PHYSICS 323

23 April, 2003 Midterm 1

Name:

There are three problems in this exam. Please do not open the test until the starting time is announced. This is an open book test.

1. (a) (10) The electric field associated with an electromagnetic plane wave moving in the $+z$-direction, in a medium with index of refraction $n = 1$, is given by

$$\mathbf{E}^{(\lambda)}(z,t) = E_0 \; e^{i(kz-\omega t)} \; (\hat{\mathbf{e}}_x + \lambda \hat{\mathbf{e}}_y)\,,$$

where λ is a given real number. Determine the polarization of the wave.

(b) (15) Determine the intensity of the wave of part (a).

2. Consider light of angular frequency ω traveling from $z = -\infty$ incident normally on a plate of glass with thickness a and real index of refraction n. The plate is parallel to the xy plane, with one face at $z = 0$ and the other at $z = a$. The index of refraction is unity for $z < 0, z > a$. The electromagnetic field for $z < 0$ is a superposition of incident and reflected waves. In the region $0 \leq z \leq a$ there are waves moving in the positive and negative z directions. In the region $z > a$ there is only a transmitted wave.

(a) (15) Write equations for $\mathbf{E}(z, t)$ and $\mathbf{B}(z, t)$ in all three regions of z. The direction of polarization of the incident wave is $\hat{\mathbf{j}}$, and its amplitude is E_0, a real number. Express your answers in terms of 4 unknown electric field amplitudes.

(b) (10) List the quantities that are continuous at $z = 0$ and $z = a$

(c) (15) Write the boundary conditions that the amplitudes of the electric field obey at $z = 0, a$.

3. A plasma is an ionized gas consisting at least partly of free electrons and positive ions; it is therefore a conducting material. The sun and stars are largely plasmas. Consider the classical model discussed in class and in sect. 9.4.3, with $\gamma = 1/\tau$ and the electrons are not bound. Take the electric field to have the form $\mathbf{E}(t) = \mathbf{E}_0 e^{-i\omega\, t}$.

(a) (15) The density of electrons in the plasma is N, and the current density $\mathbf{J} = -eN\mathbf{v} = \sigma\mathbf{E}$. Show that the conductivity σ is given by $\sigma(\omega) = \frac{i(\omega e^2/m)\, N}{\omega^2 + i\omega\gamma} = \frac{i\epsilon_0 \Omega^2}{\omega + i\gamma}$, with $\Omega^2 \equiv Ne^2/(m\epsilon_0)$.

(b)(10) For a dilute plasma the damping factor γ is small, so for the remainder of this problem, take $\gamma = .01\Omega$, and for a typical plasma $\Omega = 1.5 \times 10^{10}$ rad/s. For what frequencies is $\mathrm{Re}(\sigma) \gg \mathrm{Im}(\sigma)$? For such frequencies, is the wave damped or oscillatory? Explain.

(c)(10) For what frequencies is $\mathrm{Re}(\sigma) \ll \mathrm{Im}(\sigma)$? For such frequencies, is the wave damped or oscillatory? Explain.