Equations of Physics 323 - Electrodynamics

Maxwell's in matter: (i) $\nabla.\mathbf{D}=\rho_f$ (ii) $\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}$ (iii) $\nabla.\mathbf{B}=0$ (iv) $\nabla\times\mathbf{H}=\mathbf{J}_f+\frac{\partial\mathbf{D}}{\partial t}$

Boundary conditions: $\hat{\mathbf{n}}.(\mathbf{D}_2-\mathbf{D}_1)=\sigma_f$ $\hat{\mathbf{n}}\times(\mathbf{E}_2-\mathbf{E}_1)_{//}=0$ $\hat{\mathbf{n}}.(\mathbf{B}_2-\mathbf{B}_1)=0$ $\hat{\mathbf{n}}\times(\mathbf{H}_2-\mathbf{H}_1)_{//}=\mathbf{K}_f$

Poynting: $\mathbf{S}=\mathbf{E}\times\mathbf{H}$ Momentum density $\frac{d\mathbf{P}_{em}}{dV}=\frac{\mathbf{S}}{c^2}$ In linear medium, $u_{em}=\frac{1}{2}(\mathbf{D}.\mathbf{E}+\mathbf{B}.\mathbf{H})$

In linear dielectric $\nabla^2\mathbf{E}=\frac{1}{v^2}\frac{\partial^2\mathbf{E}}{\partial t^2}$ $v=\frac{1}{\sqrt{\varepsilon\mu}}=\frac{c}{\sqrt{\varepsilon_r\mu_r}}=\frac{c}{n}$ $\mathbf{D}=\varepsilon_r\varepsilon_0\mathbf{E}$ $\mathbf{B}=\mu_r\mu_0\mathbf{H}$

Plane wave: $\mathbf{E}=(E_{0x}\hat{\mathbf{x}}+E_{0y}\hat{\mathbf{y}})e^{i(\mathbf{k}.\mathbf{r}-\omega t)}$ $\mathbf{k.E}=\mathbf{0}=\mathbf{k.B}$ $\mathbf{B}=\frac{\mathbf{k}\times\mathbf{E}}{\omega}$ $I=\langle S\rangle=\frac{\varepsilon_0 cE_0{}^2}{2}=u_{em}c$

Derive: $n_1\sin\theta_1=n_2\sin\theta_2$ Brewster: $\tan\theta_B=n_2/n_1$, critical angle: $\sin\theta_c=n_2/n_1$ and Fresnel equations

Derive $\nabla^2\mathbf{E}=\mu\varepsilon\frac{\partial^2\mathbf{E}}{\partial t^2}+\mu\sigma\frac{\partial\mathbf{E}}{\partial t}$ Derive $\delta=\sqrt{\frac{2}{\mu\sigma\omega}}$ $v_{phase}=\omega/k$ $v_g=d\omega/dk$

General complex dispersion: $k(\omega)=k'(\omega)+i\alpha(\omega)/2$ where $k'=2\pi/\lambda$, $n=ck'/\omega$ and intensity $I\propto e^{-\alpha z}$

Dielectric dispersion: $\varepsilon_r=1+\sum_j\frac{N_jq_j{}^2}{\varepsilon_0m_j}\frac{1}{\omega_j{}^2-\omega^2-i\gamma\omega}$ where the sum is over all sets of oscillators

Plasma dispersion: $k^2=(\omega^2-\omega_p{}^2)/c^2$ where $\omega_p{}^2=e^2N/m\varepsilon_0$ Waveguide: $k^2=(\omega^2-\omega_{mn}{}^2)/c^2$

Lorentz gauge: If $\nabla.\mathbf{A}=-\frac{1}{c^2}\frac{\partial V}{\partial t}$ then $\Box^2V=-\rho/\varepsilon_0$ $\Box^2\mathbf{A}=-\mu_0\mathbf{J}$ where $\Box^2=\nabla^2-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}$

Retarded potentials: $V=\frac{1}{4\pi\varepsilon_0}\int\frac{\rho(\mathbf{r}',t_r)}{|\mathbf{r}-\mathbf{r}'|}d^3r'$ $\mathbf{A}(\mathbf{r},t)=\frac{\mu_0}{4\pi}\int\frac{\mathbf{J}(\mathbf{r}',t_r)}{|\mathbf{r}-\mathbf{r}'|}d^3r'$ $t_r=t-|\mathbf{r}-\mathbf{r}'|/c$

Approximations: $r\gg\lambda$, radiation zone; $\lambda\gg d$ long-wavelength (compact source); $r\gg d$ dipole limit

Electric dipole radiation fields, be familiar with $\mathbf{B}\propto\frac{-\mu_0}{4\pi rc}\left(\hat{\mathbf{r}}\times\ddot{\mathbf{p}}(t_r)\right)$ $\mathbf{E}=c\hat{\mathbf{r}}\times\mathbf{B}$ $dP/d\Omega\propto\ddot{p}^2\sin^2\theta$

For a source with $\mathbf{p}=\mathbf{p_0}\sin\omega t$, $\mathbf{B}=\frac{\mu_0\omega^2\hat{\mathbf{r}}\times\mathbf{p}_0}{4\pi rc}\cos\omega(t-r/c)\propto\hat{\varphi}$ $\mathbf{E}\propto\hat{\theta}$ $dP/d\Omega=\omega^4p_0{}^2\sin^2\theta$

Fields for magnetic dipole have **B** and **E** interchanged and are smaller by $\sim d/\lambda$

Derive Larmor formula, $P=\frac{\mu_0q^2a^2}{6\pi c}$ using $\mathbf{p}=q\mathbf{w}(t)$ Rayleigh scattering cross-sec $\sigma\propto\omega^4$

Understand the Lienard-Wiechert potentials: $V(\mathbf{r},t)=q/4\pi\varepsilon_0(s-\mathbf{s}.\mathbf{v}/c)$, $\mathbf{A}(\mathbf{r},t)=(\mathbf{v}/c^2)V(\mathbf{r},t)$ for charge q moving along trajectory $\mathbf{w}(\mathbf{r},t)$, where $\mathbf{v}=\dot{\mathbf{w}}(t_r)$, $\mathbf{s}=\mathbf{r}-\mathbf{w}(t_r)$, and t_r is specified by $c(t-t_r)=|\mathbf{s}|$

$x^\mu=(ct,\mathbf{r})$ $x_\mu=(-ct,\mathbf{r})$ $\gamma=(1-v^2/c^2)^{-1/2}$ $d\tau=\frac{1}{c}\sqrt{-dx^u dx_\mu}=dt/\gamma$ $\eta^\mu=dx^\mu/d\tau$ $p^\mu=m_0\eta^\mu=(E/c,\mathbf{p})$

4-potential $A^\mu=(V/c,\mathbf{A})$ 4-current $J^\mu=(c\rho,\mathbf{J})$ transformation of 4-vectors: $x'^\mu=\Lambda^\mu{}_\nu x^\nu$ $A'^\mu=\Lambda^\mu{}_\nu A^\nu$

$a^\mu b_\mu=$ scalar $-\partial J^\mu/\partial x^\mu=\nabla.\mathbf{J}-\partial\rho/\partial t=0$; $E^2/c^2-p^2=m_0{}^2c^2$; $\Delta x^\mu\Delta x_\mu=\Delta\mathbf{r}^2-c^2\Delta t^2$ = invariant interval

$$\Lambda^\mu{}_\nu=\begin{pmatrix}\gamma & -\gamma v/c & 0 & 0\\ -\gamma v/c & \gamma & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}$$ recognise: $\partial F^{\mu\nu}/\partial x^\nu=\mu_0J^\mu$ and $$F^{\mu\nu}=\begin{pmatrix}0 & E_x/c & E_y/c & E_y/c\\ -E_x/c & 0 & B_z & -B_y\\ -E_y/c & -B_z & 0 & B_x\\ -E_z/c & B_y & -B_x & 0\end{pmatrix}$$

$F'^{\mu\nu}=\Lambda^\mu{}_\alpha\Lambda^\nu{}_\beta F^{\alpha\beta}$ –> derive $E'_{//}=E_{//}$ $B'_{//}=B_{//}$ $\mathbf{E}_\perp=\gamma(\mathbf{E}_\perp+\mathbf{v}\times\mathbf{B}_\perp)$ $\mathbf{B}_\perp=\gamma(\mathbf{B}_\perp-\mathbf{v}\times\mathbf{E}_\perp/c^2)$

More scalar invariants: $F^{\mu\nu}F_{\mu\nu}=2(E^2/c^2-B^2)$ $F^{\mu\nu}G_{\mu\nu}=-4\mathbf{E}.\mathbf{B}/c$

Derive field or potentials of charge in uniform motion by transforming Coulomb field or potential

$\Box^2A^\mu=\frac{\partial}{\partial x^\nu}\frac{\partial}{\partial x_\nu}A^\mu=-\mu_0J^\mu$ Lorentz gauge $\partial A^\mu/\partial x^\mu=0$ Lorentz force $dp^\mu/d\tau=q\eta_\nu F^{\mu\nu}$