PHYSICS 323

16 May, 2003 Midterm 2

Name:

There are two problems in this exam, each with five parts. Each part counts the same. Please do not open the test until the starting time is announced. This is an open book test.

"If tachyons do exist, and if they do go faster than the speed of light, then I'm determined to find something that goes faster than tachyons."

1. Consider a wave guide with a hollow cross sectional area in the form of a right isosceles triangle as shown in the figure. The direction of propagation is the z direction, with z axis perpendicular to the page. This entire problem is concerned with TM modes of angular frequency ω.

(a) What is B_z?

(b) For waves propagating in the positive z direction, we may write the z component of the electric field as $E_z(x, y)e^{i(kz-\omega t)}$. State the boundary conditions that $E_z(x, y)$ must obey.

(c) Derive a wave equation for $E_z(x, y)$.

Problem 1 continued

(d) Determine the set of functions $E_z(x, y)$ that satisfy the wave equation and boundary conditions. Make sure that you determine the allowed values of k. Hint: **if** $x = y$, $e^{i(\alpha x+\beta y)} - e^{i(\alpha y+\beta x)} = 0$.

(e) Determine the lowest cutoff frequency.

2. This problem concerns radiation from a system of charges and currents that is localized near an origin, surrounded by empty space. Suppose that, for positions $\mathbf{r}$ in the radiation zone, the vector potential can be represented by the expression $\mathbf{A}(\mathbf{r},t) = e^{i(kr-\omega t)}\,\frac{\mathbf{a}(k,\widehat{r})}{r}$, with $\mathbf{a}(k,\widehat{r})$ a given but unspecified function. This problem is concerned with fields in the radiation zone. ($\widehat{r} = \frac{\mathbf{r}}{\mathbf{r}}$).

(a) Determine $\mathbf{B}(\mathbf{r},t)$.

(b) Determine $\mathbf{E}(\mathbf{r},t)$.

(c) Show that the angular distribution of (time averaged) radiated power, $\frac{dP}{d\Omega}$ is given by $\frac{dP}{d\Omega} = \frac{k^2 c}{2\mu_0}|\widehat{r} \times \mathbf{a}|^2$.

Problem 2 continued

(d) A system consists of a particle of charge Q moving along a trajectory given by $\mathbf{r}_Q(t) = \widehat{\mathbf{x}}L\cos(\omega_0 t)$ and a wire loop of radius $R > L$. This loop is located in the xy plane and is centered at the origin with current $I(t) = I_0\cos(\omega_0 t)$. Suppose $\omega R/c \ll 1$. Determine the vector function $\mathbf{a}(k, \widehat{r})$.

(e) For the situation of part (d) determine $\frac{dP}{d\Omega}$.