 Name Solutions

Electrodynamics, Physics 323 **Spring 2008**

Second midterm Instructor: David Cobden

8.20 am, Wednesday May 21, 2008

Do not turn this page until the buzzer goes at 8.20. You must hand your exam to me before I leave the room at 9.25.

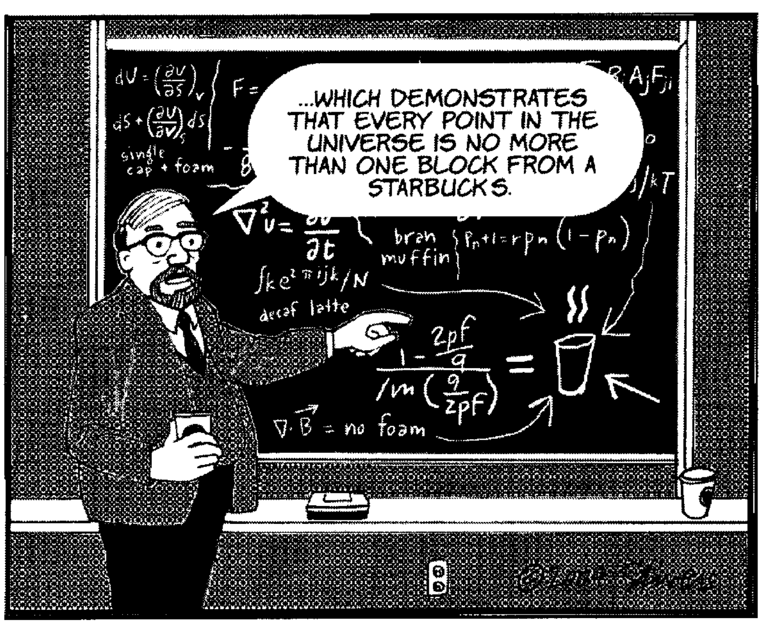
Attempt all the questions.

Please write your name on every page and your SID on the first page.

Write all your working on these question sheets. Use this front page for extra working. It is important to show your calculation or derivation. Some of the marks are given for showing clear and accurate working and reasoning.

Watch the blackboard for corrections or clarifications during the exam.

This is a closed book exam. ***No books, notes or calculators allowed.***



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1. [4] The (Lienard-Wiechert) scalar potential for a particle of charge q moving along trajectory $\mathbf{w}(t)$ is

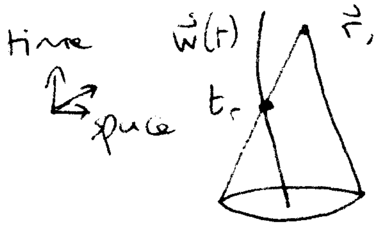
$$V(\mathbf{r},t) = \frac{q}{4\pi\epsilon_0 \mathcal{R}\left(1 - \hat{\boldsymbol{\mathcal{R}}}.\frac{\mathbf{v}}{c}\right)}$$

where $\boldsymbol{\mathcal{R}} \equiv \mathbf{r} - \mathbf{w}(t_r)$. Is this specific to a particular gauge, and if so, which one?

Yes, Lorentz gauge.

2. [4] Give the equation defining the retarded time t_r. $|\vec{r} - \vec{w}(t_r)| = c(t - t_r)$

3. [4] Show using a spacetime diagram why the retarded point is unique.



$\dot{w} < c$

$\therefore$ slope of trajectory < slope of backwards light cone

$\therefore$ only one point of intersection

4. [4] Is it possible to choose a gauge in which $V(\mathbf{r},t) = \frac{q}{4\pi\epsilon_0\mathcal{R}}$, ie, where there is no correction factor on the bottom, and why (not)?

Yes. You can add $-\frac{\partial\lambda}{\partial t}$ to V, which could be (almost) anything analytic.

5. [5] Give the equations for $V(\mathbf{r},t)$ and $\mathbf{A}(\mathbf{r},t)$ in terms of $\rho(\mathbf{r},t)$ and $\mathbf{J}(\mathbf{r},t)$ in the Lorentz gauge.

$$V(\vec{r},t) = \int \frac{\rho(\vec{r}\,', t_r)}{4\pi\varepsilon_0|\vec{r}-\vec{r}\,'|}\,d^3r' \quad \vec{A}(\vec{r},t) = \int \frac{\mu_0 \vec{J}(\vec{r}\,',t_r)\,d^3r'}{4\pi\,|\vec{r}-\vec{r}\,'|} \quad t_r = t - \frac{|\vec{r}\,'-\vec{r}|}{c}$$

6. [6] By considering the current density due to the point charge, find the (Lienard-Wiechert) expression for $\mathbf{A}(\mathbf{r},t)$ for the moving point charge.

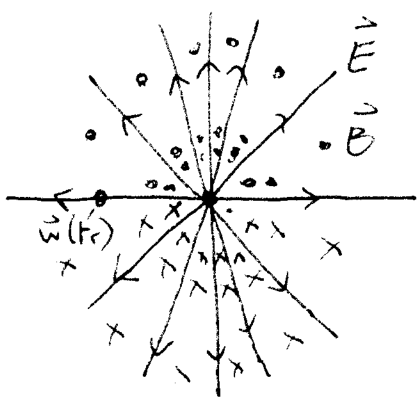
$\vec{J}(\vec{r},t) = \rho(\vec{r},t)\vec{v}(t)$ for a point charge

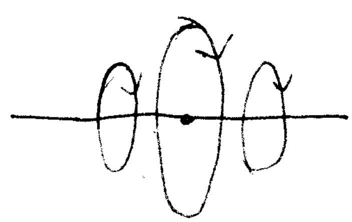
$$\vec{A}(\vec{r},t) = \int \frac{\mu_0\rho(\vec{r}\,',t_r)\vec{v}(t_r)\,d^3r'}{4\pi\,|\vec{r}-\vec{r}\,'|} = \mu_0\epsilon_0\vec{v}(t_r)\int\frac{\rho(\vec{r}\,',t_r)}{4\pi\varepsilon_0|\vec{r}-\vec{r}\,'|} = \frac{\vec{v}(t_r)}{c^2}V(\vec{r},t)$$

7. [8] A proton moves with constant velocity 0.8c along the x-axis. Sketch the electric field lines around it. Sketch also the magnetic field, using either dots and crosses on the same diagram or another diagram.

$\vec{B} = \frac{\hat{\mathcal{R}} \times \vec{E}}{c}$ $\quad |B| \sim \frac{1}{r}$

$E \sim \frac{1}{r^2}$





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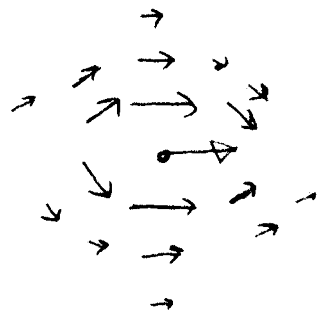
8. [6] Draw a diagram showing the flow of electromagnetic energy around the proton, using arrows of variable length to indicate the direction and amplitude of the flow field.



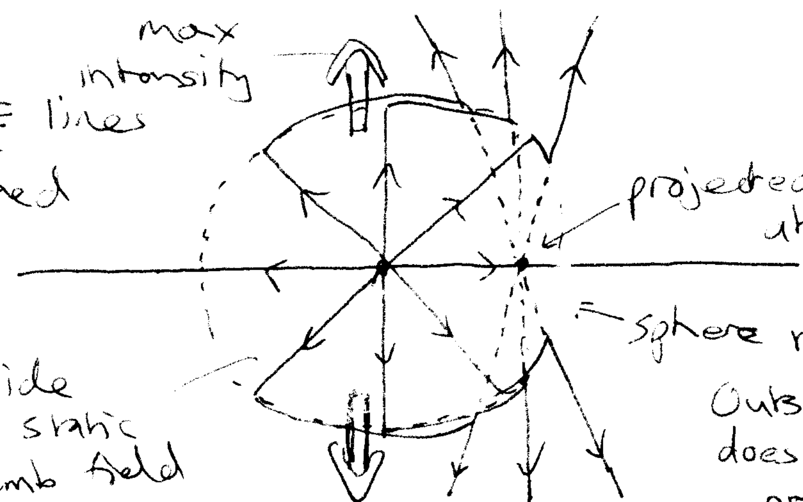
$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$ $|S| \sim \frac{1}{r^4}$

largest in plane $\perp r$ $\vec{v}$ containing proton, where E is largest.

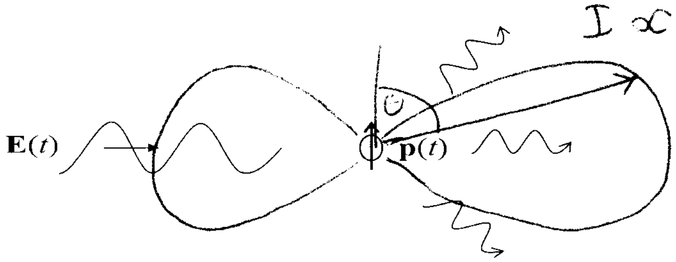
9. [4] What is the total rate of flow of energy out through a surface enclosing the proton, and why?

0 – no radiation because no acceleration (cannot radiate in inertial rest frame)

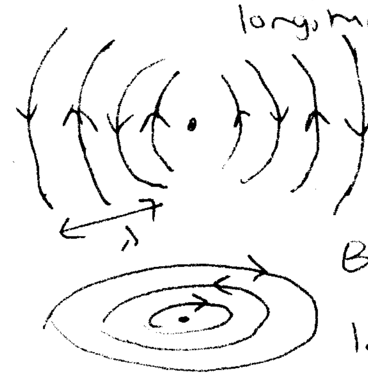
10. [12] The proton hits a target at $t = 0$ and stops abruptly. Sketch the electric field lines around the electron a short time later, illustrating the propagation of radiation out from the impact point, and in what directions it has maximum intensity.



11. [12] A small dust particle of size much less than a wavelength scatters radiation from the sun by the process of Rayleigh scattering: an oscillating dipole moment **p** is induced in the particle by the incident radiation, and this then emits dipole radiation: this is the basic reason why the sky is blue. For the component of radiation from the sun that is vertically polarized, sketch the distribution of scattered radiation and give its dependence on angle to the vertical and on distance from the scattering particle. Sketch also the electric and magnetic field lines for the scattered radiation.



$I \propto \frac{\sin^2\theta}{r^2}$



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12. [9] List the approximations involved in deriving the above radiation field pattern, say what each of them means, and check that each is applicable in this situation.

$r \gg d$ lowest order multipole (dipole) dominates ✓

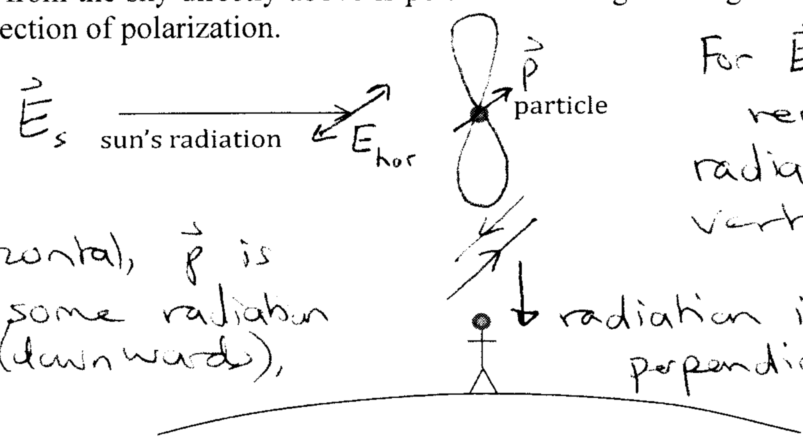
$r \gg \lambda$ radiation terms dominate static ones ✓

$\lambda \gg d$ phase variation over particle $\ll 2\pi$ ✓

$\lambda \sim 0.5\,\mu m$, $r \sim 10\,km$, $d \ll \lambda$ specified.

$\Rightarrow r \gg d$

13. [8] The light from the sky directly above is polarized. Using the diagram below, explain this and determine the direction of polarization.



For $\vec{E}_s$ vertical, $\vec{p}$ is vertical → no radiation/scattering vertically.

For $\vec{E}_s$ horizontal, $\vec{p}$ is horizontal, some radiation is vertical (downwards),

radiation is polarized perpendicular to the direction to the sun.

14. [6] Why is the sky blue (ie, why are predominantly shorter wavelengths scattered) ?

$p = \alpha E$ doesn't depend strongly on ω if you're away from resonances but radiated power $\propto \omega^4$

$\therefore$ Scattered intensity $\propto \omega^4 \propto \lambda^{-4}$ $\therefore$ blue (400nm) is scattered more than red (800nm)

15. [10] Define the polarizability α of the dust particle by $\mathbf{p} = \alpha\mathbf{E}$. If the particle can be modeled as an assembly of N molecular oscillators with natural frequency ω_0, damping constant γ, charge q, and mass m, show that the response to incident radiation at frequency ω is given roughly by

$$\alpha(\omega) \approx \frac{Nq^2}{m(\omega_0^2 - \omega^2 - i\gamma\omega)}$$

Eq of motion: $\ddot{x} + \gamma\dot{x} + \omega_0^2 x = \frac{F}{m} = \frac{qE_0 e^{-i\omega t}}{m}$

$\therefore x = \frac{qE_0 e^{-i\omega t}}{m(\omega_0^2 - \omega^2 - i\gamma\omega)}$ $\quad \therefore \alpha = \frac{p}{E} = \frac{Nqx}{E} = \frac{Nq^2}{m(\omega_0^2 - \omega^2 - i\gamma\omega)}$