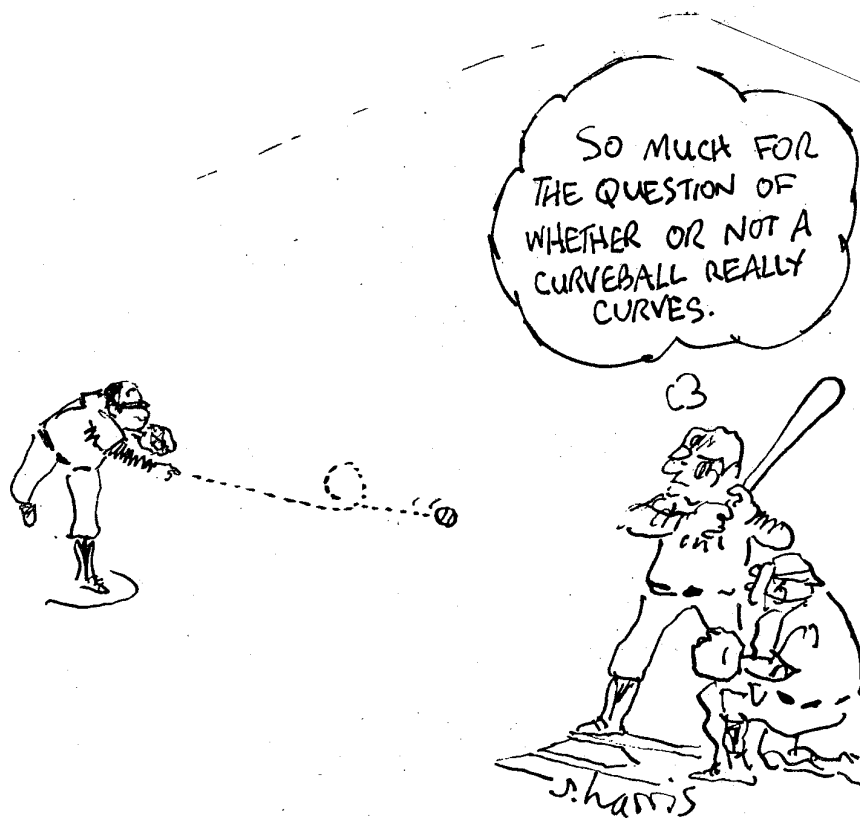


PHYSICS 323

10 June, 2003 Final Exam

Name:

There are six problems in this exam. All parts count the same, 10 pts, for a total maximum score of 200. Please do not open the test until the starting time is announced. This is an open book test.



1. Suppose light is propagated from air ($n = 1$) into a medium of index of refraction $n = 4/3$, with $\mu = \mu_0$.

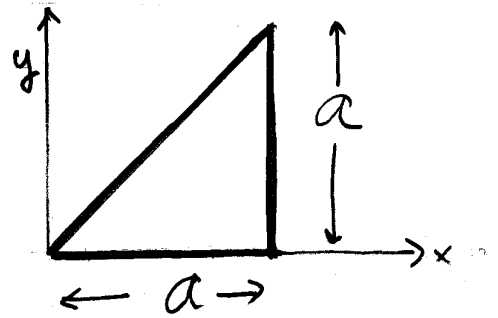
(a) Determine ϵ/ϵ_0 for the medium.

(b) In most situations, there will be a reflected wave and a transmitted wave. Explain the physical origin of the reflected wave.

(c) Suppose the incident light is polarized parallel to the plane of incidence and has an amplitude E_{0I} and a frequency ω . Determine the tangent of the angle of incidence θ_I for which the reflection coefficient vanishes.

(d) Provide an explanation for the vanishing of part (c). Your explanation should involve geometry and the radiation fields produced by electric dipoles.

2. Consider a wave guide with a hollow cross sectional area in the form of a right isosceles triangle as shown in the figure. The direction of propagation is the z direction, with z axis perpendicular to the page. This entire problem is concerned with **TE** modes of angular frequency ω .



(a) What is E_z ?

(b) For waves propagating in the positive z direction, we may write the z component of the magnetic field as $B_z(x, y)e^{i(kz - \omega t)}$. State the specific boundary conditions that $B_z(x, y)$ must obey.

(c) Determine the set of functions $B_z(x, y)$ that satisfy the wave equation (9.181) and the boundary conditions.

(d) Determine the lowest cutoff frequency.

$$\mathbf{A}(\mathbf{r}, t) = \frac{-i\mu_0\omega}{4\pi r} \mathbf{p} e^{i(kr-\omega t)}, \quad \mathbf{A}(\mathbf{r}, t) = \frac{-i\mu_0\omega}{4\pi r} \frac{1}{c} \mathbf{m} \times \hat{\mathbf{r}} e^{i(kr-\omega t)}$$

3. A system consists of a particle of charge Q moving along a trajectory given by $\mathbf{r}_Q(t) = \hat{\mathbf{z}}L\sin(\omega_0 t)$ and a wire loop of radius $R > L$. This loop is located in the xy plane and is centered at the origin with current $I(t) = I_0 \cos(\omega_0 t)$. Suppose $\omega R/c \ll 1$.

(a) Determine the vector potential $\mathbf{A}(\mathbf{r}, t)$ for positions in the radiation zone.

(b) For the situation of part (a) determine the electric field.

Problem 3 continued

- (c) Measurements at positions in the radiation zone along the y axis determine that the light is circularly polarized. Determine R in terms of L and other given quantities.

4. In a reference frame K two almost evenly matched sprinters are lined up a distance d apart on the x axis for a race parallel to the y axis. Two starters, one beside each sprinter, will fire their starting pistols at slightly different times, giving a handicap to the better of the two sprinters: the slower runner will start at a time T before the faster runner.

(a) For what range of values of T is there a reference frame \bar{K} for which there is no handicap.

(b) Suppose T is such that there is no handicap. Find the frame K .

5. The threshold kinetic energy T_{th} in the laboratory (target particle at rest) for a given reaction is the kinetic energy of the incident particle on a stationary target just sufficient to make the center of mass energy W equal to the sum of the rest energies of the particles in the final state. (W is the sum of the energies of the particles in the frame in which the sum of the particle's three momentum vanishes, for two particles $-W^2/c^2 = (p_1 + p_2)_\mu (p_1 + p_2)^\mu$. (Griffiths's convention))

(a) Calculate the threshold kinetic energy for the process of neutral pion production in a collision between a proton beam and a target nucleus, A : $pA \rightarrow pA\pi^0$. Take the proton mass to be M . The nucleus A consists of a total of N neutrons plus Z protons, with $A = N + Z$. The mass of a neutron is close to that of the proton, so take the mass of a nucleus to be MA . The pion mass is m_π .

(b) Suppose $Mc^2 = 1000 \text{ MeV}$ and $m_\pi c^2 = 150 \text{ MeV}$ and you have a beam of protons of kinetic energy 180 MeV . Can this beam be used to produce pions in proton proton collisions?

(c) For the situation of (b), how large must A be for pion production to be allowed?

6. In a frame \mathcal{S} , the vector potential \mathbf{A} is given by $\mathbf{A} = A_0 \hat{z} \exp(-r^2/R^2)$, where R is a given length and A_0 is a given potential strength. The electric potential $V(\vec{r}, t) = 0$.

(a) What is the charge density ρ in the frame \mathcal{S} ?

(b) Determine $\mathbf{E}(\mathbf{r}, \mathbf{t})$ and $\mathbf{B}(\mathbf{r}, \mathbf{t})$ in \mathcal{S} .

Problem 6 continued

(c) The frame $\bar{\mathcal{S}}$ moves relative to \mathcal{S} at a velocity $\mathbf{v} = v\hat{x}$, and its origin overlaps that of \mathcal{S} at $t = 0$. Determine $\bar{\mathbf{E}}(\bar{\mathbf{r}}, \bar{t})$ and $\bar{\mathbf{B}}(\bar{\mathbf{r}}, \bar{t})$ in the frame $\bar{\mathcal{S}}$.

(d) Determine the charge density $\bar{\rho}(\bar{\mathbf{r}}, \bar{t})$ in the frame $\bar{\mathcal{S}}$.