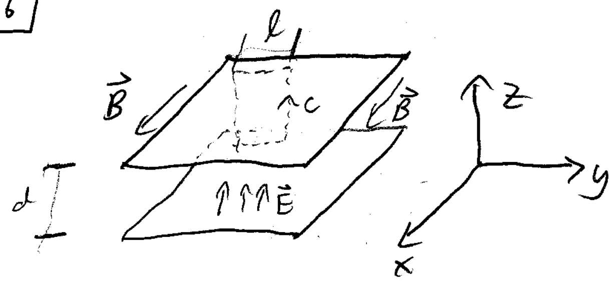
8.6



$\vec{E} = E\hat{z}$

$\vec{B} = B\hat{x}$

a) $\wp_{EM} = \varepsilon_0 \vec{E}\times\vec{B} = \varepsilon_0 EB\,\hat{y}$

"curly p" = momentum density

$P_{em} = \int d^3x\, \wp_{em} = \int d^3x\, \varepsilon_0 EB\hat{y} = \boxed{\varepsilon_0 EBAd\,\hat{y}}$

where A is the area of the plates

b) $\vec{I}$ = impulse

$\vec{I} = \int_0^\infty \vec{F}\,dt = \int_0^\infty I(\vec{l}\times\vec{B})\,dt = \int_0^\infty IBd\,\hat{z}\times\hat{x}\,dt = Bd\,\hat{y}\int_0^\infty \frac{dQ}{dt}\,dt$

current again

$\boxed{\vec{I} = BdQ\,\hat{y}}$

but $E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A} \Rightarrow Q = \varepsilon_0 AE$

$\Rightarrow \boxed{\vec{I} = \varepsilon_0 BEAd\,\hat{y}} = P_{em}$ ✓

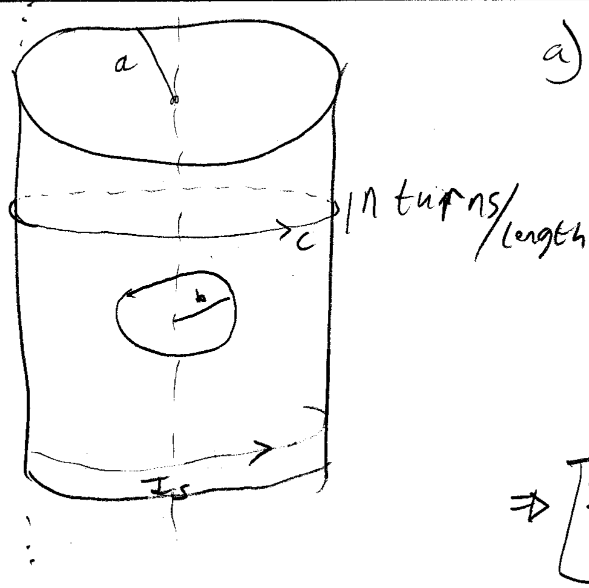
c) $\oint_c \vec{E}\cdot d\vec{l} = -\frac{d\Phi}{dt} = -\frac{dB}{dt}ld$, l = length of loop c in y-direction

$-lE(d) + lE(0) = -ld\frac{dB}{dt} \Rightarrow E(d) - E(0) = d\frac{dB}{dt}$

$\vec{F} = -\sigma AE(d)\hat{y} + \sigma AE(0)\hat{y} = -\sigma A(E(d)-E(0))\hat{y} = -\sigma Ad\frac{dB}{dt}\hat{y}$

$\vec{I} = \int_0^\infty \vec{F}\,dt = -\sigma Ad\,\hat{y}\int_0^\infty \frac{dB}{dt}\,dt = \boxed{+\varepsilon_0 EBAd\,\hat{y}}$ – still same!

8.9



a) $\vec{B} = \mu_0 n I_s \hat{z}$

$\Phi = \pi a^2 B$

$\mathcal{E} = -\frac{d\Phi}{dt} = -\mu_0 n \pi a^2 \frac{dI_s}{dt}$

$\mathcal{E} = IR$

$$\Rightarrow \boxed{I_r = -\frac{\mu_0 n \pi a^2}{R} \frac{dI_s}{dt}}$$

b) $P_r = I_r^2 R = +\frac{\mu_0^2 n^2 \pi^2 a^4}{R} \left(\frac{dI_s}{dt}\right)^2$

$\vec{E}$-field just outside: $\oint_c \vec{E} \cdot d\vec{l} = 2\pi a E = -\frac{d\Phi}{dt} \Rightarrow$

$$\vec{E} = -\frac{1}{2} \mu_0 a n \frac{dI_s}{dt} \hat{\phi}$$

$\vec{B}$ from ring: $\vec{B} = \frac{\mu_0 I_r}{2} \frac{b^2}{(b^2+z^2)^{3/2}} \hat{z}$ (à la Eq. 5.38)

$z=0$ at ring.

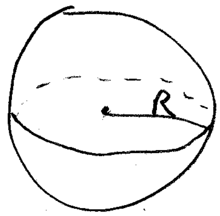
$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} = -\frac{1}{4} \mu_0 I_r \frac{dI_s}{dt} \frac{a b^2 n}{(b^2+z^2)^{3/2}} \hat{r}$$

$$P = \int \vec{S} \cdot d\vec{a} = \int_{-\infty}^{+\infty} S(2\pi a)\, dz = -\frac{1}{2} \pi \mu_0 a^2 b^2 n I_r \frac{dI_s}{dt} \underbrace{\int_{-\infty}^{+\infty} \frac{1 \cdot dz}{(b^2+z^2)^{3/2}}}_{= \frac{1}{b^2} - \frac{-1}{b^2} = \frac{2}{b^2}}$$

$$= -\left(\pi \mu_0 a^2 n \frac{dI_s}{dt}\right) I_r = (R I_r) I_r$$

$$= I_r^2 R$$

8.10



uniform $\vec{P}$ + $\vec{M}$.

Eq'ns. 3.104, 4.14, 5.87, + 6.16 tell us the fields:

$$\vec{E} = \begin{cases} -\frac{1}{3\varepsilon_0}\vec{P} & ; r<R \\ \frac{1}{4\pi\varepsilon_0}\frac{1}{r^3}\left[3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}\right] & ; r>R \end{cases}$$

"little p" $\vec{p} = \frac{4}{3}\pi R^3 \vec{P}$ (total electric dipole moment)

$$\vec{B} = \begin{cases} \frac{2}{3}\mu_0 \vec{M} & ; r<R \\ \frac{\mu_0}{4\pi}\frac{m}{r^3}\left[3(\vec{m}\cdot\hat{r})\hat{r} - \vec{m}\right] & ; r>R \end{cases} \quad ; \quad \vec{m} = \frac{4}{3}\pi R^3 \vec{M}$$

EM momentum: $P_{EM} = \varepsilon_0 \int \vec{E}\times\vec{B}\, d^3x$

– 2 regions, treat separately.

$$\vec{P}_{in} = \varepsilon_0 \int \left(-\frac{1}{3\varepsilon_0}\vec{P}\right)\times\left(\frac{2}{3}\mu_0\vec{M}\right) d^3x = -\frac{2}{9}\mu_0 \vec{P}\times\vec{M}\left(\frac{4}{3}\pi R^3\right)$$

$$= \frac{8}{27}\mu_0 \pi R^3 \vec{M}\times\vec{P}$$

$$\vec{P}_{out} = \varepsilon_0 \int \frac{1}{4\pi\varepsilon_0}\frac{\mu_0}{4\pi}\frac{1}{r^6}\left\{\left[3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}\right]\times\left[3(\vec{m}\cdot\hat{r})\hat{r} - \vec{m}\right]\right\} d^3x$$

$$= \frac{\mu_0}{16\pi^2}\int \frac{1}{r^6}\left(-3(\vec{p}\cdot\hat{r})\hat{r}\times\vec{m} + 3(\vec{m}\cdot\hat{r})\hat{r}\times\vec{p} + \vec{p}\times\vec{m}\right) d^3x$$

Note: $\hat{r}\times(\vec{p}\times\vec{m}) = \vec{p}(\hat{r}\cdot\vec{m}) - \vec{m}(\hat{r}\cdot\vec{p})$

$$\hat{r}\times(\hat{r}\times(\vec{p}\times\vec{m})) = (\hat{r}\times\vec{p})(\hat{r}\cdot\vec{m}) - (\hat{r}\times\vec{m})(\hat{r}\cdot\vec{p}) \quad \text{(using above)}$$

$$\parallel$$

$$\hat{r}(\hat{r}\cdot(\vec{p}\times\vec{m})) - (\vec{p}\times\vec{m})(\hat{r}\cdot\hat{r}) \quad \text{(using BAC-CAB directly)}$$

So,

$$\vec{P}_{out} = \frac{\mu_0}{16\pi^2}\int\frac{d^3r}{r^6}\left(3\left((\hat{r}\times\vec{p})(\hat{r}\cdot\vec{m}) - (\hat{r}\times\vec{m})(\hat{r}\cdot\vec{p})\right) + \vec{p}\times\vec{m}\right)$$

$$= \frac{\mu_0}{16\pi^2}\int\frac{d^3r}{r^6}\left(3\left\{\hat{r}[\hat{r}\cdot(\vec{p}\times\vec{m})] - \vec{p}\times\vec{m}\right\} + \vec{p}\times\vec{m}\right)$$

$$= \frac{\mu_0}{16\pi^2}\int\frac{1}{r^6}\left[-2\vec{p}\times\vec{m} + 3\hat{r}(\hat{r}\cdot(\vec{p}\times\vec{m}))\right] r^2\sin\theta\, dr\, d\phi\, d\theta$$

To do integral, align z-axis with $\vec{p}\times\vec{m}$ (since it's constant),

Then: $\hat{r}\cdot(\vec{p}\times\vec{m}) = |\vec{p}\times\vec{m}|\cos\theta$

$$\hat{r} = \sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\theta\,\hat{z}$$

$\rightarrow \int_0^{2\pi} d\phi$ will kill the $\hat{x}$ + $\hat{y}$ terms $\left(\int_0^{2\pi}\cos\phi\, d\phi = \int_0^{2\pi}\sin\phi\, d\phi = 0\right)$

$$\Rightarrow \vec{P}_{out} = \frac{\mu_0}{16\pi^2}\underbrace{\left[\int_R^\infty \frac{1}{r^4}dr\right]}_{=\frac{1}{3R^3}}\left[-2\vec{p}\times\vec{m}\underbrace{\int\sin\theta\, d\theta\, d\phi}_{=4\pi} + 3|\vec{p}\times\vec{m}|\hat{z}\underbrace{\int\cos^2\theta\sin\theta\, d\theta\, d\phi}_{=\frac{4\pi}{3}}\right]$$

$$\vec{P}_{out} = -\frac{\mu_0}{12\pi R^3}(\vec{p}\times\vec{m}) = \frac{4\mu_0}{27}R^3\,\vec{M}\times\vec{P}$$

8.10 ct'd

$$\boxed{\vec{P}_{tot} = \left(\frac{8}{27} + \frac{4}{27}\right) \mu_0 R^3 \vec{M} \times \vec{P} = \frac{4}{9} \mu_0 R^3 \vec{M} \times \vec{P}}$$

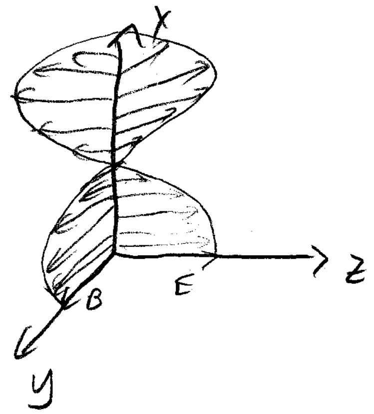
hopefully all "p's" are clear from context.

9.9

a) $\vec{k} = -\frac{\omega}{c} \hat{x}$ $\qquad \vec{k} \cdot \vec{r} = -\frac{\omega}{c} x$

$\hat{n} = \hat{z}$ $\qquad \hat{k} \times \hat{n} = \hat{y}$

$$\boxed{\vec{E} = E_0 \cos\left(\frac{\omega}{c} x + \omega t\right) \hat{z} \qquad \vec{B} = \frac{E_0}{c} \cos\left(\frac{\omega}{c} x + \omega t\right) \hat{y}}$$



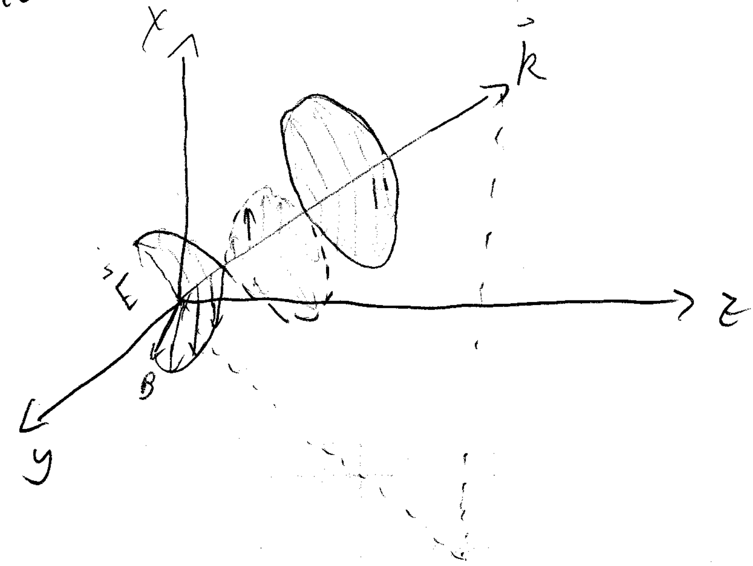
b) $\vec{k} = \frac{\omega}{c}\left(\frac{\hat{x}+\hat{y}+\hat{z}}{\sqrt{3}}\right)$ $\qquad \hat{n} = \frac{\hat{x} - \hat{z}}{\sqrt{2}}$ $\longrightarrow$ to satisfy $\hat{n} \cdot \hat{n} = 1$ & $\hat{n} \cdot \vec{k} = 0$ & in x-z plane

$\vec{k} \cdot \vec{r} = \frac{\omega}{\sqrt{3} c}(x + y + z)$; $\hat{k} \times \hat{n} = \frac{1}{\sqrt{6}}\left(-\hat{x} + 2\hat{y} - \hat{z}\right)$

$$\vec{E} = E_0 \cos\left[\frac{\omega}{\sqrt{3} c}(x + y + z) - \omega t\right]\left(\frac{\hat{x} - \hat{z}}{\sqrt{2}}\right)$$

$$\vec{B} = \frac{E_0}{c} \cos\left[\frac{\omega}{\sqrt{3} c}(x + y + z) - \omega t\right]\left(\frac{-\hat{x} + 2\hat{y} - \hat{z}}{\sqrt{6}}\right)$$

9.9) b) ctd



9.10)

perfect absorber: $P_{abs} = \frac{I}{c} = \frac{1.3\times10^{3}\ W/m^2}{3\times10^{8}\ m/s} = \boxed{4.3\times10^{-6}\ \frac{N}{m^2}}$

perfect reflector is twice as great:

$$\boxed{P_{ref} = 8.6\times10^{-6}\ \frac{N}{m^2}}$$

$1\,atm = 1.03\times10^{5}\ \frac{N}{m^2} \Rightarrow \frac{P_{ref}}{P_{atm}} = \frac{8.6\times10^{-6}}{1.03\times10^{5}} = \boxed{8.3\times10^{-11}}$