

Electrodynamics, Physics 323
Spring 2005
Final exam
 Instructor: David Cobden

8.20 am, Tuesday June 8, 2005

You have 120 minutes. End on the buzzer at 10.20. Answer all questions.

You are strongly recommended to read through all the questions before you begin.

Write your name on every page and your ID on the first page.

Watch the blackboard for corrections or clarifications during the exam.

In this exam you are allowed **no books, no note sheets, and no calculators!**

Write all your working on these question sheets. You will get credit for it. It is important to show your calculation or derivation, and where appropriate to write a few words to indicate the reasoning. You usually won't get full marks just for stating the correct answer.

Do not turn this page until the buzzer goes.

$$\Lambda^\mu_\nu = \begin{pmatrix} \gamma & -\gamma v/c & * & * \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{pmatrix} \quad F^{\mu\nu} = \begin{pmatrix} * & E_x/c & E_y/c & E_z/c \\ * & * & B_z & -B_y \\ * & * & * & B_x \\ * & * & * & * \end{pmatrix}$$

$c = 3 \times 10^8 \text{ ms}^{-1}$ (hope you don't need to be reminded of that.)



(possible bonus mark: spot the errors)

Warm up:

1. [10] Write down the continuity equation for charge (a) in normal form and (b) in manifestly Lorentz-covariant form.

2. [10] Write down the differential relationships between the scalar and vector potentials and the charge density and current density, (a) in normal form (2 equations) and (b) in covariant form (one equation).

I. Radiation. The power radiated by a changing charge distribution is approximately $P(t) = \frac{\mu_0 \ddot{p}(t)^2}{6\pi c}$, where $p(t)$ is the instantaneous net electric dipole moment at time t .

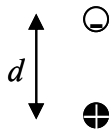
3. [6] Derive from this a formula for the average power radiated by an oscillating electric dipole with angular frequency ω .

4. [12] Hence, with the help of a diagram, explain why the daytime sky is blue (allegedly).

5. [10] Estimate the typical size of a simple broadcasting antenna for 100 MHz radio waves.

6. [10] An oscillating dipole consists of two oppositely charged particles which oscillate in their relative coordinate at angular frequency ω as indicated below. Sketch below the electric field patterns at the three different phases shown if the amplitude d of the oscillation is similar to c/ω , indicating how the pattern evolves towards the spherical dipole radiation pattern at larger distances.

$t = 0$



$t = \pi/2\omega$



$t = \pi/\omega$



7. [6] A solid sphere spinning about an axis through its center has a net charge Q spread evenly over its surface. Does it radiate electromagnetic energy? If so, what kind(s) of radiation, and which is dominant? (ie, magnetic dipole, electric quadrupole, etc?) If not, why not?

8. [6] Answer the same question for the case that the spin axis is offset from the center of the sphere but is close to it.

II. Relativity concepts. The ‘worldline’ of a particle of charge q , rest mass m_0 is specified by $x^\mu(\tau)$.

9. [5] Define the 4-momentum p^μ of the particle in terms of the parameters given.

10. [5] What is the definition and meaning of the proper time τ ?

11. [5] Write x^μ in component form in terms of the trajectory of the particle $\mathbf{w}(t)$ observed in a frame S.

12. [15] If in frame S the 4-momentum has components $p^\mu = (E/c, \mathbf{p})$, show that $E = \gamma m_0 c^2$ and that $E^2 = p^2 c^2 + m_0^2 c^4$.

13. [8] An event X occurs at the origin at $t = 0$ in frame S , at which time the particle is *not* at the origin. Sketch the worldline of the particle on a space-time diagram in frame S . Leave room for answers to Q15.

14. [4] Define the invariant interval between a point on the trajectory and X .

15. [8] With reference to the invariant interval, indicate on the above diagram which parts of the trajectory are timelike and which are spacelike with respect to X , and say what the consequences are for causality and simultaneity.

III. Transforming the potentials. Let us find the potentials of a charge in uniform motion, by considering a charge which is stationary in inertial frame S and moving at $\mathbf{v} = v\hat{\mathbf{x}}$ in frame S' .

16. [6] Show that $\frac{\partial}{\partial x^\mu} A^\mu = 0$ written in component form specifies the Lorentz gauge.

17. [8] Write down the components of A^μ for a charge q which is stationary at the origin in frame S .

18. [12] Apply the Lorentz transformation to find the components A'^{μ} of the 4-potential in a frame S' . At this point your answer should be in terms of the coordinates in S .

19. [12] Now find the coordinates (x, y, z, t) in frame S in terms of the coordinates (x', y', z', t') in S' .

20. [12] Hence deduce the scalar potential $V(x', y', z', t')$ and vector potential $\mathbf{A}'(x', y', z', t')$ of a charge moving at constant velocity $\mathbf{v} = v\hat{\mathbf{x}}$ in frame S' .

III. Transforming the fields. An infinite line charge is at rest with charge density λ along the x -axis in inertial frame S .

21. [8] Find the electric and magnetic fields in S , or just state them.

22. [10] Construct the electromagnetic field tensor $F^{\mu\nu}$ in Cartesian coordinates in frame S .

23. [12] In another frame S' the line charge moves at constant velocity $\mathbf{v} = v\hat{\mathbf{x}}$ along the x -axis. Find the electric field (using Gauss) and the magnetic field (using Ampere) in S' . Don't forget to take account of length contraction.

24. [10] Construct the field tensor $F^{\mu\nu}$ in frame S' .

25. [10] Show that the B_y' component of $F^{\mu\nu}$ is related by the appropriate Lorentz transformation to the components of $F^{\mu\nu}$.