Name __________solutions____________________

Electrodynamics, Physics 323 **Second midterm** **8.20 am, May 19, 2004**
Spring 2004 Instructor: David Cobden

You have 60 minutes. End on the buzzer at 9.20. Answer all 12 questions.

Write your name on every page and your ID on the first page.

Write all your working on these question sheets. Use this cover page for extra working (you might get credit for it.)

It is important to show your calculation or derivation. You may not get full marks just for stating the correct answer if you don't show how you get it.

Watch the blackboard for corrections or clarifications during the exam.

This is a **closed book** exam. **No notes allowed. No calculators.**

$c = 3 \times 10^8$ m/s.

Don't turn this page until I say 'go'.

Why the Sky is Blue *by John Ciardi*

I don't suppose you happen to know
Why the sky is blue? It's because the snow
Takes out the white. That leaves it clean
For the trees and grass to take out the green.
Then pears and bananas start to mellow,
And bit by bit they take out the yellow.
The sunsets, of course, take out the red
And pour it into the ocean bed
Or behind the mountains in the west.
You take all that out and the rest
Couldn't be anything else but blue.
Look for yourself. You can see it's true.

 Name solutions

I. 1. [6] The vector potential sometimes obeys the following equation: $\Box^2 \mathbf{A}(\mathbf{x},t) = -\mu_0 \mathbf{J}(\mathbf{x},t)$.
Write $\Box^2$ explicitly in terms of derivatives. What is the condition for this equation to be valid?

$$\Box^2 = \nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$$

(3)

Lorentz gauge 2

$$\vec{\nabla}\cdot\vec{A} = -\frac{1}{c^2}\frac{\partial V}{\partial t}$$ 1

2. [14] A circular wire loop centered on the *z*-axis carries a current I(t). Find the (retarded) vector potential $\mathbf{A}(x,t)$ and the scalar potential $V(x,t)$ *along the z-axis only.*

(3)

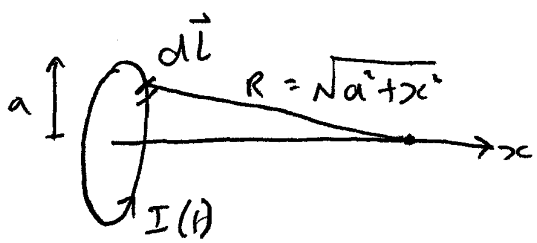
$$V(\vec{x},t) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\vec{x}', t_r)\, d^3x'}{|\vec{x}-\vec{x}'|}$$

but $\rho = 0$ always (2)

$\therefore V = 0$ always.

(3)

$$\vec{A}(\vec{x},t) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{x}', t_r)\, d^3x'}{|\vec{x}-\vec{x}'|} \qquad t_r = t - \frac{|\vec{x}-\vec{x}'|}{c}$$



On x-axis

$$\vec{A} = \frac{\mu_0}{4\pi}\oint \frac{I\left(t-\frac{R}{c}\right) d\vec{l}}{R}$$ 4

$$= \frac{\mu_0 I\left(t-\frac{R}{c}\right)}{R}\oint d\vec{l}$$

$= 0$ (2) R is same for all elements of loop

3. [8] Hence show that the electric field along the axis of the loop is always zero.

$$\vec{E} = -\vec{\nabla}V - \frac{\partial\vec{A}}{\partial t}$$

$$= 0 - 0$$

$$= 0$$

Name solutions

II. Which of the following are true? 3 points for each correct answer.

-1 points for each wrong answer.

0 points for no answer.

4. [9] A charge radiates whenever:

(a) it is moving in whatever manner false

(b) it is being accelerated true

(c) it is bound in an atom false

5. [9] A radially pulsating charged sphere:

(a) emits electromagnetic radiation false

(b) creates a static magnetic field false

(c) can set a nearby electrified particle (which never touches the sphere) in motion true

6. [9] Radiation emitted by an antenna has the angular distribution characteristic of dipole radiation when:

(a) the wavelength is long compared with the antenna true

(b) the wavelength is short compared with the antenna false

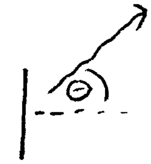
(c) the antenna has the appropriate shape false

 Name solutions

III. A radio station has an antenna on top of Queen Anne Hill which is a vertical wire. It emits a power of 100 kW at 90 MHz.

7. [5] What is the intensity of radiation emitted directly upwards into space?

5 Zero – it goes like $\sin^2\theta$



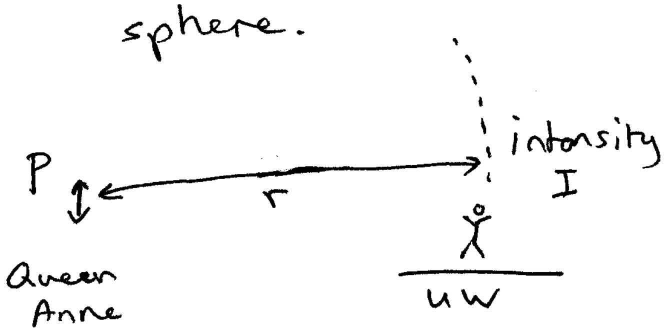
8. [5] What length, in meters, should the antenna have to work best?

half-wave antenna: (2)

$$L = \frac{\lambda}{2} \overset{(2)}{=} \frac{c}{2f} = \frac{3\times10^8\,ms^{-1}}{2\times 9\times10^7 s^{-1}} = \frac{10\,m}{6} \overset{(1)}{\doteq} 1.7\,m$$

9. [15] Obtain a rough estimate of the strength of the electric field on the UW campus, approximately 4 km away, in V/m.

Assume power is radiated uniformly over a sphere.



$$\therefore \quad I \doteq \frac{P}{4\pi r^2} \quad (5)$$

Like a plane wave, $I \overset{(5)}{=} \frac{1}{2}\varepsilon_0 c E_0^2 \doteq \frac{P}{4\pi r^2}$

$$\therefore \quad E_0 \doteq \left(\frac{P}{2\pi r^2 \varepsilon_0 c}\right)^{1/2}$$

$$= \left[\frac{10^5\,W}{2\pi.(4\times10^3 m)^2 \times 10^{-11} Fm^{-1} \times 3\times10^8 ms^{-1}}\right]^{1/2} \quad (3)$$

$$\doteq \left(\frac{10^{5-6+11-8}}{2.3.4^2.3}\right)^{1/2} Vm^{-1}$$

$$\doteq \left(\frac{10^2}{300}\right)^{1/2} Vm^{-1} \doteq \frac{1}{\sqrt{3}}\,Vm^{-1} \sim 1\,Vm^{-1}$$

(2)

contd.

Name solutions

10. [4] Is the radiation on campus polarized, and if so, in which direction?

Yes, approximately vertically (ignoring scattering/ reflection from ground, buildings etc).

$\uparrow E$

11. [6] *Estimate* the magnetic field on campus also, and describe its direction.

$$B_0 = \frac{E_0}{c} \text{ for plane wave } \sim \frac{1/\sqrt{3}\ \mathrm{Vm^{-1}}}{3\times10^8\ \mathrm{ms^{-1}}} \simeq 1.7\times10^{-9}\ \mathrm{T} \sim 10^{-9}\ \mathrm{T}$$

$\vec{B} = \dfrac{\vec{k}\times\vec{E}}{\omega}$ so if $\vec{E}$ is vertically polarized, $\vec{B}$ is approximately horizontal.

12. [10] Using the fact that the total power radiated by an oscillating electric dipole p is $P = \dfrac{\mu_0 \ddot{p}^2}{\cancel{12}\,6\pi c}$, or otherwise, *estimate roughly* the average current I flowing in the antenna.

bonus mark for correcting this to 6!

Estimate: $\dot{p}_\omega = I_\omega L$ ← antenna length

$I = I_\omega \cos\omega t$

$\therefore \ddot{p}_\omega = \omega I_\omega L$ ← $= \frac{\lambda}{2}$ from above

$$\therefore P \simeq \frac{\mu_0 \left(\omega I_\omega \frac{\lambda}{2}\right)^2}{12\pi c} = \frac{\mu_0}{48\pi c} \cdot I_\omega^2 \lambda^2 \omega^2$$

should be 6

$$= \frac{\mu_0 I_\omega^2 (2\pi c)^2}{48\pi c}$$

$$= \mu_0 c \cdot \frac{\pi}{12} I_\omega^2$$

μ_0

$$\therefore I_\omega \simeq \left(\frac{12}{\pi}\frac{P}{\mu_0 c}\right)^{1/2}$$

$$\mu_0 c = Z_0 = 4\pi\times10^{-7}\times3\times10^8 = 120\pi\ \Omega = 377\ \Omega$$

$$= \left(\frac{12P}{120\pi^2}\right)^{1/2} = \frac{1}{\pi}\left(\frac{P}{10}\right)^{1/2}$$

$$= \frac{1}{\pi}\cdot(10^4)^{1/2} \simeq 30\ \mathrm{A}.$$