

Equations of Physics 322 - Magnetism

By the end of the course you should know all these equations, their interrelationships, and whether they are empirical or derivative. Other equations will be given or their derivation may be required.

grad, div, curl, and ∇^2 in *Cartesian coordinates only*.

relations between unit vectors in Cartesian, cylindrical polars, and spherical polars

Lorentz force law $\mathbf{F} = m\ddot{\mathbf{v}} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$ $d\mathbf{F} = I d\mathbf{l} \times \mathbf{B}$ $d\mathbf{F} = \mathbf{J} \times \mathbf{B} d^3 r$ derive $\omega_c = \frac{qB}{m}$

Biot-Savart: $d\mathbf{B}(\mathbf{r}) = \frac{\mu_0 I d\mathbf{l}' \times (\mathbf{r} - \mathbf{r}')}{4\pi |\mathbf{r} - \mathbf{r}'|^3}$ Continuity equation $\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}$

$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$ or $\oint_C \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{enc}$ Ampere's law $\nabla \times \mathbf{A} = \mathbf{B}$ $\mathbf{B} = \frac{\mu_0 NI}{l} \hat{\mathbf{k}}$ inside solenoid

$\nabla \cdot \mathbf{A} = 0$ $\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}$ $\mathbf{A}(\mathbf{r}) = \int \frac{\mu_0 \mathbf{J}(\mathbf{r}') d^3 r'}{4\pi |\mathbf{r} - \mathbf{r}'|}$ (Coulomb gauge only)

$\mathbf{m} = I \int_S d\mathbf{S} = I \mathbf{S}$ for current loop. Recognise $\mathbf{m} = \frac{1}{2} \int \mathbf{r} \times \mathbf{J}(\mathbf{r}) d^3 r$ and $\mathbf{A}_{dip}(\mathbf{r}) = \frac{\mu_0}{4\pi} \frac{\mathbf{m} \times \hat{\mathbf{r}}}{r^2}$

$U_m = -\mathbf{m} \cdot \mathbf{B}$ $\mathbf{N} = \mathbf{m} \times \mathbf{B}$ $\mathbf{F} = -\nabla U$ $\nabla \times \mathbf{M} = \mathbf{J}_b$ $\mathbf{M} \times \hat{\mathbf{n}} = \mathbf{K}_b$

$\mathbf{H} = \frac{1}{\mu_0} \mathbf{B} - \mathbf{M}$ $\nabla \times \mathbf{H} = \mathbf{J}_f$ $\mathbf{M} = \chi_m \mathbf{H}$ $\mathbf{B} = (1 + \chi_m) \mu_0 \mathbf{H} = \mu_r \mu_0 \mathbf{H} = \mu \mathbf{H}$

$I = GV = \frac{V}{R}$ $\mathbf{J} = \sigma \mathbf{E} = \frac{1}{\rho} \mathbf{E}$ $R = \frac{\rho l}{A}$ $G = \frac{1}{R} = \frac{\sigma A}{l}$ Joule heating $P = I^2 R$ $P/volume = \mathbf{J} \cdot \mathbf{E} = \sigma E^2$

$\epsilon = -\frac{d\Phi}{dt}$ $\Phi_C = \int_S \mathbf{B} \cdot d\mathbf{S} = \oint_C \mathbf{A} \cdot d\mathbf{l}$ $\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_C}{dt}$ $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ Faraday's law

$\Phi = LI$ $V = L \frac{dI}{dt}$ solenoid: derive $L = \frac{\mu_r \mu_0 N^2 A}{l}$

$U = \frac{1}{2} LI^2 = \frac{1}{2} \int \mathbf{A} \cdot \mathbf{J} d^3 r = \frac{1}{2\mu_0} \int_{\text{all space}} B^2 d^3 r$ $\Phi_2 = M_{21} I_1$ $V_2 = -M_{21} \frac{dI_1}{dt}$ $M_{12} = M_{21} = M$

Poynting: $\mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}$ $u_{em} = \frac{\epsilon_0 E^2}{2} + \frac{B^2}{2\mu_0}$ Momentum density $\frac{d\mathbf{P}_{em}}{dV} = \frac{\mathbf{S}}{c}$

Maxwell's equations (i) $\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$ (ii) $\nabla \cdot \mathbf{B} = 0$ (iii) $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ (iv) $\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$

Wave equation in vacuum: $\nabla^2 \mathbf{E} = \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2}$, $c^2 = \frac{1}{\mu_0 \epsilon_0}$

Monochromatic plane wave: $\mathbf{E} = \mathbf{E}_0 e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)} = (E_{0x} \hat{\mathbf{x}} + E_{0y} \hat{\mathbf{y}}) e^{i(kz - \omega t)}$ if $\mathbf{k} = k \hat{\mathbf{z}}$

$\mathbf{k} \cdot \mathbf{E} = \mathbf{k} \cdot \mathbf{B}$ $\mathbf{B} = \frac{\mathbf{k} \times \mathbf{E}}{\omega}$ intensity $I = \langle S \rangle = \left\langle \frac{\epsilon_0 c E^2}{2} \right\rangle = u_{em} c$ pressure $P = \frac{(2)I}{c}$ (2 if perfectly reflected)

Maxwell's in matter: $\nabla \cdot \mathbf{D} = \rho_f$ $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ $\nabla \cdot \mathbf{B} = 0$ $\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}$

Boundary conditions: $\hat{\mathbf{n}} \cdot (\mathbf{D}_2 - \mathbf{D}_1) = \sigma_f$ $\hat{\mathbf{n}} \times (\mathbf{E}_2 - \mathbf{E}_1)_{||} = 0$ $\hat{\mathbf{n}} \cdot (\mathbf{B}_2 - \mathbf{B}_1) = 0$ $\hat{\mathbf{n}} \times (\mathbf{H}_2 - \mathbf{H}_1)_{||} = \mathbf{K}_f$

In linear materials only: $\mathbf{D} = \epsilon_r \epsilon_0 \mathbf{E}$ $\mathbf{B} = \mu_r \mu_0 \mathbf{H}$ $\nabla^2 \mathbf{E} = \frac{1}{v^2} \frac{\partial^2 \mathbf{E}}{\partial t^2}$ $v = \frac{c}{n} = \frac{c}{\sqrt{\epsilon_r \mu_r}}$