heat in the resistance of the wire.

$$\text{heat} = W = \frac{1}{2} L I_0^2 = \frac{1}{2} \frac{\mu_0 \pi a^2 N^2}{l} I_0^2$$

9. [10] A circular wire loop of radius $b > a$ and resistance R_{loop} is placed around the solenoid coaxial with it. The solenoid current is then ramped from zero to I_0. Find the total charge Q which flows around the loop during this process.



$$I_{loop} = \frac{\mathcal{E}_{loop}}{R_{loop}}$$

$$\mathcal{E}_{loop} = \frac{-d\Phi_{loop}}{dt} = \frac{-d}{dt}\left(\pi a^2 B_{inside}\right)$$

$$= -\pi a^2 \frac{d}{dt}\left(\frac{\mu_0 N I}{l}\right) = -\frac{\mu_0 N \pi a^2}{l} \frac{dI}{dt}$$

$$\therefore \; Q_{loop} = \int_{I=0}^{I_0} I_{loop}\, dt = -\frac{\mu_0 N \pi a^2}{l R_{loop}} \int_0^{I_0} dI$$

$$= (-1) \frac{\mu_0 N \pi a^2}{l R_{loop}} I_0$$

irrelevant — depends on definition

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10. [10] The solenoid current is now set back to zero, and a small battery with emf V_0 is incorporated into the loop. Using the properties of mutual inductance, find the resulting magnetic flux linked by the solenoid coil.

$$\Phi_{solenoid} = M I_{loop} = M \frac{V_0}{R_{loop}}$$

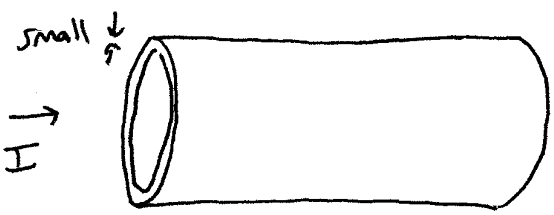
Get M from (Neumann) symmetry $M_{12} = M_{21}$

$$M = \frac{\Phi_{loop}}{\left(I_{solenoid} \text{ with } I_{loop}=0\right)} = \frac{\pi a^2 \mu_0 N I / L}{I} = \frac{\pi a^2 \mu_0 N}{L}$$

$$\therefore \quad \Phi_{solenoid} = \frac{\pi a^2 \mu_0 N V_0}{L R_{loop}}$$

IV. A long, straight, empty, thin-walled metal pipe of radius a carries a current I uniformly.

11. [10] Find the magnetic energy stored per unit length of the pipe.



Outside pipe $\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\phi}$ as usual.

Inside pipe, $\oint \vec{B} \cdot d\vec{l} = 0$ so $\vec{B} = 0$.

Energy stored $U = (\text{length}) \times \int \frac{B^2}{2\mu_0} d^3r$

$= (\text{length}) \times \int_a^\infty \left(\frac{\mu_0 I}{2\pi r}\right)^2 \frac{2\pi r \, dr}{2\mu_0}$

$$\therefore \frac{\text{Energy}}{\text{unit length}} = \frac{\mu_0 I^2}{4\pi} \int_a^\infty \frac{dr}{r} = /$$

12. [10] Hence find the self-inductance per unit length of the pi

energy stored $U = \frac{1}{2} L I^2$

$\therefore$ inductance/unit length $= \frac{L}{\text{len}}$

Sorry - there is a horrible mistake in this problem which some of you have noticed! The answer is infinite!