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Electrodynamics, Physics 323 Spring 2004 **Final exam** Instructor: David Cobden

You have 120 minutes. End on the buzzer at 10.20. Attempt all the questions if you can.

Write your name on every page and your ID on the first page.

Write all your working on these question sheets. Use this cover page for extra working (you might get credit for it.)

It is important to show your calculation or derivation. You won't get full marks just for stating the correct answer if you don't show how you get it.

Watch the blackboard for corrections or clarifications during the exam.

This is a **closed book** exam. **No notes allowed. No calculators.** 

Do not turn this page until the buzzer goes at 8.20.

$$\Lambda_{n}^{m} = \begin{pmatrix} \mathbf{g} & -\mathbf{g} v/c & 0 & 0 \\ -\mathbf{g} v/c & \mathbf{g} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} F^{m} = \begin{pmatrix} 0 & E_{x}/c & E_{y}/c & E_{y}/c \\ -E_{x}/c & 0 & B_{z} & -B_{y} \\ -E_{y}/c & -B_{z} & 0 & B_{x} \\ -E_{z}/c & B_{y} & -B_{x} & 0 \end{pmatrix}$$
$$A^{m} = (c\mathbf{r}, \mathbf{J}) \qquad A^{m} = \sum_{n=0}^{3} \Lambda_{n}^{m} A^{n} \qquad F'^{m} = \sum_{a=0}^{3} \sum_{b=0}^{3} \Lambda_{a}^{m} \Lambda_{b}^{n} F^{ab}$$



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**I.** Say whether each of the following is true or false. [5 pts for the right answer, -1 pt for the wrong answer, 0 pts for no answer.]

1. [5] Maxwell's equations are not Lorentz invariant unless they are written solely in terms of Lorentz tensors (of various ranks).

2. [5] A sphere of uniform charge density  $\mathbf{r}$  which is stationary in inertial frame S will appear as an ellipsoid of charge density  $(1-u^2/c^2)^{-1/2}\mathbf{r}$  in frame S' traveling with velocity u relative to S.

3. [5] If event  $X_2$  occurs later than event  $X_1$  in frame S, it is never possible to find a frame S' in which  $X_2$  occurs earlier than  $X_1$ .

**II.** The electric field in a medium of conductivity **s** obeys the equation  $\nabla^2 \mathbf{E} = \mathbf{m} \mathbf{e} \frac{\partial^2 \mathbf{E}}{\partial t^2} + \mathbf{m} \mathbf{s} \frac{\partial \mathbf{E}}{\partial t}$ . We are concerned with the propagation of electromagnetic waves at frequency **w**.

4. [5] Why is  $\mathbf{w} \ll \mathbf{s}/\mathbf{e}$  said to be the criterion for a 'good' conductor?

5. [15] Show that in a good conductor, the wave decays exponentially with length scale  $\delta = (2/msw)^{1/2}$ .

6. [10] Show that the amplitude of the wave decreases by a factor  $e^{-2\pi}$  over a distance of one wavelength in the propagation direction.

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**III**. The power radiated by an electric dipole **p** is  $P = m_0 \ddot{p}^2 / 6pc$ .

7. [5] Obtain from this the Larmor formula for the power radiated by a moving charged particle.

8. [5] Under what conditions does the Larmor fomula break down?

9. [5] Give a simple reason for why a particle moving at constant velocity does not radiate.

10. [20] A particle of charge q and mass m is released from rest and falls under gravity for a time T. What fraction of the potential energy lost is radiated away? (Make a reasonable approximation).

11. [10] Sketch the distribution of the radiation emitted during this process.

IV. When a point charge q moves along a trajectory  $\mathbf{w}(t)$ , the resulting scalar potential is given by the Lienard-

Wiechert formula,  $V(\mathbf{x}, t) = \frac{q}{4\boldsymbol{p}\boldsymbol{e}_0(X - \mathbf{X}.\mathbf{v}/c)}$ .

12. [5] Is this equation correct at relativistic velocities?

13. [10] What *precisely* is the displacement vector  $\mathbf{X}$  in this equation, and how precisely is the velocity vector  $\mathbf{v}$  specified?

14. [5] Write down the standard solution for the retarded potential  $V(\mathbf{x},t)$  resulting from a charge density  $\mathbf{r}(\mathbf{x},t)$  in the Lorentz gauge.

15. [5] Briefly, why is it *wrong* just to put the retarded position in this equation, which would lead to the *incorrect* result  $V(\mathbf{x}, t) = \frac{q}{4\mathbf{p}\mathbf{e}_0 X}$ ?

16. [10] Evaluate the Lienard-Wiechert formula above for the case of a charge at rest at position  $\mathbf{w}_0$ .

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**V.** In the laboratory frame S there exist two stationary infinite conducting surfaces at y = 0 and y = d, separated by vacuum. The lower plate is grounded while the upper is held at potential  $V_0$ . A particle of charge *q* moves along with velocity  $\mathbf{v} = v\hat{\mathbf{x}}$  in between the plates. The rest frame of the particle is S'. 17. [5] What is the force **F** on the particle in the lab frame S?

18. [6] Write a manifestly covariant equation which relates  $A^m$  and  $J^m$ , and state the Lorentz gauge condition in corresponding covariant form.

19. [14] Find the components in S of:

- (a) the 4-potential  $A^m$ ,
- (b) the corresponding 4-current  $J^{m}$  (use a Dirac delta function for the charge density), and
- (c) the electromagnetic field tensor  $F^{\mathbf{m}}$ .

20. [20] By Lorentz transformation (or otherwise), find the components of the three tensors,  $A'^m$ ,  $J'^{\mu}$ , and  $F'^{\mu\nu}$ , in the particle's rest frame S'.

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21. [10] Hence, or otherwise, find the potentials V' and  $\mathbf{A}'$  and the fields  $\mathbf{E}'$  and  $\mathbf{B}'$  experienced by the particle in S'.

22. [10] Show that E' and B' are generated by the current and charge density on the plates in S'.

23. [5] Find the (Lorentz) force  $\mathbf{F'}$  on the particle in S'.

24. [5] Why does  $\mathbf{F}'$  differ from  $\mathbf{F}$ ?